Acoustic Tomography: Promise versus Reality

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Abstract—**Imaging with acoustic waves has made great advances in recent decades. In opposing limits of wavelength, acoustics have played a major role in geophysical applications on the one hand and in medical ultrasound imaging on the other. In contrast to X-rays, acoustic waves interact strongly with materials through which they propagate, through processes such as refraction, reflection and diffraction. The interactions can be very strong in heterogeneous media such as human tissue. Tomographic reconstructions of acoustic data therefore require much more sophisticated modeling of acoustic wave propagation often involving highly non-linear inversions. These factors have impeded progress in this otherwise promising methodology.**

The advancement of computing power and the rise of high-throughput data acquisition hardware have made acoustic tomography (AT) feasible in recent years. The objective of this paper is to relate these developments to practical applications of AT, particularly in the area of medical imaging.

Today, a number of laboratory groups are collecting data with AT prototypes and some projects have become commercial ventures. This paper reviews the status of AT imaging, particularly in the area of breast cancer detection, where some of the most recent advances have taken place. It is shown that parallel developments in AT methodologies have given rise to exciting new possibilities for acoustic tomography, *at all wavelengths***, with potential applications in areas as diverse as seismic exploration, non-destructive testing and cancer detection.**

Index Terms—**acoustic tomography, ultrasound, non-destructive testing, medical imaging.**

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I. INTRODUCTION

Acoustic tomography (AT) is a technique that uses computed tomography (CT) methods to solve an inverse problem involving sound signals. It is well suited for inferring acoustic properties of a volume of material from measurements made along a surface surrounding the material. Applications can be found in seismology, process flow, industrial non-destructive testing (NDT) and, increasingly in medical imaging and even in meteorology.

Unlike the X-rays used in conventional CT, sound is purely wave-like and tomographic techniques that process sound data must, therefore, take into account wave propagation phenomena such as reflection, refraction and even diffraction. In an inhomogeneous medium, ultrasound pulses do not travel in straight lines, thereby complicating the tomographic inversion and placing extra burden on the computational requirements. The need for a high level of computing power and associated data processing has been a major historical factor in limiting the development of AT compared to CT and other tomographic methods. For this reason, AT has largely not met its promise. However, in recent years, thanks to increasing processing power of both computers and electronics, the landscape has changed dramatically thanks to the exponential increase of computing power which has largely followed Moore's Law.

Over the past 30 to 40 years, computing power has increased by a factor of 10 million. This development has enabled sophisticated physics-based inversion algorithms such as waveform tomography. At the same time, the processing power of electronics has also increased about 10 million fold leading to massive parallelization of data acquisition and the ability to process large amounts of data. These two parallel trends have enabled the development of tomographic systems containing large numbers of sensors. In the area of medical imaging, for example, large transducer arrays are bringing about the realization of ultrasound tomography (UST) systems that are gaining clinical applications.

AT is riding Moore's law into relevancy and this review will attempt to capture the AT concept, current status and future status as a viable imaging technology.

II. IMAGE RECONSTRUCTION METHODS

In an inverse problem one wishes to find m such that

$$d = G(m)$$

where G is an operator describing the explicit relationship between the data d and the model parameters m, and is a representation of the physical system. The operator G is often called the forward operator.

In most practical applications, the object can be confined either to a 2-D plane or a 3-D volume while the measurements are made in 1-D or 2-D space respectively. In the case of AT, a typical setup would involve surrounding the object with either an array of transducer elements or rotating a transducer around the object to both probe the object with sound waves and to measure the resulting interaction between the sound waves and the object. The measurements would then be recorded and used to construct an image tomographically.

Generally, QT sensors consist of sound or ultrasound transducers that are highly efficient at converting electrical energy into mechanical energy and vice versa. Often, the transducers can be used either as receivers or emitters and the same transducer can be switched between the two modes.

A transducer can be mechanically rotated around the object to provide many points of insonofication and measurement or a transducer consisting of an array of elements can be placed around the object or over some portion of the object. An array of transducers is generally more expensive to build and field but it offers the possibility of either electronic multiplexing or a parallelized system that provides data channels for many or all elements, thereby greatly accelerating the data acquisition process. There is therefore a trade-off between the cost and speed associated with any AT implementation. The exact amount of trade-off is governed by the application and in the case of clinical imaging favors the high speed implementations as described later.

A. *Solving the inverse problem*

The quality of the reconstructed image depends on the quality of the signals acquired by the AT system and by the sophistication of the reconstruction algorithm. The latter is defined by how well the physics of the sound propagation are modeled. Generally, the simpler the wave-based assumptions the faster an algorithm can run but the lower the quality of the final image. Therefore, a trade-off exists between reconstruction speeds and image quality. This trade-off can be understood by discussing the wave propagation theory and its computational implementations as summarized below.

Sound propagates according to the acoustic wave equation as:

$$\nabla^2 p - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = 0$$

where ∇^2 is the Laplace operator, p is the acoustic pressure (the local deviation from the ambient pressure), and where c is the speed of sound. The latter can also be expressed as:

$$c^2 = \frac{K}{\rho},$$

where ρ is the material density and K is the compressibility constant. The solution for a spherical wave in a homogenous medium is given by:

$$p(r,k) = \frac{A}{r}e^{\pm ikr},$$

where k is the wave number and A is the amplitude of the wave which falls off with radial distance traveled, r. It is evident that the propagation of the acoustic wave is sensitive to the material properties of density and stiffness. Therefore, for a heterogeneous medium, the solution is much more complex and requires algorithmic computations. Furthermore, both transverse and longitudinal waves are supported in any real system. However, most AT implementations rely on measurements of the longitudinal wave since it propagates much more rapidly and decays relatively slowly. The longitudinal waves can also undergo mode conversions creating surface waves, such as those referred to as the "whispering gallery". A full solution to the wave equation is therefore computationally daunting given the complexity of the physics being described. Most reconstruction methods therefore make simplifying assumption to make the problem tractable.

