Changes in the spectrum of light arising on propagation through a linear time-invariant system ☆

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Received 17 September 1990

The spectrum of light, in general, changes when the light interacts with a dispersive medium or when it is diffracted. It may also change on propagation, even in free space, because of the coherence properties of the source. The difference and some similarities between these processes of spectral modification are analyzed.

It has been demonstrated in the last few years, both theoretically [1–5] and experimentally [6–10] that, in general, the spectrum of light generated by a partially coherent source changes on propagation, even in free space. Spectral changes also arise when a polychromatic light passes through a dispersive optical element, such as a prism or a diffraction grating or is diffracted at an aperture. The distinction between these two effects has been rather obscure until now. The present note is concerned with clarifying this situation.

Let us consider the propagation of light through a linear, deterministic, time-invariant system. We assume that the light is statistically stationary, at least in the wide sense, and we confine our attention to the optical field in an input plane $z=z_0$ and an output plane $z=z_1$ (see fig. 1). According to the coherence theory in the space–frequency domain [11], we may represent the fields in the two planes by ensembles $\{U_0(\boldsymbol{\rho}', \omega)\}$ and $\{U_1(\boldsymbol{\rho}, \omega)\}$ of frequency-dependent realizations. Here $\boldsymbol{\rho}'$ and $\boldsymbol{\rho}$ are position vectors of typical points P_0 and P_1 in the input and the output plane respectively. Corresponding realizations will be related by a formula of the form

$$U_1(\boldsymbol{\rho}, \omega)=\int U_0(\boldsymbol{\rho}', \omega)\, K(\boldsymbol{\rho}, \boldsymbol{\rho}', \omega)\, \mathrm{d}^2\boldsymbol{\rho}' , \qquad (1)$$

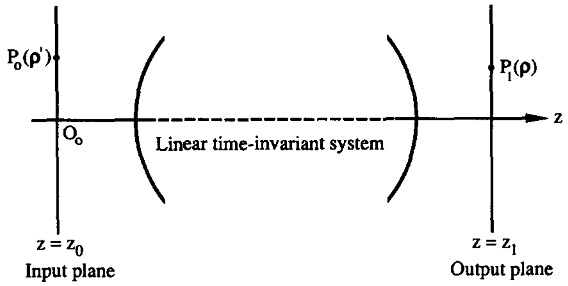

Fig. 1. Illustration of the notation relating to eq. (1).

where $K(\boldsymbol{\rho}, \boldsymbol{\rho}', \omega)$ is the impulse response function of the system. We stress that the system need not be an imaging system. It could be, for example, a prism or a diffraction grating or even free space.

The cross-spectral densities at frequency ω of the fields in the two planes may be expressed in the form [11]

$$W_0(\boldsymbol{\rho}'_1, \boldsymbol{\rho}'_2, \omega)=\langle U_0^*(\boldsymbol{\rho}'_1, \omega)\, U_0(\boldsymbol{\rho}'_2, \omega)\rangle , \qquad (2a)$$

☆ Research supported in part by the Army Research Office.
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$$W_1(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\omega)=\langle U_1^*(\boldsymbol{\rho}_1,\omega)\,U_1(\boldsymbol{\rho}_2,\omega)\rangle\,, \tag{2b}$$

where the asterisk denotes the complex conjugate and the angular brackets denote the ensemble average.

On substituting from eq. (1) into eq. (2b) and making use of eq. (2a) we obtain the following expression for the cross-spectral density of the field in the output plane in terms of the cross-spectral density in the input plane:

$$W_1(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\omega)=\int\int W_0(\boldsymbol{\rho}'_1,\boldsymbol{\rho}'_2,\omega)\times K^*(\boldsymbol{\rho}_1,\boldsymbol{\rho}'_1,\omega)\,K(\boldsymbol{\rho}_2,\boldsymbol{\rho}'_2,\omega)\,\mathrm{d}^2\rho'_1\,\mathrm{d}^2\rho'_2\,. \tag{3}$$

If we set $\boldsymbol{\rho}_1=\boldsymbol{\rho}_2=\boldsymbol{\rho}$, the expression on the left reduces to the spectral density $S_1(\boldsymbol{\rho},\omega)$ of the field in the output plane:

$$S_1(\boldsymbol{\rho},\omega)=\int\int W_0(\boldsymbol{\rho}'_1,\boldsymbol{\rho}'_2,\omega)\times K^*(\boldsymbol{\rho},\boldsymbol{\rho}'_1,\omega)\,K(\boldsymbol{\rho},\boldsymbol{\rho}'_2,\omega)\,\mathrm{d}^2\rho'_1\,\mathrm{d}^2\rho'_2\,. \tag{4}$$

