Image synthesis from nonimaged laser-speckle patterns

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Received June 8, 1987; accepted August 14, 1987

We demonstrate that unspeckled images of coherently illuminated, diffuse objects can be formed from measurements of backscattered laser-speckle intensity. The theoretical basis for this imaging technique is outlined, and results of computer experiments that successfully construct images from digitally simulated laser-speckle measurements are presented.

It is well known that the spatial structure of a fully developed laser-speckle pattern—produced by the coherent interference of light backscattered from a sufficiently diffuse, reflecting surface—is dependent on the macroscopic features of the illuminated surface.[1] In this Letter we demonstrate that measurements of the backscattered speckle intensity are sufficient to (uniquely) construct a high-resolution, unspeckled, incoherent image (or brightness distribution) of the coherently illuminated object.

Our approach to image synthesis is based on the fact that from the average energy spectrum of a laser-speckle intensity pattern one can obtain the autocorrelation function of the illuminated object's brightness distribution.[2] Here, the object's brightness distribution corresponds to the object's reflectance function or, alternatively, to its irradiance distribution had the object been illuminated by an incoherent light source. Since the Fourier transform of the autocorrelation of the object brightness function is equivalent to the squared modulus of the Fourier transform of the brightness function,[3] an image of the object can be obtained if the phase associated with this Fourier transform can be determined. Fortunately, a practical solution to this phase-retrieval problem has been demonstrated by Fienup,[4–6] in which an iterative transform algorithm can be used to recover the phase associated with the modulus of the Fourier transform of a real, nonnegative object function, provided that certain boundedness and nonnegativity constraints are continually reinforced throughout the iteration process. The iterative transform algorithm, together with certain digital preprocessing operations (which are described below) permit us to recover complete, unspeckled images from nonimaged speckle data.

Let us suppose that a diffuse object is flood illuminated with a laser whose coherence length is at least twice as long as the object is deep. An array of photodetectors measures the backscattered light intensity in a far-field plane some distance from the object. We assume that the object is optically rough, so that its microscale surface height variations are random and comparable in size with the wavelength of light. This being the case, the reflected laser light is randomly (and coherently) dephased, and the photodetectors in the observation plane record a fully developed laser-speckle pattern.

Each realization of the observed speckle intensity $I_n(u)$ may be expressed as the squared modulus of the Fourier transform of the complex object field:

$$I_n(u) = |F_n(u)|^2 = |\mathcal{F}\{f_n(x)\}|^2,$$

where $\mathcal{F}$ denotes a Fourier transform, $f_n(x) = |f_0(x)| \exp[i\phi_n(x)]$ is the field reflected by the object, $|f_0(x)|$ is the object's field amplitude reflectivity, and $\phi_n(x)$ is the (random) phase of the nth realization of the reflected object field associated with the object's surface height profile. In the above expression, x represents a two-dimensional spatial (or angular) coordinate vector in object space; u represents a two-dimensional coordinate in the measurement plane. The inverse Fourier transform of the observed speckle pattern is proportional to the autocorrelation of the object field, which may be written as

$$\begin{aligned} r_n(x) &= \mathcal{F}^{-1}\{|F_n(u)|^2 H(u)\} \\ &= [f_n(x) \star f_n(x)] * h(x), \end{aligned}$$

where $\mathcal{F}^{-1}$ denotes an inverse Fourier transform, $*$ denotes a convolution operation, and $\star$ denotes an autocorrelation. The aperture function $H(u)$ denotes the region of the measurement plane over which the speckle pattern is observed: $H(u) = 1$ for points within the measurement aperture, and $H(u) = 0$ elsewhere. The function $h(x) = \mathcal{F}^{-1}\{H(u)\}$ is the (diffraction-limited) coherent impulse response; hence $r_n(x)$ is a diffraction-limited (albeit speckled) autocorrelation of the laser-illuminated object.