B. *Ray tomography*

For finite bandwidth sound waves used in acoustic imaging, energy travels from transmitter to receiver along a hollow banana shaped volume which can be represented as a "Banana – Donut" [1]-[7]. The center width of the "Banana - Donut" for dominant the frequency is the width of the first Fresnel zone, $\sqrt{\lambda L}$, where λ is the wavelength and L is the distance between transmitter and receiver. In ray theory, this volume is collapsed into an infinitesimal line (ray path) by assuming the infinite frequency approximation, similar to what is assumed for geometrical optics. The straight ray approximation is similar to the assumption made for CT reconstructions which assume X-rays travel in straight lines. Thus, every transmitter is connected to every receiver by a straight line. The spatial resolution of the reconstructed images is poor because the straight ray approximation does not take into account the refraction of the waves as they pass through an inhomogeneous medium. This blurring can be reduced by taking refraction into account when reconstructing the images by allowing for rays to bend as they propagate from transmitter to receiver.

Bent ray tomography relies on the knowledge that refraction is governed by changes in sound speed. The initial model of sound speed can be homogeneous or heterogeneous. The model is used to bend the rays as they propagate from one pixel to the next. The traced ray path and predicted arrival times are used to generate the next sound speed model which allows for more accurate bending. The process is repeated until convergence is achieved. The net effect of bending the rays is to compensate for the refractive effects and thereby reduce artifacts and improve the spatial resolution. More studies on bent-ray tomography can be found in references [8]-[17].

Ray-based Transmission algorithms
A typical transmission algorithm has 3 components:

Data processing. Before performing ray-based sound speed tomography on the acquired acoustic data, the time-of-flight (TOF) for each received waveform needs to be picked. In other words, the onset time of the signal arriving at receiver needs to be determined. The picked TOFs are used to reconstruct sound speed images. To determine the TOF for each waveform, either manual picking or some forms of automatic picking can be exploited. Automatic pickers are required when large volumes of data are collected, as in medical imaging [18]-[27].

Before performing ray-based attenuation tomography on the acquired data, the attenuation data for each received waveform needs to be calculated. Various authors [28]-[33] present different ways to determine attenuation data from the received waveforms.

Forward model. In straight ray tomography, the propagation paths of sound waves are assumed to be straight lines. In bent ray tomography, 2-D wave propagation is governed by the *eikonal equation*. The eikonal equation can be obtained from the wave equation in the limit of infinite frequency.

To solve the inverse problem, a regular rectangular grid model is created on the image plane, whose boundaries enclose the acquisition geometry. The Eikonal equation is solved to obtain a travel-time map for each source (transmitter) position which is later used to calculate travel-time gradients for ray tracing. A ray is back-propagated from receiver to transmitter based on either a straight ray path (straight ray tomography) or a travel-time gradient method (bent ray tomography) [6]. The traced ray paths serve as a sensitivity matrix in the inverse process.

Inversion. This is a linear problem for straight ray tomography since the matrix is fixed through the whole inverse process. The problem becomes nonlinear when we take the ray bending into consideration, in which case the matrix depends on the current sound-speed model.

Starting with a homogeneous sound-speed model, the optimization is performed iteratively. For bent ray tomography, ray paths are traced on the updated sound speed model after each iteration. The iteration continues until the solution converges. There are excellent discussions in [34]-[36] on convergence rate and stopping criteria for the iterative inverse process. A simple stopping criterion is that the cost function for the current iteration is not significantly improved from the previous iteration.

Diffraction Tomography

As noted above, waveform tomography is computationally intensive while ray tomography is fast but provides inferior spatial resolution. Investigators have sought simplified forms of the wave equation to reduce the computational burden while avoiding the ray approximation. The most common simplifying assumptions are known as first Born approximation and first Rytov approximation [37]-[43].

First Born approximation: In this approach, the wave equation is simplified by assuming that scattering is weak and there is no multiple scattering as the wave propagates from the transmitter to the receiver. The first Born approximation assumes the heterogeneity in the propagating medium perturbs the total wavefield. It consists of taking the incident wavefield in place of the total wavefield as the driving wavefield at each scatterer. This approximation is accurate enough if the scattered wavefield is small, compared to the incident wavefield. It breaks down if the scattered wavefield becomes large relative to the reference wavefield. Consequently, this method achieves high resolution but fails to properly reconstruct images with more than a few percent contrast differences. In most clinical applications, it is tantamount to assuming that the object being imaged can be inhomogeneous but with very small contrast variations.

Distorted Born Method: The distorted Born method is a high order Born approximation. It computes iterative solutions to nonlinear inverse scattering problems through successive linear approximations. By decomposing the scattered field into a superposition of scattering by an inhomogeneous background and by a material perturbation, large or high-contrast variations in medium properties can be imaged through iterations that are each subject to the distorted Born approximation. However, the repeated numerical computation of forward solutions (Green's function) imposes a very heavy computational burden, which limits its real-world application.

