Phase retrieval from experimental far-field speckle data

J. N. Cederquist, J. R. Fienup, J. C. Marron, and R. G. Paxman

Optical Science Laboratory, Advanced Concepts Division, Environmental Research Institute of Michigan, P.O. Box 8618, Ann Arbor, Michigan 48107

Received March 11, 1988; accepted May 20, 1988

Phase retrieval from experimental (laboratory) data has been successfully demonstrated. A diffuse object was coherently illuminated and Fourier intensity data were collected by a charge-coupled device detector and a video digitizer. By using the data and an *a priori* triangular image support constraint, an iterative Fourier-transform algorithm was used to estimate the phase of the Fourier transform of the object. The reconstructed image compares favorably with a conventional image with the same spatial-frequency bandwidth.

Phase retrieval from Fourier intensity data and *a priori* object constraint information is known to have many applications.[1] Iterative Fourier-transform algorithms have demonstrated phase retrieval from computer-simulated data both for real-valued, nonnegative objects[2–4] and for certain types of complex-valued objects.[5] The effect of noise on phase retrieval has been investigated both empirically through computer simulations[2,6] and theoretically through Cramer–Rao lower bounds.[7] The research reported here demonstrates phase retrieval from Fourier intensity (far-field speckle pattern) data collected in a laboratory experiment using a coherently illuminated diffuse (i.e., complex-valued) object. In this Letter, we describe the experimental optical system, data collection hardware and procedures, and data-processing and image-reconstruction steps. An image reconstructed from Fourier intensity data and a comparison with a conventional image are shown.

The optical system used in performing the experiment is shown in Fig. 1. An argon-ion laser beam of wavelength $\lambda = 0.5145\ \mu m$ is spatially filtered, collimated, and used to illuminate a transmissive object. The object consists of a binary mask placed in contact with a ground glass [Fig. 2(a)]; thus the transmittance of the object is binary in intensity and random in phase. An *a priori* image support constraint is introduced by giving the transmissive region of the mask a known overall triangular shape. The lens L_1 of focal length f_1 produces the Fourier transform of the complex-valued object transmittance in its back focal plane. There, an aperture A selects a portion of the Fourier transform and lenses L_2 and L_3 (of focal lengths f_2 and f_3, respectively) image this portion, with suitable magnification, to the detector for collection of Fourier intensity data. When the removable mirrors M_1 and M_2 are in place, the light is diverted through lens L_4, which produces an image of the object at the detector. Because of the placement of aperture A, this conventional image provides a reference for comparison to the image reconstructed by phase retrieval. A polarizer was placed just before the detector to ensure detection of only a single polarization.

The base and height, d, of the triangular mask and the lens focal lengths must be chosen so that the speckle intensity in the Fourier transform is adequately sampled by the detector. Assuming a speckle size at the detector equal to $\lambda f_1 f_3/f_2 d$, to sample the intensity at the Nyquist rate the detector pixel spacing, Δs, must equal $\lambda f_1 f_3/2 f_2 d$. In the experiment, d was about 16 mm and f_1, f_2, and f_3 were 500, 50, and 300 mm, respectively, giving $\Delta s \simeq 48\ \mu m$. The charge-coupled device (CCD) detector used has horizontal and vertical pixel spacings of 30 and 18 μm, respectively, so the data were sampled at a rate greater than the Nyquist rate.

Since the Fourier intensity has twice the spatial-frequency bandwidth of the complex-valued Fourier transform, measurement of $2N_1$ by $2N_2$ Nyquist-spaced samples of the Fourier intensity enables reconstruction of a complex-valued image of N_1 by N_2 resolution elements. The number of horizontal and vertical pixels in the detector array thereby sets an upper limit on the size, in resolution elements, of the reconstructed image. In this experiment, the central 180 pixels × 256 pixels (over a 5.4 mm × 4.6 mm region) of the detector were used. The aperture A was therefore 0.9 mm × 0.77 mm, and the reconstructed and conven-

