Synthetic-aperture radar autofocus by maximizing sharpness

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To focus a synthetic-aperture radar image that is suffering from phase errors, a phase-error estimate is found that, when it is applied, maximizes the sharpness of the image. Closed-form expressions are derived for the gradients of a sharpness metric with respect to phase-error parameters, including both a point-by-point (nonparametric) phase function and coefficients of a polynomial expansion. Use of these expressions allows for a highly efficient gradient-search algorithm for high-order phase errors. The effectiveness of the algorithm is demonstrated with an example. © 2000 Optical Society of America

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The maximization of image sharpness was originally developed to correct phase errors in incoherent optical imagery.[1] Paxman and Marron[2] showed that similar approaches could be used for speckled, coherent imagery, including synthetic-aperture radar (SAR). However, use of sharpness maximization on coherent imagery has been computationally inefficient and restricted to lower-order phase errors.[2,3] In this Letter a computationally efficient implementation of image sharpening is described that uses closed-form expressions for the gradient of the sharpness with respect to the phase-error parameters. Nonlinear optimization algorithms that use the gradient expressions can efficiently maximize the sharpness for arbitrarily high-order phase errors.

Suppose that we have a SAR image degraded by a spatially invariant phase error, $\phi_e(v)$. We model the measured range-compressed SAR signal, degraded by phase errors, as

$$G_d(x,v) = F(x,v)\exp[i\phi_e(v)], \qquad (1)$$

where $F(x,v)$ is the ideal SAR range-compressed signal history without phase errors, x is the range coordinate, and v is the pulse number (or slow-time coordinate). If there is significant motion through range bins during the integration time, as commonly occurs for inverse SAR's, then we assume that the pulses have already been range aligned.[4] The inverse Fourier transform of $G_d(x, v)$ in the slow-time dimension gives the complex-valued SAR image, $g_d(x, y)$, where y is the azimuth (or cross-range) coordinate. Because the phase error is only in the v direction, the image is smeared only in the azimuth direction.

For the case of imaging a ground scene with SAR platform motion errors, it is possible to determine the phase error for the entire image from one or more patches of the image. One would choose the more favorable (higher-contrast) areas. This would also allow for a more efficient computation of the phase error. We may, optionally, break up the SAR image into K patches, $g_{dk}(x, y)$, each of length N pixels in azimuth. We Fourier transform each in the azimuth dimension to arrive at their degraded range-compressed signal histories:

$$
\begin{aligned}
G_{dk}(x,v) &= \mathcal{F}[g_{dk}(x,y)] \\
&= \sum_{y=0}^{N-1} g_{dk}(x,y)\exp(-i2\pi vy/N). \qquad (2)
\end{aligned}
$$

We assume that each patch is blurred by the same one-dimensional phase error:

$$G_{dk}(x,v) = F_k(x,v)\exp[i\phi_e(v)], \qquad (3)$$

where F_k is the ideal range-compressed signal history (without phase errors) for patch k and ϕ_e is the phase error. If we estimate the phase error to be $\phi(v)$, then our estimates of the signal histories are

$$G_k(x,v) = G_{dk}(x,v)\exp[-i\phi(v)], \qquad (4)$$

and our corrected image patches are

$$g_k(x,y) = \mathcal{F}^{-1}[G_k(x,v)]. \qquad (5)$$

The goal is to form an accurate estimate, $\phi(v)$, of $\phi_e(v)$ and compute a corrected image. We accomplish this by seeking a value of $\phi(v)$ that maximizes the sharpness of the image, using the sharpness metric

$$
\begin{aligned}
S_1 &= \sum_k \sum_{x,y} w_k(x,y)\,|g_k(x,y)|^4 \\
&= \sum_k \sum_{x,y} w_k(x,y)\,[|g_k(x,y)|^2]^2, \qquad (6)
\end{aligned}
$$

where $w_k(x,y)$ is an optional weighting function that allows one to place more weight on different patches and on different areas within a patch, the summation over k is for $k = 1, \ldots, K$, and the summation over (x, y) is for all values of (x, y) within each image patch. The summation over k allows us to optimize simultaneously over all the patches jointly. Because $|g_k(x,y)|^2$ is the intensity of the image, this metric is a generalization of the classic Muller–Buffington squared-intensity sharpness metric.[1,2] Alternatively, one can choose to optimize over one patch at a time ($K = 1$), average the phase estimates or compute the principal component among the different phase-error estimates, and employ that net phase-error estimate

for the entire image. One may also optimize over the entire image ($K = 1$).

In some instances in coherent imaging the phase errors are two dimensional. For example, because phase errors are proportional to instantaneous frequency, when the product of the SAR fractional bandwidth and the size of the phase error is sufficiently large it is not accurate to represent the phase as a simple one-dimensional function. The algorithm can be adapted to this circumstance, and the Fourier transforms must then be two dimensional instead of one dimensional.