The physical significance of this formula becomes clearer if we express the cross-spectral density $W_0(\boldsymbol{\rho}'_1,\boldsymbol{\rho}'_2,\omega)$ in terms of the spectrum $S_0(\omega)$ (assumed for simplicity to be the same at every point in the input plane) and the degree of spatial coherence [12]

$$\mu_0(\boldsymbol{\rho}'_1,\boldsymbol{\rho}'_2,\omega)=W_0(\boldsymbol{\rho}'_1,\boldsymbol{\rho}'_2,\omega)/S_0(\omega) \tag{5}$$

of the input field. On substituting for W_0 from this formula in eq. (4) we obtain the following expression for the spectrum $S_1(\boldsymbol{\rho},\omega)$:

$$S_1(\boldsymbol{\rho},\omega)=S_0(\omega)\int\int \mu_0(\boldsymbol{\rho}'_1,\boldsymbol{\rho}'_2,\omega)\times K^*(\boldsymbol{\rho},\boldsymbol{\rho}'_1,\omega)\,K(\boldsymbol{\rho},\boldsymbol{\rho}'_2,\omega)\,\mathrm{d}^2\rho'_1\,\mathrm{d}^2\rho'_2\,. \tag{6}$$

The expression (6) shows that the spectrum in the output plane will, in general, differ from the spectrum in the input plane for two different reasons:

(i) because of the spatial coherence properties of the input field, characterized by its degree of spatial coherence $\mu_0(\boldsymbol{\rho}'_1,\boldsymbol{\rho}'_2,\omega)$; and

(ii) because of the transmission properties of the system, characterized by the impulse response function $K(\boldsymbol{\rho},\boldsymbol{\rho}',\omega)$.

Let us consider two extreme cases. Suppose first that light from a partially coherent planar secondary source occupying a finite region σ of the input plane $z=z_0$ propagates in free space to the far zone. The impulse response function is then given by [13]

$$K(\boldsymbol{\rho},\boldsymbol{\rho}',\omega)=-\frac{ik}{2\pi}\cos\theta\,\frac{\exp(ikr)}{r}\exp(-ik\boldsymbol{s}\cdot\boldsymbol{\rho}')\,,\quad (kr\to\infty)\,, \tag{7}$$

where $r=[(z_1-z_0)^2+\boldsymbol{\rho}^2]^{1/2}$ is the distance from the origin O_0 in the input plane to a field point $P_1(z_1,\boldsymbol{\rho})$ in the far zone, $\boldsymbol{s}$ is the unit vector pointing from O to P_1 and θ is the angle which the line OP_1 makes with the positive z-axis (fig. 2). On substituting from eq. (7) in to eq. (6) we obtain the following expression for the spectrum of the light at the point P in the far zone:

$$S_1(r\boldsymbol{s},\omega)=S_0(\omega)\left(\frac{k\cos\theta}{2\pi r}\right)^2\int_\sigma\int_\sigma \mu_0(\boldsymbol{\rho}'_1,\boldsymbol{\rho}'_2,\omega)\times\exp[-ik\boldsymbol{s}_\perp\cdot(\boldsymbol{\rho}'_2-\boldsymbol{\rho}'_1)]\,\mathrm{d}^2\rho'_1\,\mathrm{d}^2\rho'_2 \tag{8}$$

$(r\boldsymbol{s}\equiv z_1,\boldsymbol{\rho})$, $\boldsymbol{s}_\perp$ being the projection, considered as a two-dimensional vector, of the unit vector $\boldsymbol{s}$ on the source plane. In particular if the source is quasi-homogeneous [14], its degree of spatial coherence depends on its two spatial arguments only through the difference $\boldsymbol{\rho}'_2-\boldsymbol{\rho}'_1$ and we will then write $\mu_0(\boldsymbol{\rho}'_2-\boldsymbol{\rho}'_1,\omega)$ in place of $\mu_0(\boldsymbol{\rho}'_1,\boldsymbol{\rho}'_2,\omega)$. In this case the formula (8) gives, to a good approximation

$$S_1(r\boldsymbol{s},\omega)=\left(\frac{\omega\cos\theta}{cr}\right)^2 A\tilde{\mu}_0(k\boldsymbol{s}_\perp,\omega)\,S_0(\omega)\,, \tag{9}$$

where $\omega=kc$ (c being the speed of light in vacuo) and $\tilde{\mu}_0(\boldsymbol{f},\omega)$ is the two-dimensional spatial Fourier transform of the degree of spatial coherence of the light in the source plane $z=z_0$ viz.,

$$\tilde{\mu}_0(\boldsymbol{f},\omega)=\frac{1}{(2\pi)^2}\int_\sigma \mu_0(\boldsymbol{\rho}',\omega)\exp(-i\boldsymbol{f}\cdot\boldsymbol{\rho}')\,\mathrm{d}^2\rho'\,, \tag{10}$$

A being the source area.