First Rytov approximation: The first Rytov approximation starts by assuming the heterogeneity in the medium perturbs the phase of the scattered wavefield. This approximation is valid under a less restrictive set of conditions than the first Born approximation [37],[38]. The validity of first Rytov approximation is governed by the change in scattered phase over one wavelength not the total phase. In other words, first Rytov approximation is valid when the phase change over a single wavelength is small (a few percent). Unfortunately, most applications violate this assumption.

Hybrid Method (Wavepath TOF tomography): Diffraction tomography has usually been presented in the Fourier domain, for a single frequency source [40]-[42]. Woodward formulated diffraction tomography as a multi-frequency inverse problem in the space domain, and proposed the wavepath concept for wave-equation tomography to account for the finite-frequency effects[1]. For band-limited wave propagation through a non-dispersive medium, the phase shifts experienced by each frequency are equivalent. For the Rytov approximation which naturally separates the amplitude (real part) from the phase (imaginary part) of wave, this means that the imaginary part of the Rytov wavepaths can be summed over all frequencies without loss of information. This summation over frequencies yields a narrow wavepath resembling a banana-donut like volume that runs from a source to a receiver. The TOF perturbation of a sound wave is linearly related to its phase shift that is closely related to the medium sound speed, This linear relationship provides a natural way to reconstruct the medium sound speed by applying the wavepath TOF tomography (WTFT) technique. WTFT has been investigated in Geophysics but, to the best of our knowledge, not yet in medical imaging.

Reflection Tomography

In contrast to the use of transmitted signals, reflection tomography relies on the reflected echoes to construct images of relative echo amplitudes. Since most AT data do not generally employ beam-forming on the front end, the Kirchoff migration technique does so "after the fact", in other words, after all the data have been gathered. Consequently, there is a great deal of flexibility on how the data are used. In particular, it is possible to use apertures of arbitrary size to reconstruct a reflection image. Unlike B-mode medical imaging this method allows receive aperture sizes that are limited only by the overall locus of the receiving transducer elements. Finally, since the Kirchhoff Migration method operates on raw data there is more flexibility in terms of how the signals are processed before and after the migration process is initiated. Fig 7 shows a reflection image of a clinical phantom, made in this way.

Generally, migration methods rely on the assumptions that every point in the object is a scattering object, independent of each other and that only one scatter occurs on a path that connects that point to a emitter-receiver pair. Consequently, traditional migration methods do not take into account diffraction and other wave properties. Recent developments in wave-based migration methods promise to overcome these barriers [44]. Some migration methods have been adapted to correct for refraction and for attenuation. Using the reconstructed data sets of other modalities, sound speed and attenuation, allows for proper time delays to be calculated for signal alignment and variable signal amplification to correct for inhomogeneous energy loss. The resulting effects to the image include sharper boundaries, higher contrast, less background noise, and, in some cases, the resolution of objects otherwise lost due to constant media assumptions. The ease of implementing these corrections is unique to the data acquisition method and the geometry of the problem.

C. Waveform tomography

With ever-increasing computational power the ability to solve the wave equation is being realized. Solutions are now possible, for both sound speed and attenuation [45]. The advantage of this approach in light of the computational burden is that it allows for diffraction as well as a better correction for refractive effects. Furthermore, by utilizing all of the recorded wave information (as opposed to the arrival time of the signal) the method has the potential to increase image contrast while suppressing artifacts. The limiting resolution of $\lambda/2$ is up to an order of magnitude better than ray tomography. Fig 1 shows a waveform reconstruction of a numerical model of a geophysical field.. We can observe that the quality of the reconstruction is significantly enhanced compared to ray-based reconstructions. In particular, the heterogeneities in the image are well resolved and have sharp boundaries.

Waveform tomography reconstruction methods have been formulated in the time domain [46],[47] and in the frequency domain [48],[49]. The latter usually allows for a simpler formulation of the problem since convolution and differential operators are mapped to multiplications. The reconstruction process is similar to ray tomography. We start from an initial model of the unknown parameters (sound speed, attenuation) and solve a forward problem. The solution of this forward problem is a set of simulated waveforms recorded at the transducer locations. The residual between the recorded waveforms and the measured ones is then used to update iteratively the unknown parameters until convergence.

Forward modeling is usually achieved by means of finite difference or finite element methods. These methods must be accurate enough to avoid numerical dispersion and to properly account for the boundaries of the simulation area (e.g., absorbing boundary conditions) [50],[51]. For a given accuracy, the size of the model typically scales linearly with the frequency of the probing pulse. Complexity can be lowered using approximations. It can also be addressed by means of efficient parallel implementations [52],[53]. However, this issue remains a challenging one, especially in medical imaging applications where reconstruction time must be kept at a minimum to keep a high patient throughput.

Convergence to the correct cycle of the waveform requires an accurate initial model, especially at high frequencies. One approach is to start from an initial model obtained using ray tomography, and to sequentially drive the iterative algorithm using waveform components from low to high frequencies.

III. LABORATORY AND COMMERCIAL SYSTEMS

Today, there are a number of tomographic systems performing imaging in a wide variety of applications. Examples of real-world applications of AT are listed in TABLE I.

TABLE I

Area of application	Imaging Targets	Transducer Frequency
Civil Infrastructure	Flaws	1 – 500 KHz
Industrial NDT	Flaws	0.5 – 150 MHz
Forestry	Tree decay	10-100 KHz
Oceanography	Ocean monitoring	100 – 200 KHz
Meteorology	Temperature and wind velocity	40 KHz
Agriculture	Spoilage / insects	0.5 - 3 KHz
Process	Air/liquid flows	1-3 MHz
Wildlife	Vocalizing animals	2 Hz – 30 KHz
Geophysical	Oil /ore deposits	1 Hz – 2 KHz
Medical	Tissues /lesions	1 – 5 MHz

The table lists the applications, along with the range of typical operating frequencies. The AT systems used in these areas have generated real-world laboratory data and in some cases the systems have been commercialized. Examples of some systems and their imaging results are provided below.

