DICTIONARY LEARNING FOR SPARSITY-DRIVEN SAR IMAGE RECONSTRUCTION

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ABSTRACT

We consider the problem of synthetic aperture radar (SAR) image formation, where the underlying scene is to be reconstructed from undersampled observed data. Sparsity-based methods for SAR imaging have employed overcomplete dictionaries to represent the magnitude of the complex-valued field sparsely. Selection of an appropriate dictionary with respect to the features of the particular type of underlying scene plays an important role in these methods. In this paper, we develop a new reconstruction method that is based on learning sparsifying dictionaries and using such learned dictionaries in the reconstruction process. Adaptive dictionaries learned from data have the potential to represent the magnitude of complex-valued field more effectively and hence have the potential to widen the applicability of sparsity-based radar imaging. We demonstrate the performance of the proposed method on both synthetic and real SAR images.

***Index Terms*—** synthetic aperture radar (SAR), image reconstruction, dictionary learning, compressed sensing (CS), sparse representation.

1. INTRODUCTION

Synthetic aperture radar (SAR) images are used in a variety of applications ranging from target recognition to urban monitoring and land cover classification. Conventionally, SAR image reconstruction is performed using techniques such as the polar format algorithm [1, 2]. However, such conventional reconstruction techniques suffer from speckle, limited resolution, and sidelobe artifacts due to the limited bandwidth of SAR systems. Over the last decade, sparsity-driven methods have been developed for SAR imaging, leading to various improvements over conventional imaging especially in scenarios involving limited or low-quality data. For example, the feature-enhanced imaging framework of [3] imposes sparsity on the magnitudes of the complex-valued reflectivities or their gradients, leading to enhancement of spatially-localized point scatterers or piecewise smooth regions. This corresponds to solving an analysis-based sparse representation problem with complex-valued variables with sparsity constraints on the magnitudes. While this approach has been shown to produce high quality images in challenging scenarios, it applies sparsity on limited, fixed features. A synthesis-based approach for sparsity-driven SAR imaging has been developed in [4]. This approach is able to preserve multiple types of features appearing in different parts of the scene simultaneously. The phase and the magnitude of the SAR reflectivity are separated and a joint optimization problem is solved by optimizing one group of variables while keeping others constant. Experimental results demonstrate successful use of various dictionaries in this framework. However, this approach uses pre-determined, fixed dictionaries. The focus of our paper is to develop a framework that lets us replace such fixed dictionaries with dictionaries learned from data.

Dictionaries learned from training or test data have been used in a variety of sparse image restoration and reconstruction problems [5, 6]. One of the most widely used dictionary learning method is K-SVD [5] that jointly updates the dictionary and the sparse coefficients. This method has been used for many applications such as image denoising [7], color image restoration [8], and medical image reconstruction [9, 10].

While existing sparsity-based methods for SAR imaging have produced appealing results, we feel exploiting sparsity with demonstrable benefits in a wider diversity of SAR scenes in various applications requires the sparse representation dictionaries to be adapted to the particular context in a data-driven fashion. Motivated by this observation, in this paper we propose an approach that expands the idea of dictionary learning to the complex-valued SAR image formation problem. This is the main contribution of our paper. To the best of our knowledge, there exists no prior work that considers the complex-valued inverse problem for SAR imaging and uses a dictionary learning-based approach for sparse representation of reflectivity magnitudes. We propose a framework for dictionary learning and SAR image formation, in which dictionaries can be learned from training data in an offline manner, or from the test data in an online manner. For dictionary learning we use K-SVD. The image formation piece of our framework involves updates for the magnitude and the phase of the complex-valued reflectivities. Preliminary experimental results on synthetic and real scenes demonstrate the potential of the proposed approach.