Using the iterative transform algorithm, one could attempt to reconstruct a complex-valued, speckled image of $f_n(x)$ from $|F_n(u)|^2 H(u)$ or equivalently from $r_n(x)$. However, at present the practical reconstruction algorithm is effective only for certain classes of complex-valued objects if the object's support is known *a priori*[7] and for even more restrictive classes of complex-valued objects if the object support is un-

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known. (The support is the closed set of points outside which the object is zero.) For this reason we concentrate, in this Letter, on a method that allows us to reconstruct a real, nonnegative image—a case for which the iterative transform algorithm is effective for a broad class of objects.

Image recovery begins by estimating the average energy spectrum of the observed speckle pattern by averaging together the squared moduli of many independent speckled autocorrelations $r_n(x)$. This may be referred to as noncoherent averaging of the coherent autocorrelations. Independent realizations of the speckle pattern can be obtained, for example, by laterally displacing the observation plane with respect to the object or by measuring the speckle pattern for slightly different rotations of the object. One can show that as the number N of independent speckle realizations increases, the computed average energy spectrum converges to[8]

$$\lim_{N\to\infty} N^{-1} \sum_{n=1}^{N} |r_n(x)|^2 = b|h(x)|^2 + cr_0(x) * |h(x)|^2,$$

where

$$b = c\left[\int |f_0(x')|^2 \mathrm{d}^2x'\right]^2$$

is the square of the total measured irradiance,

$$r_0(x) \triangleq |f_0(x)|^2 \star |f_0(x)|^2$$

is the autocorrelation of the object brightness function, and c is a constant. Thus the average energy spectrum converges to the sum of an autocorrelation of the desired incoherent image plus a dc term $b|h(x)|^2$, where the dc term is simply the (incoherent) point-spread function of the collecting aperture, possessing a strength b. On subtracting the dc term from the averaged energy spectrum, we obtain a diffraction-limited autocorrelation of the incoherent object. The square root of the Fourier transform of this incoherent object autocorrelation, then, provides us with an estimate of the modulus of the Fourier transform of the object's brightness function. Note that one can obtain the same results by subtracting a bias from an average of the autocorrelations of $I_n(u)$ and then taking the square root. One can see that the latter approach is analogous to a highly redundant, multichannel intensity interferometer.[9]

We conducted a series of computer experiments to demonstrate that phase retrieval can be used to recover imagery from speckle data processed in this way. Original object data for these experiments were contained in a digitized photograph of a satellite model illuminated with incoherent light. These data comprised approximately 40 × 60 pixels embedded in a 128 × 128 discrete array. Each realization of a coherent (speckled) image of the object was obtained from the digitized photograph by (1) replacing each pixel with a circular-complex Gaussian random variable whose real and imaginary parts possessed variances equal to half of the pixel intensity value and (2) low-pass filtering the result. The filter used to smooth the complex object data corresponds to the aperture function $H(u)$, which was represented by a 64 × 64 square of detector pixels embedded in 128 × 128 array. Multiple realizations of the coherent object data were obtained by using different random-number seeds in the computation of the complex Gaussian random variables. Each coherent image autocorrelation $r_n(x)$ was computed by inverse Fourier transforming the squared modulus of the apertured Fourier transform of the simulated coherent image. Averages of both the speckled autocorrelations and their squared moduli (i.e., the energy spectrum of the speckle-intensity patterns) were then taken. A function proportional to the square of the former, an estimate of the dc term, was subtracted from the latter (the noncoherent average) to arrive at an estimate for the autocorrelation of the incoherent image.

The process of noncoherently averaging object-field autocorrelations and subtracting the dc term is illustrated in Fig. 1. The first column contains averages of the squared inverse Fourier transforms of N simulated speckle measurements providing estimates of the speckle energy spectrum, where N is the number of independent speckled autocorrelations noncoherently averaged. The second column shows the corresponding dc term, which, for the case of a square aperture, is a squared sinc$[(\pi x)^{-1} \sin(\pi x)]$ function. The third column shows the results when the dc term is subtracted from the noncoherently averaged autocorrelations of the first column. Note that the speckle artifacts in the averaged autocorrelations (in the first and third columns of Fig. 1) disappear as N increases.