A. Seismic Imaging

In Fig 1 we show the application of both ray-based and waveform tomography of a lithospheric simulation. Using acoustic signals that are emitted and subsequently refracted from one set of sensors to another, it is possible to perform transmission tomography. As the figure indicates, the waveform tomography provides much richer detail compared to traditional ray-based imaging.
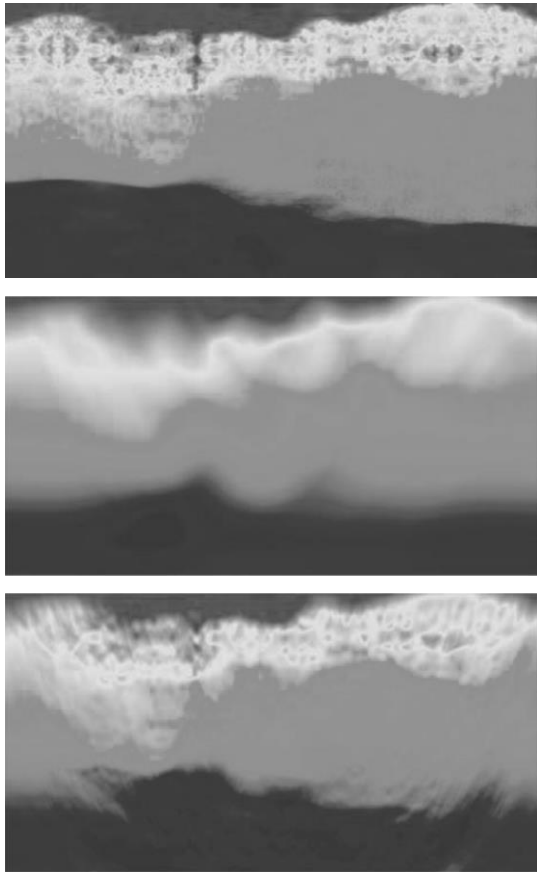

Fig 2. An array of sensors is used to perform transmission tomography for imaging the inside of a tree trunk. This commercial product is branded as PiCUS Tomography, Argus Electronic, Rostock, Germany.

Fig 1. Top: The numerical model represents a section of continental lithosphere 250 km in length and 40 km in depth. Middle: The ray-based reconstruction shows a relatively smooth reconstruction. Bottom: The waveform tomography reconstruction shows much finer details. A. Brenders, R. Pratt. Geophy. J Int. Vol 168, p. 133–151, (2007).
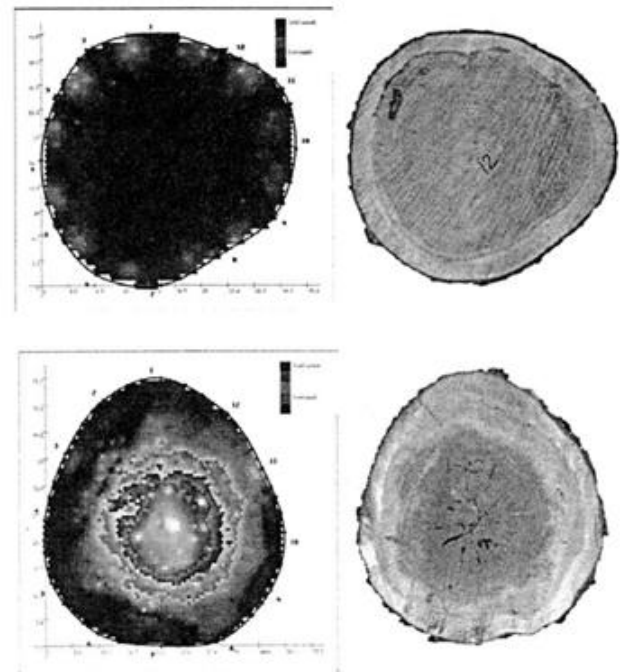

Fig 3. Sound speed reconstructions using the sensor array shown in Fig 2. A sound speed image and its corresponding optical image of a healthy tree cross-section are shown at the top. A decaying tree is shown at the bottom. From Liang, Wang, Wiedenbeck, Cai,Fu. "Evaluation of Acoustic Tomography for Tree Decay Detection". Proceedings of the 15th International Symposium on Nondestructive Testing of Wood, September 10-12, 2007, Duluth, Minnesota. Madison : Forest Products Society, 2008., p. 49-54.

B. Forestry

Acoustic tomography is increasingly used in forestry to measure properties of individual trees (Fig 2). Fig 3 shows cross-sectional sound speed reconstructions of a cherry tree trunk. The presence of rot is clearly indicated by the large differences in sound speed relative to a healthy tree.

