Phasing sparse arrays of heterodyne receivers

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ABSTRACT

We describe two new methods for phasing arrays of heterodyne receivers. Both can be used when the arrays are sparse and distributed. One is based on the iterative transform algorithm using a support constraint and the other on maximizing image sharpness. Both work well: they require a modest number of speckle realizations with the same aberrations and are relatively immune to measurement noise.

1. INTRODUCTION

If we measure an optical field, scattered from a laser-illuminated object, with an array of heterodyne receivers,[1] then we can reconstruct an image of the object by performing a suitable propagation transformation (typically a Fresnel or Fourier transform) in the computer.[2] However, if phase errors are present in the measurements, then a blurred image will result. Phase errors can be induced by many things, including atmospheric turbulence and path-length errors between the field from the object and the local oscillator within the receiver. Approaches to phasing the array without a beacon either require the array of heterodyne receivers to be on a filled regular grid,[3,4] or to have a special pattern or alternatively require a low-resolution image be available.[5]

In this paper we describe two new algorithms for phasing heterodyne arrays that work well for sparse distributed arrays (and will work for filled arrays as well), requires no low-resolution image, and is relatively insensitive to noise. One is based on the iterative transform algorithm using a support constraint[6,7] and the other on maximizing image sharpness.[8,9]

2. IMAGING MODEL

We measure the optical fields (ignoring measurement noise)

$$G_{dk}(u) = F_k(u) \exp[i\phi_e(u)] , \qquad (1)$$

for $k = 1, \ldots, K$, where F_k is the ideal complex field (without phase errors) for realization k, ϕ_e is the phase error, and u is a 2-D coordinate (the pixel index) in the measurement plane. Note that we assume the phase error is the same for all K realizations. Different speckle realizations are obtained if the object rotates slightly or translates. If we estimate the phase error as $\phi(u)$, then our estimates of the optical fields are

$$G_k(u) = G_{dk}(u) \exp[-i\phi(u)] \qquad (2)$$

our coherent images are

$$g_k(x) = \mathcal{F}^{-1}[G_k(u)] = N^{-2}\sum_u G_k(u) \exp(i2\pi u{\cdot}x/N) \ , \tag{3}$$

where $u{\cdot}x$ is a vector dot (inner) product, and our speckle-reduced averaged image is

$$g_I(x) = K^{-1}\sum_k |g_k(x)|^2 \ , \tag{4}$$

where summation over k is for $k = 1, ..., K$.

3. ITERATIVE TRANSFORM ALGORITHM

Our first method to estimate the phase error is a generalization of the iterative transform algorithm.[6] In summary, one iteration of the generalized algorithm consists of the following steps:
(1) Calculate the Fourier estimates: $G_k(u) = G_{dk}(u) \exp[-i\phi(u)]$ (for $k = 1 ... K$).
(2) Inverse FFT them to compute the $g_k(x)$.
(3) Apply the support constraint: $g_{sk}(x) = s(x) \, g_k(x)$.
(4) FFT to compute the $G_{sk}(u)$.
(5) Compute a new phase-error estimate: $\phi(u) = \arg\{\sum_k G_{dk}(u) \, G_{sk}^*(u)\}$.

For the initial iteration the phase error estimate is zero. The support constraint can be known *a priori*, estimated from the autocorrelation support (which can be calculated from the given data),[10,11] or estimated from $g_I(x)$. To estimate the support from $g_I(x)$, the current estimate of the averaged image, we could smooth it, giving $s(x) = w(x) * g_I(x)$, and possibly threshold it. The smoothing function, $w(x)$ could be, for example, a uniform disk having a radius of a few pixels. The new phase-error estimate, $\phi(u)$, given in step 5 is like a weighted average of the difference in phase between $G_{dk}(u)$, the measured data, and $G_{sk}(u)$, the current estimate of optical fields constrained by the image support constraint.