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2. BACKGROUND

2.1. Sparsity-Driven SAR Image Reconstruction

The complex-valued and potentially random-phase nature of SAR reflectivities make the formulation of a sparse representation-based framework for solving the inverse problem of SAR image formation just a bit more challenging than inverse problems involving real-valued fields, such as those appearing several medical imaging applications. The recent work in [4] proposes a synthesis-based sparse representation framework for SAR imaging that involves solving the magnitude and the phase of the reflectivities separately. This approach paves the way for using overcomplete dictionaries to represent the magnitude of the reflectivity field sparsely. In particular, introducing the notation $f = \Theta\,|f|$, where Θ is a diagonal matrix containing the unknown phase of the reflectivity in exponentiated form and $|f|$ represents the magnitude of the reflectivity with an overcomplete dictionary Ψ such that $|f| = \Psi\alpha$,[4] poses the following joint optimization problem for SAR image formation:

$$\widehat{\alpha}, \widehat{\Theta} = \arg\min_{\alpha,\Theta} \|g - H\Theta\Psi\alpha\|_2^2 + \lambda\,\|\alpha\|_p^p \tag{1}$$

where α denotes the sparse coefficients and λ is a regularization parameter balancing data fidelity and reflectivity magnitude sparsity in terms of dictionary Ψ. Using a number of dictionaries such as wavelets, and shape-based dictionaries enhances some features of the magnitude. While this approach produces very good results in certain contexts using dictionaries simultaneously representing multiple types of features, one of its limitations is that these dictionaries are pre-defined and cannot be easily adapted for a certain context in a data-driven manner.

2.2. Dictionary Learning

The general idea of patch-based dictionary learning can be explained as follows. Given and image $f \in \mathbb{C}^N$ and its $\sqrt{n}\times\sqrt{n}$ image patches column-stacked into vectors $f_s \in \mathbb{C}^n$ (where s is the patch index), the goal is to represent these patches with sparse coefficients $\alpha_s \in \mathbb{C}^K$ and dictionary $D \in \mathbb{C}^{n\times K}$ where K is the number of atoms in the dictionary. If $K > n$, the dictionary is overcomplete. Within this context, the dictionary learning problem can be expressed as follows:

$$\min_{D,\alpha_s} \sum_s \|E_s f - D\alpha_s\|_2^2 + \sum_s \mu_s\,\|\alpha_s\|_0 \tag{2}$$

where $E_s \in \mathbb{C}^{n\times N}$ is 2D patch extraction operator. The first term measures the proximity between sparse representation and training patches, and the second term measures the sparsity level. This problem is NP-hard because of the l_0 norm in the sparsity term. K-SVD solves this problem approximately by iterating between sparse coding stage and the codebook update stage. In the first stage, sparse solutions of the problem α_s are found by orthogonal matching pursuit (OMP) [11] while keeping the dictionary D fixed. In the codebook update stage, each column of D is updated sequentially. For each column and its corresponding sparse coefficients the error is calculated. In the process of minimizing the error, singular value decomposition of the error matrix is utilized and the approach involves a generalization of K-means clustering. Accordingly, this approach is called K-SVD. An important difference of K-SVD from other existing ways of generalizing K-means is that it updates both the dictionary and the sparse coefficients in the codebook update step.

Although dictionary learning methods, especially K-SVD, have been widely used in various fields of image processing, they have not yet had a significant presence in SAR imaging. In [12] incomplete SAR data are reconstructed using K-SVD approach as an image inpainting problem. In [13] a dictionary learning algorithm is used for SAR image despeckling. In [14] dictionary learning algorithm has been proposed for SAR image super-resolution. In [15], K-SVD is used in the process of decomposing a SAR image into a spatially sparse and a spatially non-sparse component.