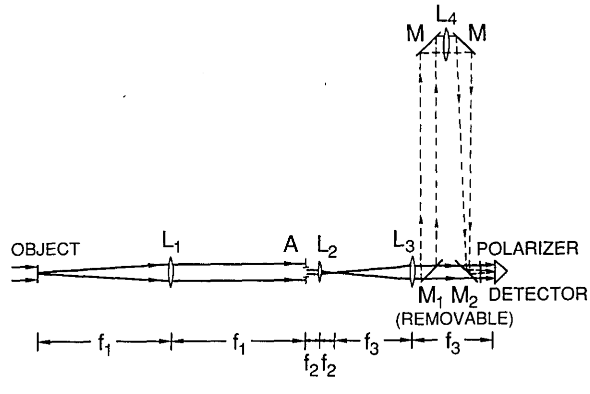
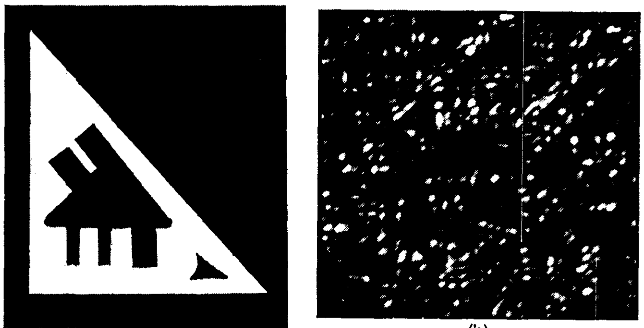


Fig. 1. Experimental optical system.

0146-9592/88/080619-03$2.00/0



(a) (b)

Fig. 2. (a) Incoherent image of test object. (b) Coherent Fourier intensity data.

tional images had 56 × 48 resolution elements of size 0.28 mm × 0.34 mm (at the object).

The diameter, D, of lens L_1 was 50 mm, which exceeds the diameter $\sqrt{2}(d + 2f_1 \tan \theta) \simeq 25$ mm (where $\sin \theta = \lambda N/2d$ and N is either N_1 or N_2) needed to avoid vignetting. The other lenses need also be only $f/20$. The focal length, f_4, of lens L_4 was 1000 mm, which resulted in a demagnification of the conventional image by a factor of 3 while still allowing more than adequate sampling by the detector.

The data were detected by a Fairchild CCD 3000F television camera with a fiber-optic faceplate. The RS170 video signal was converted to a 512 × 512, 8-bit digital image by using an Imaging Technology IP-512 video processor. For both the Fourier intensity and the image data, a single video frame of data was digitized, and a second dark frame was digitized and subtracted to remove pattern noise. The automatic gain control of the camera was disabled and the laser output was adjusted so that the brightest speckle nearly saturated the detector. Since the digitizer sampling rate was not matched to the detector pixel horizontal spacing, a Matthey MLW 401B low-pass video filter with a 3-dB width of 4.3 MHz was used to reduce the effect of CCD clock noise on the digitized data.

The support constraint was measured by increasing the size of the aperture A, digitizing the resulting high-resolution image of the object, and measuring the base and height of the triangle in the digital image. Since the focal lengths of the lenses and the digitizer pixel horizontal spacing are not known exactly, calibration measurements must also be made to determine the spatial scaling of both the Fourier intensity and the image data. This was done by using an object consisting of two circular apertures separated by a distance about four times their diameter. By orienting this object both horizontally and vertically, gathering both the Fourier intensity data and the high-resolution image data, and using the known digitizer pixel vertical spacing (which is equal to the detector pixel spacing of 18 μm), the spatial scaling of the data may be computed. For reference, the digitizer pixel horizontal spacing was determined to be 21.2 ± 0.1 μm, yielding 256 samples over a 5.4-mm width of the detector.

To estimate the signal-to-noise ratio of the data, the detector was uniformly illuminated with an extended noncoherent source, 10 frames were digitized, and the standard deviation of the digitized values was computed on a pixel-by-pixel basis. The standard deviations of the 8-bit data ranged from 0.8 to 1.3. (Part of this variation could be due to variations in light-source intensity during the collection of the 10 frames.) For signals that nearly saturate the detector, the signal-to-detector-noise ratio of the digital data is therefore about 200:1. Data for correction of spatial variations in the detector response were collected by uniformly illuminating the detector at one tenth the saturation light level, summing 10 digitized frames, and subtracting 10 dark frames. The ratio of the standard deviation to the spatial mean of the response was about 3%.