C. Atmospheric imaging

Low frequency acoustic arrays are used to image temperature distributions in the atmosphere as well as wind direction. An example of a laboratory prototype, utilizing 40 KHz transducers,   is shown in Fig 4. Images produced from prototype data are shown in Fig 5.
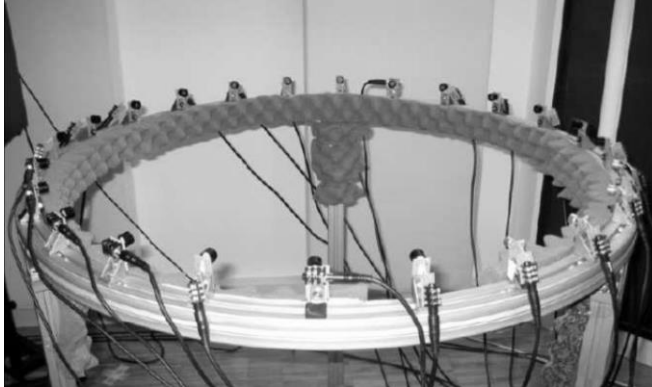

Fig 4. An   array of air coupled transducers in a laboratory setup. I. Jovanovic, "Inverse Problems in Acoustic Tomography: Theory and Applications", PhD Thesis, 2008.
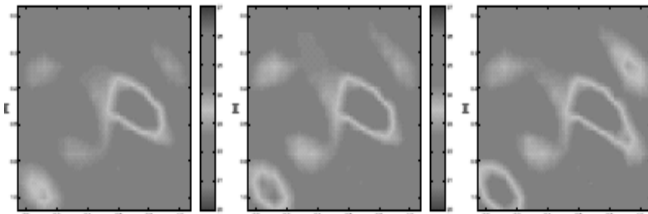

Fig 5. Reconstructions of three repeated air temperature measurements around a heat source using the setup from Fig 4. I. Jovanovic (2008).

D. Medical Imaging

The idea of solving acoustic inverse problems in medicine can be traced back to the work of Wilde and Reid [54] and Howry and Bliss [55] in the 1950's. At that time the systems used were crude mechanical scanners utilizing a single transducer that rotates on an arm and collects reflected signals using the pulse-echo technique. The first cross-sectional breast tissue images were made at that time.

However, the lack of computational power, combined with the slow rotation made it impossible to apply this technique clinically. These early methods did give birth to what is now known as B-mode clinical ultrasound. However, the tomographic aspect had to wait almost 30 years before the concept of UST was seriously re-visited.

A number of investigators developed operator-independent ultrasound scanners, based on the principles of ultrasound tomography, in an attempt to peform in-vivo scans [56]-[61]. Clinical examples include the work of Carson et al (U. Michigan),[56] Andre et al (UCSD),[57] Johnson et al (TechniScan Medical Systems),[58] Marmarelis et al (USC),[59] Liu and Waag (U. Rochester),[60] and Duric and Littrup et al (KCI)[61],[62]. More recently, Ruiter et al.[63] have reported progress on a true 3-D scanner utilizing a

hemispherical array of transducers. Although no clinical results have been reported with this system to date, clinical studies are currently being planned [63]. The clinical systems developed by these groups employed similar patient positioning systems. Patients were positioned in the prone position on a flat table with the breast suspended through a hole in the table in a water bath lying just below the table surface. The water batch is a requirement that ensures minimal distortion of the breast while allowing strong coupling of acoustic waves to the tissue.

Fig 6. An ultrasound tomography scanner located at the Karmanos Cancer Institute in Detroit, MU, USA.

In our laboratories, at the Karmanos Cancer Institute (KCI), our group has also focused on the development of ultrasound tomography for breast imaging. To that end we have been developing and testing a clinical prototype in KCI's breast center (Fig 6). The continuing development of the prototype and its associated UST methodology have been guided by clinical feedback from these studies and have led to continuing evolution in imaging performance leading to increasingly greater clinical relevance. This water bath system utilizes a solid state ring array transducer consisting of 256 elements that encircle the breast. It uses a 256-channel data acquisition system that allows single slice acquisitions in about 30 ms leading to whole breast scans of 1 minute or less. Furthermore, it utilizes bent-ray reconstructions for imaging. Although such images have lower spatial resolution compared to wave based approaches they can be run fast, in keeping
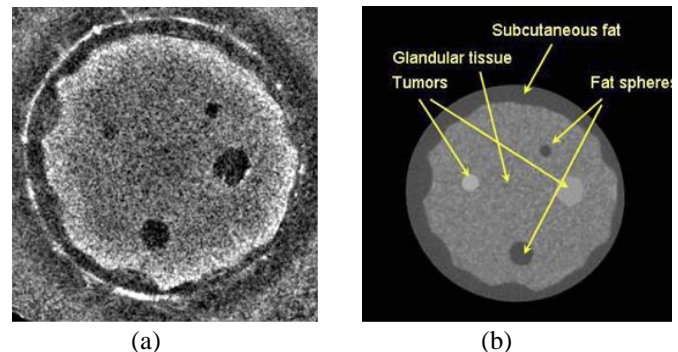

(a)   (b)

Fig 7. A reflection image of an anthropomorphic breast phantom constructed using reflection tomography (left). The truth image (right) is represented by a CT scan of an anthropomorphic breast phantom.

with the goal of a clinically fast system. A sample image constructed from data gathered with the prototype is shown in Fig 7.

The clinical prototype is currently being upgraded into a commercial system, named SoftVue, through the start-up company Delphinus Medical Technologies. The new system will have 1024 active elements and utilize a 512-channel data acquisition system.

As UST has matured its clinical relevance has begun to be tested on the clinical stage. In recent years an increasing number of studies have tested the technology under real-world clinical conditions. Techniscan Inc now has two active studies, one in Freiberg, Germany, the other at the University of California at San Diego. These studies have recruited more than 100 patients. For the past 6 years, our team at the Karmanos Cancer Institute in Detroit has undertaken multiple studies in support of scanner development. To date, more than 600 patient scans have been completed. A spin-off company from this project, Delphinus Medical technologies, is currently assembling a new generation of scanners which will also be used in international multi-center trials. The outcome of these multi-center trials will provide the first definitive assessment of UST efficacy by comparing its performance against mammography and MRI.