3. PROPOSED FRAMEWORK

In this section we describe our approach for integrating patch-based dictionary learning into sparsity-driven SAR imaging. As explained before, the random phase nature of reflectivities in SAR suggest the use of sparsifying dictionaries over the magnitude of the complex-valued reflectivity field f. Using the notation of $f = \Theta|f|$ as before, we define the following joint problem for dictionary learning and image formation.

$$\begin{aligned}\left\{\widehat{|f|}, \widehat{\Theta}, \widehat{D}, \widehat{\alpha}_s\right\} = \arg\min_{|f|,\Theta,D,\alpha_s} & \sum_s \|E_s\,|f| - D\alpha_s\|_2^2 \\ &+ \sum_s \mu_s\,\|\alpha_s\|_0 \\ &+ \lambda\,\|g - H\Theta\,|f|\|_2^2\end{aligned} \tag{3}$$

In this optimization problem; the first term measures the proximity between sparse representations and the magnitude of the image patches, the second term measures the sparsity of the image patches, and the third term measures data fidelity, where λ is the weight of the data fidelity term. This parameter depends on measurement noise. In particular, if the variance of the noise is known such as σ^2 this parameter can be expressed as $\frac{C}{\sigma^2}$ where C is positive constant. Therefore, when observation noise level is high, weight of the data fidelity term decreased.

Solution of this optimization problem needs an alternating solution procedure. In order to solve one parameter, other parameters assumed to be fixed. There are four different parameters to be solved: $\widehat{|f|}, \widehat{\Theta}, \widehat{D}, \widehat{\alpha}_s$. If the dictionary D is learned from several training patches offline, before the image reconstruction process, then the formulation decomposes into sequential steps of offline dictionary learning and online

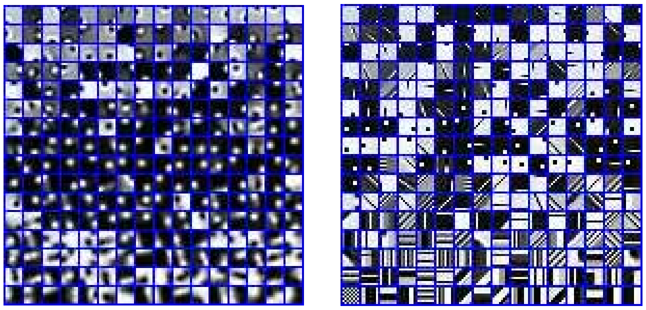

Fig. 1. Dictionary learned for real SAR images (left), synthetic images (right).

image formation. In this case, the online process would not contain the dictionary as one if its unknowns. For the sake of generality, here we describe the solution of the problem as formulated in (3). Each iteration of this process involves three steps: Dictionary learning, phase update, and magnitude update. In the first step, the dictionary D and the sparse coefficients α_s are jointly updated. If the dictionary is learned offline, then in the online process, this step only update the α_s through a pursuit algorithm. The second step minimizes the phase of the reflectivity field by using the iterative method used in [4]. The last step reconstruct the magnitude of the reflectivity field. Next we explain these three update steps in detail.

3.1. Dictionary Learning Step

This steps solves for the patch-based overcomplete dictionary as well as the sparse representation coefficients over that dictionary, while keeping $|f|$ and Θ fixed. More specifically, in this step we solve the following subproblem of (3):

$$\left\{\widehat{D}, \widehat{\alpha}_s\right\} = \arg\min_{D,\alpha_s} \sum_s \|E_s\,|f| - D\alpha_s\|_2^2 + \sum_s \mu_s \|\alpha_s\|_0 \tag{4}$$

This subproblem can be solved by K-SVD. As mentioned before, K-SVD employs alternating updates of the sparse coefficients and the dictionary. The sparse coefficients are updated using OMP. The dictionary is updated as described in Section 2.2. In the case of offline dictionary learning (4) is used in the offline learning stage as well as during the online reconstruction stage. In the offline stage, the variable f in (4) corresponds to training images. In the online stage, the dictionary learned offline is fixed and (4) is used to update the sparse coefficients only.