The incoherent autocorrelation estimate (with the dc term removed) was then Fourier transformed, and the square root was taken, to arrive at an estimate of

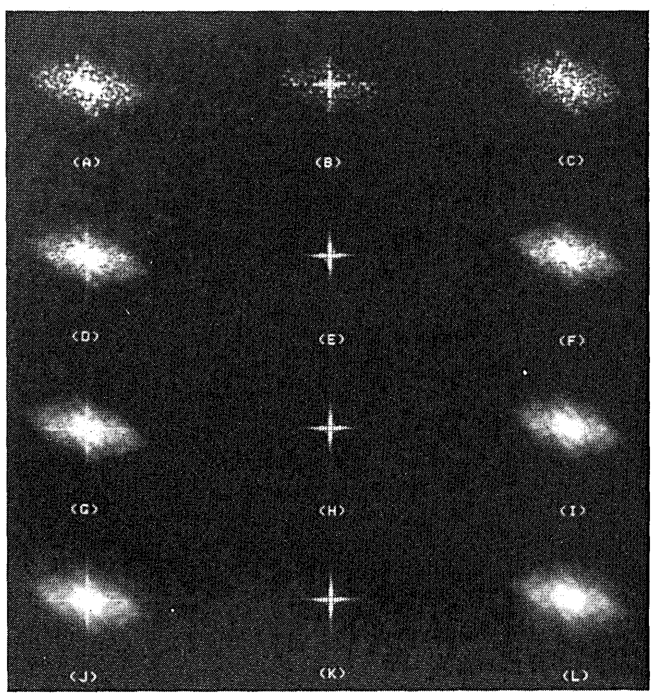

Fig. 1. Estimating the energy spectrum of speckle intensity by noncoherently averaging many coherent speckled image autocorrelations. (A) Noncoherent average of $N = 4$ autocorrelations; (B) estimate of dc term; (C), (A) minus (B); (D)–(F) $N = 32$; (G)–(I) $N = 128$; (J)–(L) $N = 1024$.

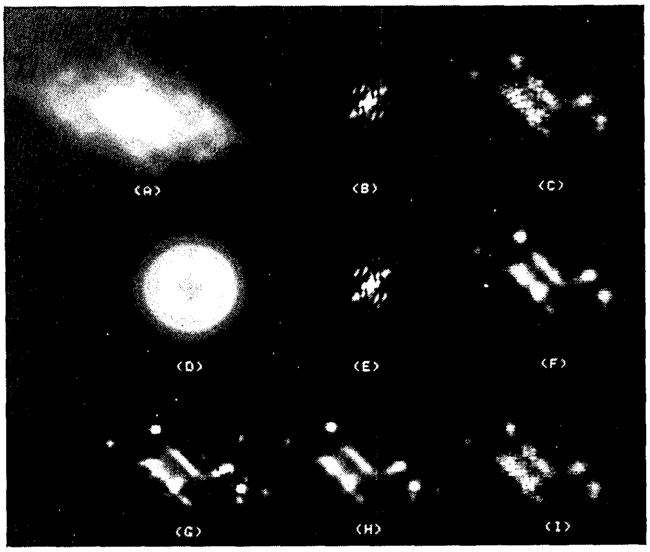

Fig. 2. Image recovery from noncoherently average autocorrelation data (N = 10,000): (A) dc-adjusted, noncoherently averaged autocorrelations, (B) estimate of the Fourier modulus of the incoherent object, (C) image reconstructed from (B) using the iterative transform (phase-retrieval) algorithm, (D) Wiener filter, (E) filtered Fourier modulus estimate, (F) image reconstructed from (E), (G) original incoherent object, (H) Wiener filtered, incoherent object, (I) result of Wiener filtering (C).

the modulus of the Fourier transform of the object brightness function. Negative numbers, resulting from noise associated with the finite-average approximation to an ensemble average, were set to zero before the square root was taken. Images were reconstructed from the Fourier modulus estimates by using the iterative Fourier-transform algorithm,[5,6] using several cycles of the hybrid input–output algorithm (using β = 0.7) and the error-reduction algorithm until the algorithm appeared to stagnate. The object-domain constraints used were nonnegativity (since an incoherent image is being reconstructed) and a loose support constraint (a rectangle half the size of the smallest rectangle enclosing the autocorrelation).