The expression (9) for the far zone spectrum is in agreement with a formula derived previously ([14], eq. (4.8)) in a different manner. It shows how the spectrum of the light is changed on free-space propagation from the source to the far zone due to the coherence properties of the source, characterized by the correlation coefficient μ_0.

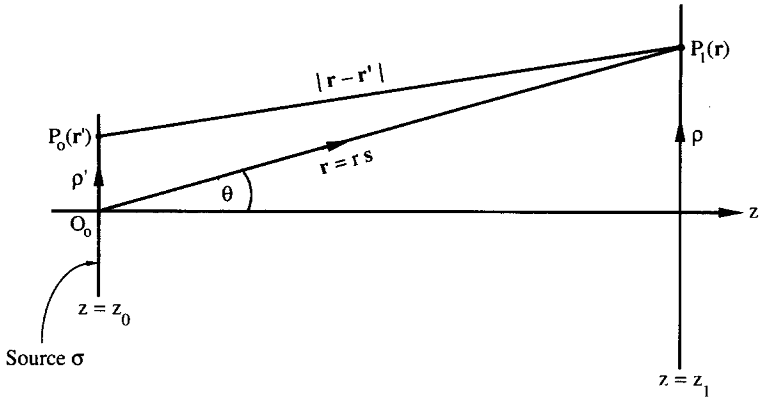

Fig. 2. Illustration of the notation relating to the formulas (8) and (9). The point P_1, located in the far zone, is specified by the position vector $\boldsymbol{r} \equiv r\boldsymbol{s} \equiv (z_1, \boldsymbol{\rho})$.

As another extreme case consider the situation when the input is a plane polychromatic wave which propagates through a linear system in the positive z-direction. The degree of spatial coherence $\mu_0(\boldsymbol{\rho}'_1, \boldsymbol{\rho}'_2, \omega)$ will then necessarily have the value unity for all value of its arguments [#1] $\boldsymbol{\rho}'_1, \boldsymbol{\rho}'_2$ and ω and the expression (6) then reduces to

$$S_1(\boldsymbol{\rho}, \omega) = S_0(\omega) \left| \int K(\boldsymbol{\rho}, \boldsymbol{\rho}', \omega)\, \mathrm{d}^2\boldsymbol{\rho}' \right|^2 . \qquad (11)$$

This formula shows that the difference between the spectra in the output plane and the input plane is now entirely caused by the response properties of the transmitting system, as might have been expected.

Eq. (11) applies to situations where the spectral changes are produced by dispersion and diffraction of a spatially completely coherent polychromatic plane wave propagating in the z-direction by a system which includes, for example, lenses, prisms, diffraction gratings and diffracting apertures. Such systems will, of course, also produce changes in the spectrum of an incident non-planar polychromatic wave of any state of coherence. However the different causes of the spectral modification cannot always be clearly separated. In some situations it may even be possible to regard the effect as arising either from the coherence properties of the source or from dispersion and diffraction. An example of such a situation is one of the experiments described in ref. [6] and illustrated in fig. 2(b) of that reference. The system employed in that experiment consisted of three planes, namely the input plane (I), a plane of a secondary source (II) and output plane (III), with a lens located between the planes (I) and (II). The change in the spectrum of the light transmitted from the input plane to the output plane was attributed to the coherence properties of the secondary source. It is, however, also possible to regard the change as being due to dispersion and diffraction of the light as it passes from the input to the output plane, without explicitly invoking the coherence properties of the light in the intermediate plane II. On the other hand, because of the basic difference in the mathematical properties of the degree of spatial coherence of sources and of the response function of linear systems it is likely that some spectral modification which might be produced by source correlations may not be realizable by dispersion and diffraction and vice versa.

[#1] This result follows from eq. (5) and the fact that the cross-spectral density of such a wave has the form $W_0(\boldsymbol{r}'_1, \boldsymbol{r}'_2, \omega) = S_0(\omega)\exp[\mathrm{i}(\omega/c)(z'_2 - z'_1)]$, where z'_1 and z'_2 are the z-components of the vector $\boldsymbol{r}'_1$ and $\boldsymbol{r}'_2$ respectively and c is the speed of light in vacuo. In particular, when $\boldsymbol{r}'_1 = \boldsymbol{\rho}'_1$, $\boldsymbol{r}'_2 = \boldsymbol{\rho}'_2$ then $z'_1 = z'_2 = z_0$ and the above expression gives $W_0(\boldsymbol{\rho}'_1, \boldsymbol{\rho}'_2, \omega) = S_0(\omega)$. Eq. (5) then implies that $\mu_0(\boldsymbol{\rho}'_1, \boldsymbol{\rho}'_2, \omega) \equiv 1$, as asserted.

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