3.2. Phase Update Step

In this step, the phase of the reflectivity field is estimated by keeping the other parameters fixed. This requires solving a subproblem of (3) involving the last term in (3) only. An algorithm for solving such a phase estimation problem has been proposed in [4, 16], which we utilize in this step. Let us introduce a vector $p \in \mathbb{C}^N$ that contains the diagonal elements of the phase matrix Θ, and the matrix $B \in \mathbb{C}^{N\times N}$ whose diagonal elements contain information about the reflectivity magnitudes. Let us also invoke the constraint that the magnitudes of the elements of p should be 1, simply because they contain phases in the form $e^{j\phi(f)}$ where $\phi(\cdot)$ denotes the phase. Then, we obtain the following optimization problem in Lagrangian form:

$$\widehat{p} = \arg\min_p \|g - HBp\|_2^2 + \lambda_2 \sum_{i=1}^{N} (|p_i| - 1)^2 \tag{5}$$

where

$$B = \text{diag}\left\{ \frac{\left(\sum_s E_s^T D\alpha_s\right)_i}{\left(\sum_s E_s^T E_s\right)_{(i,i)}} \right\} \tag{6}$$

and λ_2 is a Lagrange multiplier. As mentioned above, B contains information about the current estimate of the reflectivity magnitudes. Here we could use the estimate of $|f|$ from the previous iteration, but instead we choose to incorporate its sparse representation from the current iteration through the α_s. Since this representation is patch-based, (6) performs appropriate operations to produce an $N \times N$ matrix, whose N diagonal entries correspond to the N reflectivity magnitudes in the scene. As in [4], we solve this optimization problem through a fixed point algorithm, which can also be shown to be equivalent to a particular quasi-Newton algorithm. In particular, the non-quadratic optimization problem in (5) is solved by turning it into a series of quadratic problems. Then, each quadratic problem can be efficiently solved by the conjugate gradient algorithm.

3.3. Magnitude Update Step

In this last step, the magnitude of the reflectivity field is estimated keeping the other parameters fixed.The subproblem of (3) for updating the reflectivity magnitudes can be expressed as:

$$\widehat{|f|} = \arg\min_{|f|} \sum_s \|E_s\,|f| - D\alpha_s\|_2^2 + \lambda \|g - H\Theta\,|f|\|_2^2 \tag{7}$$

This is a quadratic optimization problem with a closed form solution. Taking the derivative with respect to $|f|$ and equating it zero gives the following equation.

$$\left(\sum_s E_s^T E_s + \lambda \Theta^{\mathrm{H}} H^{\mathrm{H}} H \Theta\right) \widehat{|f|} = \lambda \Theta^{\mathrm{H}} H^{\mathrm{H}} g + \sum_s E_s^T D\alpha_s \tag{8}$$

We solve this linear set of equations using the conjugate gradient algorithm. One important point in this step is the solution of the subproblem may produce a complex values. One can propose to convert this problem to into a constrained problem to enforce zero phase. However, this constraint will complicate the solution further. Thus, we simply take the magnitude part of the solution at each iteration.

4. EXPERIMENTAL RESULTS

We present preliminary experimental results on both synthetic and real SAR data. For the former, we constitute 128×128

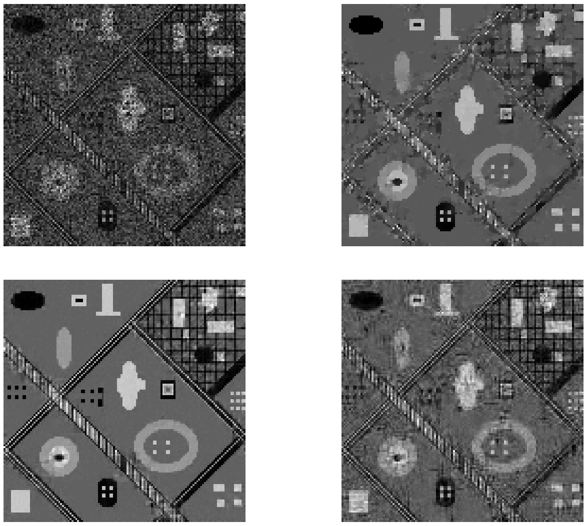

Fig. 2. Synthetic scene reconstruction experiment. Conventional reconstruction (top-left); sparsity-driven imaging with a point-region dictionary (top-right); proposed method with offline dictionary learning (bottom-left) and online dictionary learning (bottom-right).

