Localization of Oceanic Fronts & Feature Boundaries Using a Variational Technique

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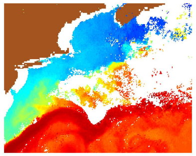
Abstract

Automatic localization of curvi-linear features (boundaries), including oceanic fronts and contours of rings such as those associated with the free-jet portion of the Gulf Stream, from satellite sea surface temperature (SST) maps is a challenging task, especially in the case of missing observations due to cloud cover. Having this as motivation, we explore whether techniques successfully used for non-oceanographic problems can be beneficial in the realm of oceanography. The goal is to apply a generalized version of the Mumford-Shah functional, a variational technique used in photographic and medical imaging applications, to an oceanographic problem. This method performs optimal smoothing jointly with localization of the feature boundaries. The feature boundary partitions the region into two or more subregions. Minimally, it establishes the location of the front. When there are rings, the boundary also separates the interior and exterior of these rings. As a by-product of our boundary localization, we estimate the field, which interpolates across areas of missing data, but maintains the discontinuity at the boundary. Optimal interpolation is commonly performed in data analysis and assimilation; however, the technique presented by us is distinctive in the sense that it incorporates information about the feature boundaries into the field estimation process. We start with an objective functional E(f,C) which we wish to minimize over both the SST field f and the boundary C, which need not be a single, connected curve. The functional has a data fidelity term, a field smoothness term, and a curve length penalty term. The curve is evolved using level set methods, which can handle changes in the topology of the boundary. This allows us to locate both rings and the front. The boundary is then identified by the discontinuity, or departure from smoothness, in the estimated field. We present experimental results of our technique on various satellite observations of SST data. Preliminary results show reasonable localization of a particular oceanic front and associated rings.

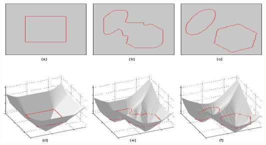
Problem Statement

- Locating oceanic fronts such as the Gulf Stream has been useful for seafarers, naval intelligence, and ecologists.
- Satellite data for sea surface temperature (SST) measurements using infrared sensors contains missing information.
  - In regions of cloud cover, the infrared sensors in the satellite record the cloud temperature rather than the SST. Thus, the temperature in these regions are effectively unobserved.
  - The swath of the satellite taking measurements does not encompass the entire earth's surface in one pass.
  - A sample image is shown here, where brown regions represent land and white regions indicate unobserved data.
- Techniques used in video processing and medical imaging to locate boundary features may apply to this oceanographic problem of locating an oceanic front in the presence of missing observations.



Background – Curve Evolution Methods

- Active contour (curve) evolution involves associating the boundary localization problem to minimizing a functional.
  - Thus, we want the functional to capture as much information as possible to achieve the desired segmentation.
  - We take the first variation of the functional to determine the direction of gradient descent.
- The contour evolution is performed using level set methods (Osher and Sethian, 1988).
  - The curve is represented as the zero level set of a surface.
  - The surface is evolved so that its zero level set moves according to the desired flow of the curve.
- Level sets are preferred over sampled contour points because the latter have the following drawbacks:
  - Nothing prevents the discretized points from moving together, requiring frequent resampling. (Sethian, 1985)
  - Stable evolution of points requires a small time-step.
  - Topological changes cannot be easily handled with points.
- The figure to the right illustrates how level set methods work.
  - Graphs (a), (b), and (c) show the curve represented by the zero level sets of the surfaces shown in (d), (e), and (f), resp.
  - We use signed distance functions for our level sets.
  - Note the topology change between (b) and (c).



Proposed Model

- The Mumford-Shah (1985) functional is a three-term functional which localizes a curve boundary and estimates the field. Mathematically,

$$E(f,\vec{C}) = \beta\int_{\Omega} h(x)(f(x)-g(x))^2\,dx + \alpha\int_{\Omega\setminus\vec{C}} |\nabla f(x)|^2\,dx + \gamma\oint_{\vec{C}} ds$$

Function of estimated field **f** and curve boundary **C**

g is the observed data

Ω is the region

h an indicator function which is 1 when data is present, 0 when there is no observation, and **β,α, γ** tunable parameters.

- This model implicitly assumes a piecewise constant field on either side of the curve. In sea surface temperature (SST) data, the temperature field is not constant. So, we modify the second term in this functional so that first and second order statistics of the region can be modeled. Namely,

$$E(f,\vec{C}) = \beta\int_{\Omega} h(x)(f(x)-g(x))^2\,dx + \alpha\int_{R_1}\int_{R_1}(f(y)-b_{R_1}(y))B_{R_1}^{-1}(y,x)(f(x)-b_{R_1}(x))\,dx\,dy$$

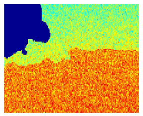
$$+\int_{R_2}\int_{R_2}(f(y)-b_{R_2}(y))B_{R_2}^{-1}(y,x)(f(x)-b_{R_2}(x))\,dx\,dy + \gamma\oint_{\vec{C}} ds$$

where $\mathbf{R_1}$ and $\mathbf{R_2}$ are the two regions separated by curve **C**, while **b** and **B** are the prior mean and covariance, respectively, of the regions.