Data along the first row of Fig. 2 illustrate a direct application of the phase-retrieval algorithm to the Fourier modulus estimate. Figure 2(A) represents the dc-subtracted autocorrelation for $N = 10^4$ independent speckle patterns. Figure 2(B) shows the corresponding Fourier modulus data produced by Fourier transforming the averaged autocorrelation [Fig. 2(A)] and then taking the square root. Figure 2(C) is the reconstructed image produced by applying the phase-retrieval algorithm as outlined above. Note that this image is very noisy compared with the original incoherent object, shown in Fig. 2(G). Noise in the reconstructed image is due to the fact that a finite number of speckle realizations were used to estimate the Fourier modulus. To reduce these noise effects, we multiplied the Fourier modulus estimate [Fig. 2(B)] by a Wiener filter of the form

$$W(u) = \frac{\mathrm{OTF}(u)E_s(u)}{|\mathrm{OTF}(u)|^2E_s(u) + E_n},$$

where $\mathrm{OTF}(u) = H(u) \star H(u)$ is the optical transfer function of the receiver aperture, $E_s(u)$ is an average energy spectrum for objects of this type (estimated by taking an angular average over the squared Fourier modulus of the object), and E_n is the energy spectrum of the noise. We approximated E_n by a constant whose value was obtained by averaging the squared Fourier modulus estimate over those higher spatial frequencies where the signal-to-noise ratio was less than one. Figure 2(D) shows the Wiener filter used for this example.

Figure 2(E) shows the filtered Fourier modulus estimate equal to the product of Figs. 2(B) and 2(D). Figure 2(F) shows the image reconstructed from the Wiener-filtered Fourier modulus estimate using the phase-retrieval algorithm. Note that the Wiener filter has significantly improved the quality of the reconstructed image in Fig. 2(F) over that in Fig. 2(C) reconstructed without Wiener filtering. For comparison, the original object [shown in Fig. 2(G)] was passed through the Wiener filter of Fig. 2(D), with the result shown in Fig. 2(H). The image reconstructed from speckle-correlation measurements, shown in Fig. 2(F), compares favorably with the filtered object [Fig. 2(H)], indicating good performance on the part of the iterative transform reconstruction algorithm. Finally, Fig. 2(I) shows the result of applying the Wiener filter to the reconstructed image shown in Fig. 2(C). Apparently, Wiener filtering followed by image reconstruction is superior to image reconstruction followed by Wiener filtering.

These results demonstrate the possibility of recovering images from nonimaged laser speckle patterns: by averaging over many realizations of the coherent (speckle) intensity data, an estimate of the autocorrelation and Fourier modulus of the incoherent object can be obtained. And, from the Fourier modulus estimate, it is possible to reconstruct an unspeckled image by applying a phase-retrieval algorithm with a nonnegativity constraint.

This research was supported by the U.S. Air Force Weapons Laboratory, partially through U.S. Air Force Wright Aeronautical Laboratory contract F33615-83-C-1046.

References

1. J. W. Goodman, "Statistical properties of laser speckle patterns," in *Laser Speckle and Related Phenomena*, 2nd ed., J. C. Dainty, ed. (Springer-Verlag, New York, 1984), pp. 9–75.
2. L. I. Goldfischer, J. Opt. Soc. Am. **55**, 247 (1965).
3. R. N. Bracewell, *The Fourier Transform and Its Applications* (McGraw-Hill, San Francisco, 1978), p. 115.
4. J. R. Fienup, Opt. Lett. **3**, 27 (1978).
5. J. R. Fienup, Appl. Opt. **21**, 2758 (1982).
6. J. R. Fienup and C. C. Wackerman, J. Opt. Soc. Am. A **3**, 1897 (1986).
7. J. R. Fienup, J. Opt. Soc. Am. A **4**, 118 (1987).
8. S. Lowenthal and H. Arsenault, J. Opt. Soc. Am. **60**, 1478 (1970).
9. R. Hanbury Brown, *The Intensity Interferometer* (Taylor and Francis, London, 1974).