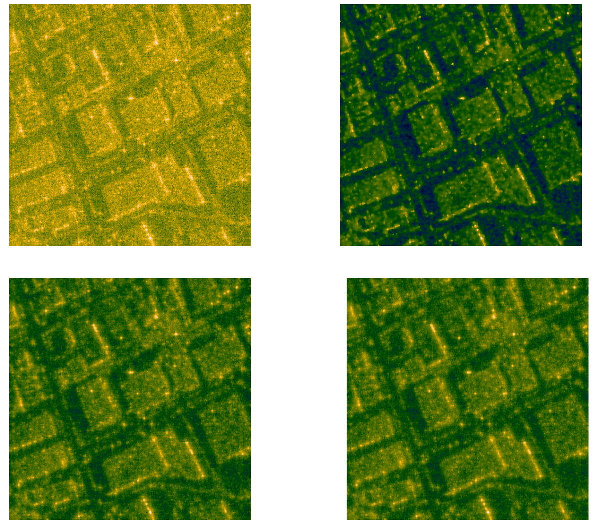

Fig. 3. Real SAR data (TerraSAR-X) reconstruction experiment. Conventional reconstruction (top-left); sparsity-driven imaging with a point-region dictionary (top-right); proposed method with offline dictionary learning (bottom-left) and online dictionary learning (bottom-right).

synthetic scenes, representing reflectivity magnitudes, and add random phase. We simulate the returns from such synthetic reflectivity fields as SAR data in the phase history domain, and our forward model involves a band-limited Fourier transform operation. Our results on such a synthetic scene are shown in Figure 2. We demonstrate the performance of our approach using both dictionaries learned offline (see Figure 1(b)) from training images, as well as dictionaries learned online from the data used for reconstruction. For the offline case, the synthetic training images we use are different from the test image. Our approaches provide enhancements over the conventional image for point and distributed objects as well as for smooth and textured regions. The performance of our offline learning approach,which utilizes high-quality synthetic scenes for dictionary learning is better than our online approach as expected. We also compare our results to that of sparsity-driven imaging with a point-region dictionary, i.e., a dictionary that aims to preserve spatially-localized point scatterers and piecewise smooth regions. These are the most widely used types of features in sparsity-driven SAR imaging. This approach performs well in parts of the scene that match the spatial structure of the used dictionaries, however it performs significantly worse than our approach particularly in regions involving textures. Some quantitative results on performance can be found in Table 1. This example demonstrates the flexibility and adaptivity of our approach to various types of features that can be learned from data. We also show preliminary results on a real SAR scene from TerraSAR-X in Figure 3[1]. For this particular scene, both our approach and the fixed-dictionary approach provide improvements over the conventional image, in terms of, e.g., speckle suppression. For this experiment, we used conventional SAR images from the same satellite for offline dictionary learning. These images suffer from noise and artifacts themselves. We expect that using improved SAR images that provide a better representation of the ground truth within our framework in future work will enable the demonstration of the full potential of our approach.

[1]Astrium TerraSAR-X sample imagery: http://www.astrium-geo.com/en/23-sample-imagery

Table 1. Performance of reconstruction methods in terms of SNR of the formed imagery for the experiment in Figure 2.

Method	SNR (dB)
Conventional	2.0907
Non-quadratic	4.0820
Offline dictionary learning	9.9730
Online dictionary learning	5.2029

5. CONCLUSION

We have proposed a new, dictionary-learning-based approach for sparsity-driven SAR imaging. Our approach considers the complex-valued nature of SAR reflectivities and incorporates learning-based dictionaries for sparsely representing the magnitude of the field, while reconstructing its phase as well. Our approach can learn dictionaries from a training set of images offline, or from the data to be used in image reconstruction online. Our preliminary experimental results suggest such learning-based approaches can widen the domain of applicability of sparsity-driven SAR imaging, and enable exploitation of context-based knowledge for more effective sparse representation in SAR.

6. REFERENCES

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