A variety of options are available for the weighting function. For example, one might choose patches or a weighting function within a patch to emphasize bright, high-contrast areas in the scene or to deemphasize or eliminate undesirable areas such as water, low-return areas, and wind-blown trees. An example of a useful weighting function is

$$w_k(x) = \frac{w_{ko}}{\left[\sum_{y'} |g_k(x, y')|^2\right]^2}, \qquad (7)$$

where w_{ko} is an optional weight for each patch. This weighting function has the same effect as normalizing the image, so each range bin has the same energy (sum of intensities) as the others, making each range bin within a patch weighted equally rather than weighting bright range bins more heavily, thereby preventing a single range bin from dominating the estimation of the phase error.

Using a normalized version of the sharpness metric,

$$S_{1n} = \frac{\sum_k \sum_{x,y} w_k(x, y) |g_k(x, y)|^4}{\left[\sum_k \sum_{x,y} w_k(x, y) |g_k(x, y)|^2\right]^2}, \qquad (8)$$

is advantageous because it is unitless and independent of any multiplicative constants. It is proportional to S_1 because the denominator, the square of the total (weighted) image energy, is independent of the phase error; consequently, when optimizing, one need not recompute the denominator. This will hold for any weighting functions that are functions of range only.

We can maximize the sharpness by using a standard gradient-search technique. Applying an approach that was previously used for phase-retrieval problems,[5] we make the computation of the gradient highly efficient by employing an expression for the gradient, which we derive as follows: If we treat each pixel of $\phi(v)$ as an independent parameter:

$$\frac{\partial S_1}{\partial \phi(v)} = 2 \sum_k \sum_{x,y} w_k(x, y)$$

$$\times |g_k(x, y)|^2 \frac{\partial |g_k(x, y)|^2}{\partial \phi(v)}. \qquad (9)$$

Using

$$\frac{\partial |g_k(x, y)|^2}{\partial \phi(v)} = -iN^{-1} g_k^*(x, y) G_k(x, v)$$

$$\times \exp(i2\pi vy/N) + \text{c.c.}, \qquad (10)$$

where c.c. denotes the complex conjugate of the term that precedes it, we find that

$$\frac{\partial S_1}{\partial \phi(v)} = 4N^{-1} \sum_k \sum_{x,y} w_k(x, y)$$

$$\times \text{Im}(G_k(x, v)\{\mathcal{F}[g_k(x, y) |g_k(x, y)|^2]\}^*), \qquad (11)$$

where the Fourier transform is in the y dimension.

So far we have described a point-by-point (or nonparametric) phase function estimation. It is also possible to compute gradients with respect to the coefficients of a polynomial-type expansion of $\phi(v)$, in the manner shown in Ref. 5. If we let

$$\phi(v) = \sum_{j=1}^{J} a_j L_j(v), \qquad (12)$$

where $L_j(v)$ is some set of basis functions (Legendre, Fourier, etc.), then we can optimize over the coefficients a_j, where the partial derivatives are

$$\frac{\partial S_1}{\partial a_j} = \sum_v L_j(v) \frac{\partial S_1}{\partial \phi(v)}, \qquad (13)$$

which involves the computation of the nonparametric gradient followed by a projection onto the basis set. This parameterization is particularly useful when the phase error is known to be a low-order polynomial, which is often the case. Then we can make the number of unknown parameters roughly match the true dimensionality of the phase function, thereby avoiding overparameterizing the problem. One could further add to the sharpness metric a penalty function of the form $\sum_j \alpha_j a_j^2$ to encourage the solution to go in the direction (as determined by the values of α_j) of some previous distribution of the expected values of a_j^2. For example, in many circumstances the quadratic phase term is by far the largest, and the magnitudes of the higher-order terms tend to decrease with increasing order j. The penalty function can be viewed as a form of regularization.

S_1 would be maximized by a nonlinear optimization algorithm that employed the objective function expression in Eq. (6) or (8) together with the one of the gradient expressions above. An example of software to perform such an optimization would be Matlab's fminu (quasi-Newton) nonlinear optimization routine. This and other off-the-shelf nonlinear optimizers, such as that of Levenberg and Marquardt, can be used if N or J is not too large. For large N or J, a purely gradient approach, such as the method of conjugate gradients, would be more efficient.