Breast Cancer Imaging

Clinical imaging with our UST prototype has been carried out with previously described tomographic reconstruction algorithms: (i) sound speed, (ii) attenuation and (iii) reflection [16],[27],[33]. Sound speed images are based on the arrival times of acoustic signals. Previous studies have shown that cancerous tumors have enhanced sound speed relative to normal breast tissue, [64]-[67] a characteristic which can aid the differentiation of masses, normal tissue, and fat. Attenuation images are tomographic reconstructions based on
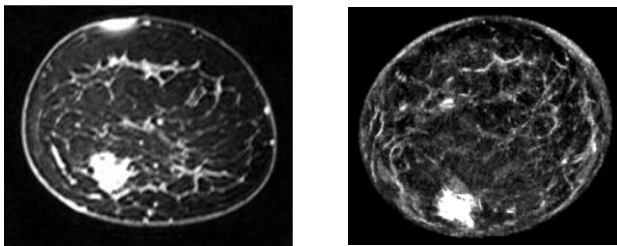

Fig 8. Left. MRI cross-sectional image showing a cancer at 7 o'clock. Right. A fused UST image (transmission sound speed image overlaid on a reflection tomography image) showing the same region. The UST images were reconstructed using bent-ray tomography.

acoustic wave amplitude changes. Higher attenuation in cancer causes greater scatter of the ultrasound (US) wave, so attenuation data in conjunction with sound speed provides a potentially effective means for determining malignancy as illustrated visually in Fig 8. The mass at 7 o'clock appears distinct in both the UST and MRI Images, indicating reliable visualization of breast lesions from UST data. .

IV. CONCLUSIONS AND FUTURE WORK

The concept of AT has been around for about half a century. However, its practical applications are only now beginning to emerge. Propelled by advances in transducer technology, data acquisition electronics and computing power, AT is now an active area of research with multiple groups and companies building imaging scanners for both research and commercialization purposes. Recent clinical studies have shown that UST is capable of imaging breast architecture and characterizing lesions. UST images generated in recent years appear at least superficially to be similar to MR images.

Finally, as computing power grows further still and as the price of electronics continues to decline, it may be possible to realize 3-D AT capable of routine volumetric imaging under a multitude of applications. Such a goal is challenging indeed, not only from a data processing perspective but also from the daunting physics required to model wave propagation in highly heterogeneous media. If history is any guide though, the evolution of both electronics and computing power will enable non-destructive volumetric evaluations with applications as diverse as cancer detection, tree decay mapping, bridge safety assessments and atmospheric characterization.

REFERENCES

[1]   Woodward, MJ, "Wave-equation tomography," Geophysics, vol. 5 (1992) 15-26.

[2]   Hung, SH, Dahlen, FA and Nolet, G, "Frechet kernels for finite-frequency traveltimes—II. Examples," Geophys. J. Int., 141 (2000) 175-203.

[3]   Spetzler, J and Snieder, R, "The effect of small-scale heterogeneity on the arrival time of waves," Geophys. J. Int., 145 (2001) 786-796.

[4]   Monetlli, R, Nolet, G, Masters, G, Dahlen, FA and Hung, SH, "Global P and PP traveltime tomography: rays versus waves," Geophys. J. Int., 158 (2004) 637-654.

[5]   Spetzler, J and Snieder, R, "Tutorial: The Fresnel volume and transmitted wave," Geophysics, 69 (2004) 653-663.

[6]   Snieder, R and Lomax, A, "Wavefield smoothing and the effect of rough velocity perturbations on arrival times and amplitudes," Geophys. J. Int., 125 (1996) 796–812.

[7]   Spetzler, J and Snieder, R, "Tutorial: The Fresnel volume and transmitted wave," Geophysics, 69 (2004) 653-663.

[8]   Schomberg, H, "An improved approach to reconstructive ultrasound tomography," J. Phys. D: Appl. Phys 11 (1978) L181-L185.

[9]   Andersen, AH, "Ray linking for computed tomography by rebinning of projection data," J. Acoust. Soc. Am. 81 (1987) 1190-1192.

[10] Norton, SJ, "Computing Ray trajectories between two points: A solution to the ray-linking problem," Optical Society of America 4 (1987) 1919-1922.

[11] Andersen, AH, "A ray tracing approach to restoration and resolution enhancement in experimental ultrasound tomography," Ultrasound Imaging 12 (1990) 268-291.

[12] Littrup PJ, Duric N, Azevedo S, Chambers DH, Candy JV, Johnson S, Auner G, Rather J and Holsapple ET, "Computerized ultrasound risk evaluation (CURE) system: Development of combined transmission and reflection ultrasound with new reconstruction algorithms for breast imaging," Acoustical Imaging 28 (2001) 175-182.

[13] Littrup PJ, Duric N, Leach Jr. RR, Azevedo SG, Candy JV, Moore T, Chambers DH, Mast JE and Holsapple ET, "Characterizing tissue with acoustic parameters derived from ultrasound data," Proceedings of the SPIE: Medical Imaging 2002; San Diego, California; Feb. 23-28, 2002. Ultrasonic Imaging and Signal Processing – 4687-43.

[14] Duric N, Littrup PJ, Babkin A, Chambers D, Azevedo S, Kalinin A, Pevzner R, Tokarev M, Holsapple E, Rama O, and Duncan R, "Development of Ultrasound Tomography for Breast Imaging: Technical Assessment," Medical Physics 32 (2005) 1375–1386.