Steps (2) to (4) can also be performed by a direct convolution in the measurement plane:

$$G_{sk}(u) = G_k(u) * S(u) = \sum_{u_1} S(u_1) \, G_k(u - u_1) \ , \tag{5}$$

where $S(u) = \mathcal{F}[s(x)]$, which would be faster to compute for the case of sparse arrays.

4. MAXIMIZING SHARPNESS

The second approach is to find the phase-error estimate $\phi(u)$ which maximize the sharpness,

$$S_1 = \sum_x [g_I(x)]^2 = \sum_x [\sum_k |g_k(x)|^2]^2 \ , \tag{6}$$

of the image estimate. Alternatively, we can maximize S_{1w}, a weighted version of the sharpness, based on a filtered version of the averaged image,

$$g_{IW}(x) = \mathcal{F}^{-1}\{W(u) \, \mathcal{F}[g_I(x)]\} \tag{7}$$

where W(u) is a weighting function (for example, a Wiener filter).

We maximize the sharpness using a standard nonlinear optimization technique which we make computationally efficient by using an analytic expression for the gradient of the sharpness with respect to the phase-error estimate:

$$\frac{\partial S_{1W}}{\partial \phi(u)} = 4N^{-2} \, \text{Im}\{\sum_k G_k(u) \, (G_k(u) * [G_I W^2])^*\} \,, \tag{8}$$

where $G_I(u) = \mathcal{F}[g_I(x)]$.

We can show that the error-reduction[6] version of our generalized iterative transform algorithm is similar in effect to a steepest-descent gradient search to maximize sharpness with a fixed step size. However, we in practice use a more powerful gradient search algorithm, such as conjugate-gradient, and we can use a more powerful version of the iterative transform algorithm as well, such as hybrid input-output.[6]

5. IMAGE RECONSTRUCTION EXAMPLE

Figure 1 shows a computer simulation example of phasing an array of sparse, distributed heterodyne receivers. Figure 1(a) shows the object, consisting of a block of 2×3 pixels and three separated points, embedded in a 64×64 array. We created complex-valued speckled images by replacing each pixel in the object by a circular complex Gaussian random number with variance equal to the square root of the original pixel value; Fourier transforming, multiplying by the a sparse aperture, and inverse transforming to produce the image. The sparse aperture we used for this experiment has 21 pixels within a 8×8 area, which is shown in Figure 1(b), magnified by a factor of four (only the central 16×16 pixels in its 64×64 array are shown). This sub-aperture pattern was designed by a genetic-based algorithm by P. Henshaw.[12] An average of ten such diffraction-limited speckled images is shown in Figure 1(c). Figure 1(d) shows a Wiener-Helstrom[13] filtered version of the averaged image, from which one can discern the major parts of the image. Figure 1(e) shows the blurred averaged image that resulted when we added, to the Fourier transform, random phase errors that were uniformly distributed Gaussian random numbers with standard deviation π radians. Figure 1(f) shows the averaged image corrected by the image sharpening algorithm and Figure 1(g) shows a Wiener-filtered version of the corrected image. The corrected image is clearly superior to the blurred image and strongly resembles the diffraction-limited image. We obtained comparable results using the generalized iterative transform algorithm. The algorithms usually produced images with better quality as the number of realizations increases. It was unreliable when using only one to four realizations. Tests with noise showed that the algorithm is not very sensitive to noise.

6. CONCLUSIONS

We have derived two new algorithms for phasing an array of heterodyne receivers, and have demonstrated that they can work well for sparse distributed arrays, require a modest number of speckle realizations, and are robust in the presence of measurement noise.