The major expense of computing S_1 is the fast-Fourier transform needed to compute the $g_k(x, y)$ from the $G_k(x, v)$. The major expense of computing the nonparametric gradient of S_1 is the fast Fourier transform
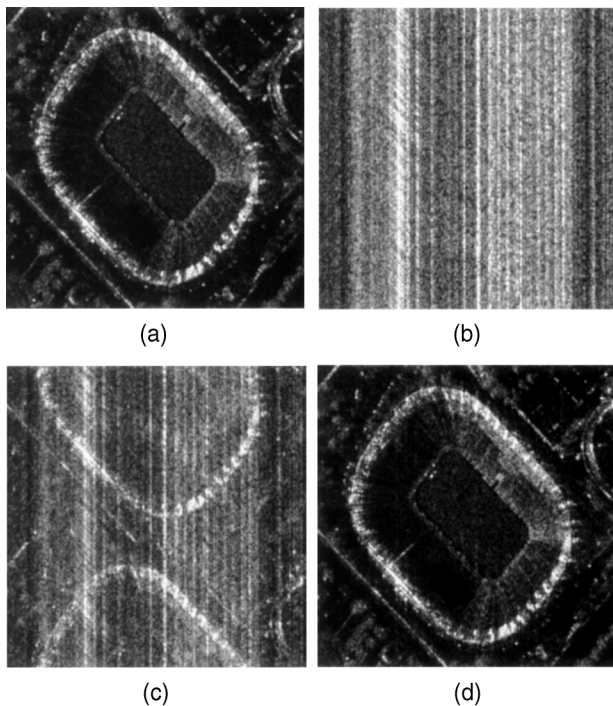
Fig. 1. Simulation example. (a) Original image, (b) image smeared in the azimuth (vertical) direction by random phase errors, (c) image after 50 iterations, (d) final focused image, with recentering.
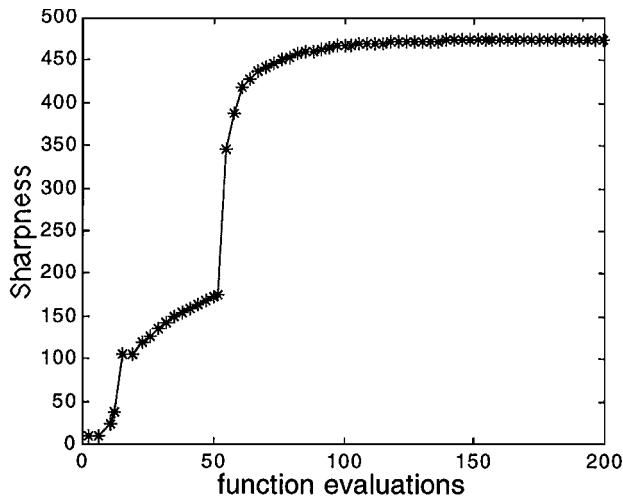

Fig. 2. Sharpness as a function of number of interations (sharpness evaluations).

needed to compute $g_k(x, y)$ and $\mathcal{F}[g_k(x, y)|g_k(x, y)|^2]$. The computational expense of the gradient is comparable with that required for computing the image.

Paxman and Marron[2] found that there can be a false maximum for a phase-error estimate $\phi(v) = -\arg[G_{dk}(x, v)]$, effectively making the phase of $G_k(x, v)$ constant for some dominant range bin x. A weighting function such as that in Eq. (7) would reduce the chance of such a false maximum because it would prevent any single range bin from dominating the sharpness. Furthermore, if we use image sharpness as a refinement technique after having corrected most of the phase error with another algorithm, then false maxima should not be much of a problem.

Figure 1(a) shows the magnitude of an example SAR image of Michigan Stadium, of size 384×384 pixels, collected by ERIM International's DCS SAR. Figure 1(b) shows the smeared image that resulted after addition of a one-dimensional white-noise random phase error, with a root-mean-squared value of 4 rad, to the phase of the signal history (Fourier transform) of the undegraded image. This image was not subdivided into patches ($K = 1$). Figure 1(c) shows the image partially corrected after 50 iterations (sharpness function evaluations) by optimization of the sharpness over a pixel-by-pixel phase function by use of a conjugate-gradient algorithm. Typically three objective function evaluations are made during the line search for each gradient calculation. Because the sharpness is insensitive to translations caused by linear phase errors, the image is shifted in azimuth. Figure 1(d) shows the final focused image, recentered in azimuth. The normalized sharpness of the reconstructed image was 475.4, compared with the sharpness of the original image, 8.8. Figure 2 shows the sharpness as a function of the number of objective function evaluations during the conjugate-gradient search. At the 50th evaluation the conjugate gradient search was reinitialized (one step of steepest descent was performed). The result changed little after iteration 100. When another popular autofocus algorithm[6] was tried on this example, it improved the image considerably but left substantial residual high-order phase errors.

In conclusion, the new SAR focusing algorithm uses a gradient search technique to maximize the sharpness of the image by employing a computationally efficient expression for the gradient. Unlike previous sharpness-based SAR focusing algorithms, it can work either with a nonparametric (point-by-point) representation of the phase or with the coefficients of a polynomial (or some other basis set) expansion of the phase. It was shown to work well for phase errors of very high order. It should be useful in other imaging systems such as medical ultrasound, MRI, and laser radar as well.

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