[15] Duric N, Littrup P, Poulo L, Babkin A, Pevzner R, Holsapple E, Rama O, Glide C, "Detection of breast cancer with ultrasound tomography: First results with the Computed Ultrasound Risk Evaluation (CURE) prototype," Medical Physics 34 (2007) 773-785.

[16] Li, C, Duric, N, "In vivo breast sound-speed imaging with ultrasound tomography," Ultrasound in Med. & Biol., 35(2009) 1615-1628.

[17] Hormati, A, Jovanovic, I, Roy, O, and Vetterli, M, "Robust ultrasound travel-time tomography using the bent ray model", in Proc. *SPIE Medical Imaging*, vol. 7629, Mar. 2010.

[18] Maeda, N, "A method for reading and checking phase times in autoprocessing system of seismic wave data," Zisin. Journal of Seismological Society of Japan 38 (1985) 365-379.

[19] Baer, M and Kradolfer, U, "An automatic phase picker for local and teleseismic events," Bull. Seism. Soc. Am. 77 (1987) 1437-1445.

[20] Ramananantoandro, R and Bernitsas, N, "A Computer Algorithm for Automatic Picking of Refraction First-arrival-time," Geoexploration 24 (1987) 147-151.

[21] Boschetti, F, Dentith, D and List, RD, "A fractal-based algorithm for detecting first-arrivals on seismic traces," Geophysics 61 (1996) 1095-1102.

[22] Molyneux, JB and Schmitt, DR, "First-break timing: Arrival onset times by direct correlation," Geophys. 64 (1999) 1492-1501.

[23] Sleeman, R and van Eck, T, "Robust automatic P-phase picking: an on-line implementation in the analysis of broadband seismogram recordings," Phys. Earth Planet. Interiors 113 (1999) pp. 265-275.

[24] Zhang, H, Thurber, C and Rowe, C, "Automatic P-wave Arrival Detection and Picking with Multiscale Wavelet Analysis for Single-Component Recordings," Bull. Seism. Soc. Am. 93 (2003) 1904-1912.

[25] Kurz, JH, Grosse, CU and Reinhardt, HW, "Strategies for reliable automatic onset time picking of acoustic emissions and of ultrasound signals in concrete," Ultrasonics 43 (2005) 538-546.

[26] Di Stefano, R, Aldersons, F, Kissling, E and Chiarabba C, "Automatic seismic phase picking and consistent observation error assessment: application to the Italian seismicity," Geophys. J. Int. 165 (2006) 121-134.

[27] Li, C, Huang, L, Duric, N, Zhang, H and Rowe, C, "An improved automatic time-of-flight picker for medical ultrasound tomography," Ultrasonics 49 (2009) 61–72.

[28] Kak, AC and Dines, KA, "Signal Processing of Broadband Pulsed Ultrasound: Measurement of Attenuation of Soft Biological Tissue," IEEE Transaction on Biomedical Engineering 25 (1978) 321-344.

[29] Parker, KJ, Lerner, RM and Waag, RC, "Attenuation of Ultrasound Magnitude and Frequency Dependence for Rissue Characterization," Radiology 153 (1984) 785-788.

[30] Parker, KJ, Lerner, RM and Waag, RC, "Comparision of Techniques for In Vivo Attenuation Measurements," IEEE Transactions on Biomedical Engineering 35 (1988) 1064-1068.

[31] Sams, M and Goldberg, D, "The validity of Q estimates from borehole data using spectral ratios," Geophysics 55 (1990) 97-101.

[32] Fujii, Y, Itoh, K, Shigeta, K, Wang, Y, Tsao, J, Kumasaki, K and Itoh, R, "A New Method for Attenuation Coefficient Measurement in the Liver," J. Ultrasound Med. 21 (2002) 783-788.

[33] Li, C, Duric, N and Huang, L, "Comparison of ultrasound attenuation tomography methods for breast imaging," Proc. of SPIE Vol. 6920 692015-2, San Diego, 2008.

[34] Paige, CC and Saunders, MA, "LSQR: Sparse linear equations and least-squares problems," ACM Trans. Math. Softw. 82 (1982) 43-71.

[35] Bissantz, N, Hohage, T, Munk, A and Ruymgaart, F, "Convergence rates of general regularization methods for statistical inverse problems and applications," SIAM J. Numer. Anal. 45 (2007) 2610–2636 (electronic).

[36] Loubes, JM and Rivoirard, V, "Review of rates of convergence and regularity conditions for inverse problems," http://www.ceremade.dauphine.fr/~rivoirar/IJAMAS_example.pdf.

[37] Chemov, LA, "Wave Propagation in a Random Medium," New York, NY:McGraw-Hill, 1960.

[38] Keller, JB, "Accuracy and validity of the Born and Rytov approximations," J. Opt.Sot. Amer., 59 (1969) 1003-1004.

[39] Devaney A, "Inverse scattering theory within the Rytov approximation," Opt. Lett. 6 (1981) 374-376.

[40] Devaney A, "Inverse formula for inverse scattering within the Born approximation," Opt Lett. 7 (1982) 111-112.

[41] Slaney, M, Kak, AC and Larsen, L, "Limitations of imaging with first-order diffraction tomography," IEEE Trans. Microwave Theory and Techniques, MTT-32 (1984) 860-873.

[42] Wu, RS and Toksoz, MN, "Diffraction tomography and multisource holography applied to seismic imaging," Geophysics, 52 (1987) 11-25.

[43] Kak, AC and Slaney, M, "Principles of Computerized Tomographic Imaging," IEEE Press, 1988.