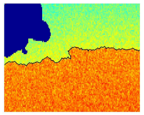
Results

Synthetic Examples

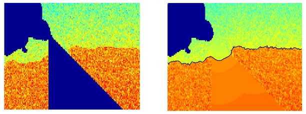
- We created a mean temperature field north and south of an arbitrarily generated curve according to the following temperature map (values were chosen to approximate actual observations of exerimental data):
  - North of the curve: T = 20 – 0.1*(distance from curve)
  - South of the curve: T = 25 + 0.02*(distance from curve)
- Additive white Gaussian noise was added to simulate noisy measurements. The figure to the right shows the synthetic data (blue region is a land mass).



- The figure (right) shows the curve localization as well as the field estimate. Note that the discontinuity was found (which is to be expected when there is no missing data and a reasonable SNR), while the field estimate is a smoothed, noise-removed version of the original.

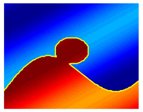


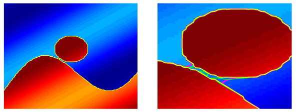
- Now, to illustrate the functional's impact on missing data, a slice was removed from original synthetic data and is shown below left. The resulting **f** and **C** are shown below right. Note that the prediction of the curve in the missing data is not a straight line (which would be the case if we did not rely on the prior estimate of the mean). Instead, the observations away from the curve impact the location of the curve itself. Also, the functional interpolates across the field in regions of missing data.



Ring Separation

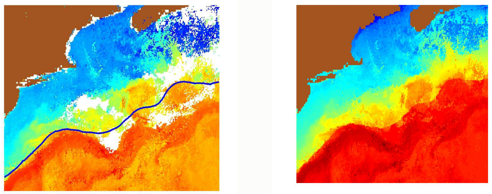
- A benefit of using level set methods is their ability to handle changes in topology. We illustrate how we can localize a warm core ring which pinches off from a front over time. The figure right shows an image prior to the warm core ring pinching off, with segmentation shown. Using this as initialization, we evolve the curve in the next frame to find both the front and the warm core ring (below left). A close up (below right) shows the new contour as well as three steps in the evolution of the front from a single initial connected curve (pink) to an intermediate curve (green) – note topology change – to the final segmentation (yellow).





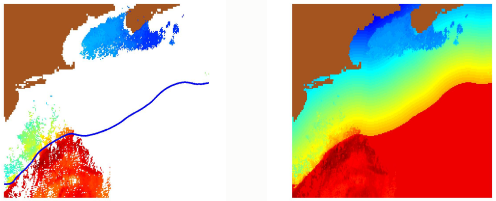
GOES Data – Little Missing Data

- Using an example from real satellite measurements (below left), we attempt to locate the discontinuity. The figure in the lower right shows the resulting field estimate, with the curve estimate shown overlaid in the original data image (below left). Note that along regions of strong gradient, the curve localizes reasonably well in the western part of the ocean. As the gradient becomes less apparent in the eastern part of the Gulf Stream, the curve seeks out the best estimate based on the available temperature measurements and the prior model.



GOES Data – Much Missing Data

- We also examine a case where most of the observations are missing (original data shown below left with the estimated location of curve front overlaid).
- In such a case, the prior mean guides the location of the curve front. The field estimate is shown below right.



Minimizing the Energy Functional

- Since we have two variables to solve in our functional E(f,C), we apply coordinate descent to minimize E (Tsai, 2000). So, each iteration is a two-step process.
- First, we fix the location of the curve and find the field **f** which minimizes the function given the curve location. The solution is written in discrete space and for the field on one side of the curve (**f** is computed on each side of the curve, with the prior error-covariance matrix **B** partitioned by the curve into $\mathbf{B_{R1}}$ and $\mathbf{B_{R2}}$ so that its influence is restricted to points on the same side only).

$$f_{R_i} = B_{R_i}H'(HB_{R_i}H' + \frac{\alpha}{\beta}I)^{-1}(g_{R_i} - Hb_{R_i}) + b_{R_i},\ i = 1,2$$

- Next, with **f** fixed, we move the curve in the direction that decreases **E** most rapidly. Creating an artificial time parameter **t** to represent the movement of **C** through iterations, we compute the first variation of **E** and find the direction that we should move the curve. [Curve evolution step]

$$\vec{C}_t = [\alpha((H_{R_1}f_{R_1} - g)^2 - (H_{R_2}f_{R_2} - g)^2) +$$

$$\beta((f_{R_1} - b_{R_1} + \Delta b)'B_{R_1}^{-1}(f_{R_1} - b_{R_1}) - (f_{R_2} - b_{R_2} + \Delta b)'B_{R_2}^{-1}(f_{R_2} - b_{R_2})) - \gamma\kappa]\vec{N}$$

where $\mathbf{C_t}$ is the change in **C** with respect to **t** and **Δb** is the change in mean temperature across the discontinuity.

- Using level set methods for this curve evolution, we evolve the surface at each iteration, with the curve front being the zero level set of the surface.

Overview of algorithm

I. Label missing data as unobserved (set h(x) = 0).

II. Initialize curve **C**

III. Coordinate descent

  A. Fix **C**, solve for **f** (closed-form solution shown above).

  B. Fix **f**, move incrementally in direction which maximally decreases the energy **E**.

  C. Repeat A and B until convergence of step B.

Conclusion

- Level set methods can be useful for this type of problem because they handle changes in the topology. We experience such a change when a warm or cold core ring develops from an oceanic front.
- The automatic localization may not outperform hand-segmentation, but it provides a reasonable estimate of the location of the oceanic front, while also providing a field estimate.
- The generalized Mumford-Shah functional allows us to incorporate prior knowledge of the first and second order field statistics for curve localization and field estimation, even if the observations are incomplete near the location of the front.

Credits

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