7. ACKNOWLEDGMENTS

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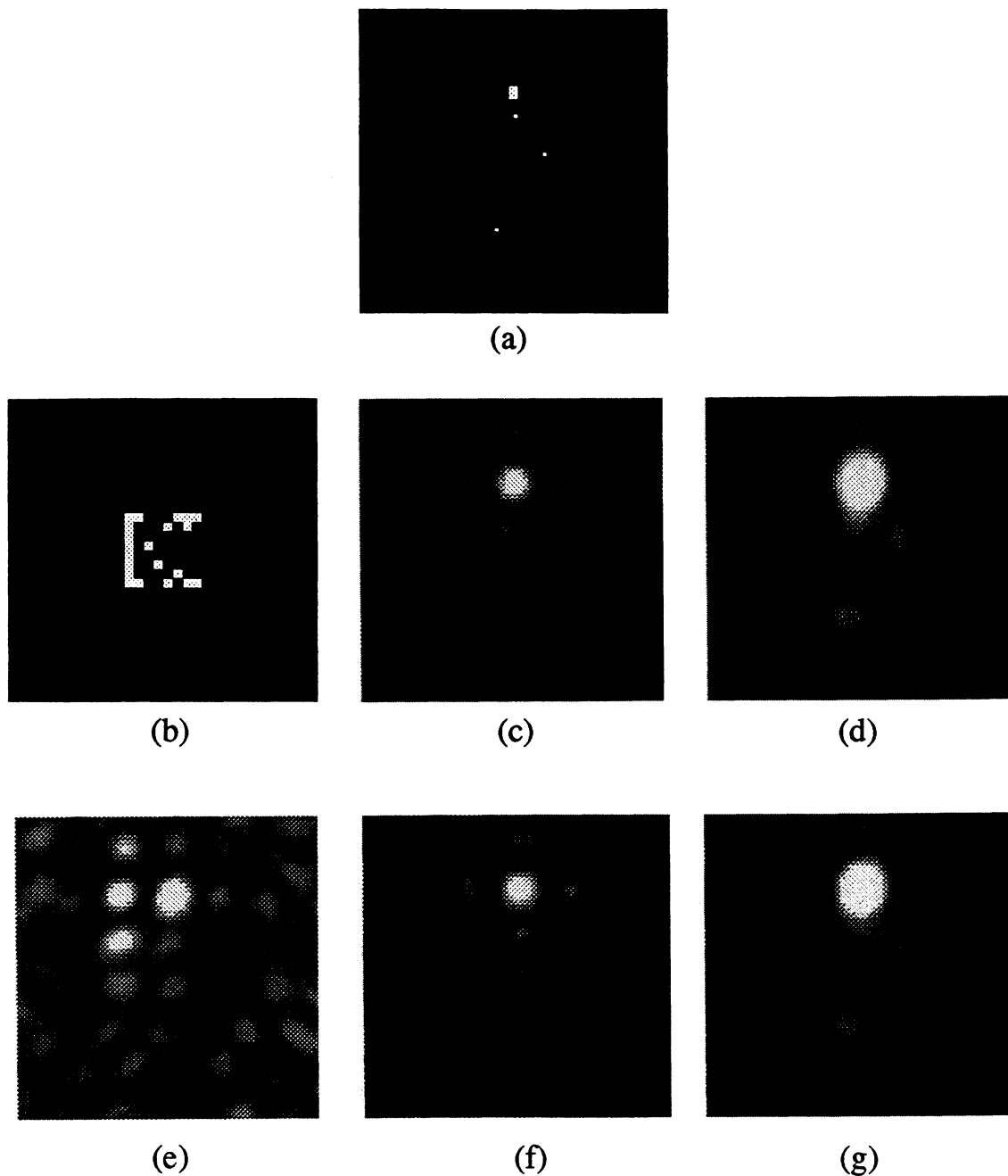
Figure 1. Computer Simulation of Sparse Heterodyne Array Phasing.
(a) Object; (b) 21-element sparse, distributed array (within an 8 x 8 -sample area);
(c) average of ten diffraction-limited speckled images; (d) Wiener-filtered version
of (c); (e) average of ten aberrated speckled images; (f) average of ten speckled
images corrected by image-sharpening phase-up algorithm; (g) Wiener-filtered
version of (f).