[44] Schmidt S., Duric N., Li C. Roy O., Huang Z.-F., Modification of Kirchhoff migration with variable sound speed and attenuation for acoustic imaging of media and application to tomographic imaging of the breast. Med. Phys. 38, 998 (2011)

[45] R. G. Pratt, L. Huang, N. Duric, and P. Littrup, "Sound-speed and attenuation imaging of breast tissue using waveform tomography of transmission ultrasound data", in Proc. SPIE Medical Imaging, vol, 6510, Mar. 2007.

[46] Tarantola, A., [Inverse Problem Theory and Methods for Model Parameter Estimation], SIAM (2005).

[47] Natterer, F. and Wubbeling, F., [Mathematical Methods in Image Reconstruction], Society for Industrial and Applied Mathematics, Philadelphia, PA, USA (2001).

[48] R. G. Pratt, "Seismic waveform inversion in the frequency domain, part 1: Theory, and verification in a physical scale model," Geophysics 64, pp. 888–901, May-June 1999.

[49] Pratt, R. G., "Seismic waveform inversion in the frequency domain, part 1: Theory and verification in a physical scale model," Geophysics 64(3), 888–901 (1999).

[50] Yang, D. H., Lu, M., Wu, R. S., and Peng, J. M., "An optimal nearly analytic discrete method for 2D acoustic and elastic wave equations," Bulletin of the Seismological Society of America 94(5), 1982–1992 (2004).

[51] Engquist, B. and Majda, A., "Absorbing boundary conditions for the numerical simulation of waves," Mathematics of Computations (139), 629–651 (1977).31

[52] O. Roy, I. Jovanovic, A. Hormati, R. Parhizkar, and M. Vetterli, "Sound Speed Estimation Using Wave-based Ultrasound Tomography: Theory and GPU Implementation", in Proc. SPIE Medical Imaging, vol. 7629, Mar. 2010.

[53] Micikevicius, P., "3D finite difference computation on GPUs using CUDA," in [2nd Workshop on General Purpose Processing on Graphics Processing Units], 79–84 (2009).

[54] Wild, JJ. and Reid, JM. (1952). "Application of echo-ranging techniques to the determination of structure of biological tissues.". *Science* **115** (2983): 226–30

[55] Howry, D.H. and Bliss, W.R. (1952) Ultrasonic visualization of soft tissue structures of the body. J. Lab. Clin. Med. 40:579-592

[56] Carson PL, Meyer CR, Scherzinger AL, Oughton TV. Breast imaging in coronal planes with simultaneous pulse echo and transmission ultrasound. Science 1981, Dec 4;214(4525):1141-3.

[57] Andre MP, Janee HS, Martin PJ, Otto GP, Spivey BA, Palmer DA, "High-speed data acquisition in a diffraction tomography system employing large-scale toroidal arrays," International Journal of Imaging Systems and Technology 1997;Vol. 8, Issue 1:137-147.

[58] Johnson SA, Borup DT, Wiskin JW, Natterer F, Wuebbling F, Zhang Y, Olsen C. Apparatus and Method for Imaging with Wavefields using Inverse Scattering Techniques. United States Patent 6,005,916 (1999).

[59] Marmarelis, V.Z., Kim, T., Shehada, R.E. Proceedings of the SPIE: Medical Imaging 2003; San Diego, California; Feb. 23-28, 2002. Ultrasonic Imaging and Signal Processing – Paper 5035-6.

[60] Liu, D.-L., and Waag, R. C. "Propagation and backpropagation for ultrasonic wavefront design," IEEE Trans. on Ultras. Ferro. and Freq. Contr. 44(1):1-13 (1997).

[61] Duric N, Littrup PJ, Babkin A, Chambers D, Azevedo S, Kalinin A, Pevzner R, Tokarev M, Holsapple E, Rama O, and Duncan R. Development of Ultrasound Tomography for Breast Imaging:

Technical Assessment. Medical Physics, Vol. 32, No. 5, pp. 1375–1386, May 2005.

[62] Duric N, Littrup PJ, Poulo L, et al. Detection of breast cancer with ultrasound tomography: First results with the Computed Ultrasound Risk Evaluation (UST) prototype. Med. Phys. 2007; 34: 773-785

[63] Nicole V. Ruiter, Georg Göbel, Lutz Berger, Michael Zapf and Hartmut Gemmeke, "Realization of an optimized 3D USCT", Proc. SPIE 7968, 796805 (2011)

[64] Duric N, Littrup P, Li C, Rama O, Bey-Knight L, Schmidt S, Lupinacci J, (2009) Detection and characterization of breast masses with ultrasound tomography: Clinical results. Proc. of SPIE: Medical Imaging 2009. Vol. 7265 72651G-1-8.

[65] N. Duric, P. Littrup, P. Chandiwala-Mody, C. Li, S. Schmidt, et al., "In-vivo imaging results with ultrasound tomography: Report on an ongoing study at the Karmanos Cancer Institute", Proceedings of SPIE Vol. 7629, 76290M (2010).

[66] Ranger B, Littrup P, Duric N, Li C, Lupinacci J, Myc L, Rama O, Bey-Knight L, (2009) Breast imaging with acoustic tomography: a comparative study with MRI. Proc. of SPIE: Medical Imaging 2009. Vol. 7265 726510-1-8

[67] B. Ranger, P. Littrup, N. Duric, C. Li, S. Schmidt, et al., "Breast imaging with ultrasound tomography: a comparative study with MRI", Proceedings of SPIE Vol. 7629, 76291C (2010).