Data processing began with the 256 × 256 array of digitized Fourier intensity data shown in Fig. 2(b). This array was divided by the response data to correct for spatial variations in detector response. The speckle contrast was then measured and found to be 80%. Since the speckle should have 100% contrast, it was assumed that some positive bias must be present in the data, possibly owing to the effect of the low-pass video filter. Therefore, a constant was subtracted from the data to increase the contrast to 90%. Data values that become negative in this process were set to zero. The Fourier transform of these data is the autocorrelation of the image. Since the image support is known, the autocorrelation support is also known. Therefore, to reduce high-frequency noise in the Fourier intensity data, it was low-pass filtered by zeroing out those parts of its Fourier transform that lay outside the support of the autocorrelation of the known triangular image support. After setting the negative values to zero, we calculated the square root of this filtered Fourier intensity data to get the Fourier modulus data.

The Fourier modulus data and the triangular image support constraint were used in the iterative Fourier-transform phase-retrieval algorithm.[2–5] Several cycles (each of 30–40 iterations of hybrid input–output with $\beta = 0.7$ followed by 10–20 error-reduction iterations) were performed for a total of 840 iterations. At this point the algorithm was making no further progress. The normalized root-mean-squared image-domain error, E_0,[2–5] was 0.112; i.e., the reconstructed image was within 11% of agreeing with the measured Fourier modulus data and the support constraint.

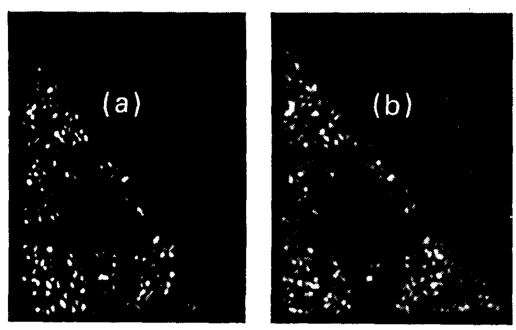


Fig. 3. (a) Intensity of coherent image reconstructed by phase retrieval. (b) Intensity of conventional coherent image.

This is an indication of the amount of noise and distortion present in the measured data.

The squared magnitude of the complex-valued reconstructed image is shown in Fig. 3(a), and a conventional image having the same spatial-frequency bandwidth is shown in Fig. 3(b). The reconstructed image was resampled to match approximately the dimensions of the conventional image. Some of the bright speckles appear in the same location in each image, indicating that some of the high-frequency information, as well as the low-frequency information, has been successfully reconstructed. Because several minutes elapsed between collection of the Fourier intensity data and collection of the conventional image, some of the difference between the two images may be attributable to speckle boiling during the time interval.

Phase retrieval from Fourier intensity data from a coherently illuminated, diffuse object collected in a laboratory experiment has been demonstrated. The image-domain constraint for the iterative Fourier-transform reconstruction algorithm was a known triangular support. The reconstructed image shows good agreement with a conventionally obtained image. With this promising beginning, further laboratory experiments are warranted. Image-reconstruction accuracy should be investigated as a function of, for example, object support shape, sharpness of the edges of support, object contrast, object surface roughness, detected data signal-to-noise ratio, data sampling rate, detector calibration, data filtering, and iterative algorithm versions used.

This research was supported by the Defense Advanced Research Projects Agency (DARPA) under U.S. Air Force Wright Aeronautical Laboratories contract F33615-83-C-1046, DARPA order 5205. The authors wish to thank S. L. Crippen, J. D. Gorman, and L. S. Redding for software development, which made the data collection and processing possible. Portions of this research were presented at the Society of Photo-Optical Instrumentation Engineers Meeting on Digital Image Recovery and Synthesis in San Diego, California, August 17–18, 1987.

References

1. H. A. Ferwerda, in *Inverse Source Problems in Optics*, H. P. Baltes, ed. (Springer-Verlag, New York, 1978), Chap. 2.
2. J. R. Fienup, Opt. Lett. **3**, 27 (1978).
3. J. R. Fienup, Appl. Opt. **21**, 2758 (1982).
4. J. R. Fienup and C. C. Wackerman, J. Opt. Soc. Am. A **3**, 1897 (1986).
5. J. R. Fienup, J. Opt. Soc. Am. A **4**, 118 (1987).
6. G. B. Feldkamp and J. R. Fienup, Proc. Soc. Photo-Opt. Instrum. Eng. **231**, 84 (1980).
7. J. N. Cederquist and C. C. Wackerman, J. Opt. Soc. Am. A **4**, 1788 (1987).