New ways to make computer-generated color holograms



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# *MÉMOIRES ORIGINAUX*

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# NEW WAYS TO MAKE COMPUTER-GENERATED COLOR HOLOGRAMS (*)

by

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MOTS CLÉS :
Holographie par ordinateur couleur

KEY WORDS :
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**Nouveaux procédés de synthèse des hologrammes en couleurs par ordinateurs**

RÉSUMÉ : On peut obtenir des images en couleurs à partir de trois hologrammes réalisés par ordinateur, chaque hologramme correspondant à une couleur fondamentale. Deux problèmes doivent être résolus : la mise en place des images en fonction de la longueur d'onde et l'élimination des images parasites. Le premier problème peut être résolu par l'ordinateur pendant la synthèse des hologrammes ou lorsque l'on reconstruit les images. Le second problème, celui des images parasites, peut être résolu soit par la géométrie de la reconstruction, soit par l'utilisation de films couleurs à émulsions multiples. L'absorption sélective des différentes longueurs d'onde par les couches du film en couleur peut être utilisée pour absorber sélectivement une radiation déterminée et transmettre la radiation voulue. Les variations de phase qui existent dans les films en couleurs, particulièrement dans le Kodachrome II, rendent possibles la réalisation de hologrammes « in-line ». De plus, les variations de phase peuvent être utilisées pour éliminer par diffraction en dehors de l'axe les couleurs non désirées. Des résultats expérimentaux sont présentés.

SUMMARY : Color images can be produced from three computer-generated holograms, one for each of the primary colors. Two basic problems must be solved : scaling the images according to wavelength and avoiding false images. The former can be solved by a computational step, during synthesis of he holograms, or during reconstruction. The latter problem, false images, can be solved by the reconstruction geometry or by using multi-emulsion color film. The wavelentgh-selective absorption of the different layers in color film can be used to selectively absorb light of an undesired color while transmitting light of the desired color at a given point on the film. This effect allows the spatial multiplexing of binary detour-phase holograms. In addition to density variations, there exist corresponding phase variations in color film, particularly Kodachrome II, making on-axis holograms possible. Furthermore, the phase variations can be used to eliminate an undesired color by selectively diffracting it off to the sides. Experimental results are presented.

## 1. — *GENERAL*

A computer-generated hologram for producing a color image would actually require three holograms, one each to control images in red, green, and blue, respectively, which are added to make a composite image with arbitrary colors. If the images are reconstructed using the 632.8 nm red line from He-Ne, the 514.5 nm green and 457.9 nm blue lines from and Argon-Ion laser, then all the colors on a C. I. E. chart bounded by the triangle defined by these three wavelengths can be produced (fig. 1). There exist two fundamental problems in the computer generation of holograms that produce color images. First, if a hologram for one color also has two other colors passing through it, there appear, in addition to the desired image, false images. Second, since the angle of diffraction is wavelength dependent, the three desired images will have three different magnifications, so somehow the three holograms must be scaled so that the images come out in their proper size relation. This scaling according to wavelength can be done in a number of different ways. Exact scaling can be accomplished by scaling the hologram transparencies during their synthesis, by changing the scale of the display device, or by changing the magnification when photographing the display ; alternatively, the images can be scaled during reconstruction. Scaling can also be done on the digitized data before Fourier transforming,

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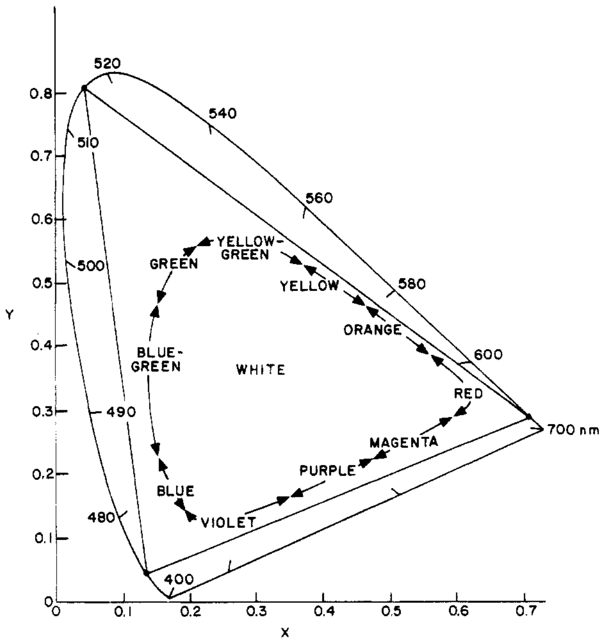


FIG. 1. — *The C. I. E. chromaticity diagram. Any color within the triangle can be obtained by mixing red* (632.8 nm), *green* (514.5 nm), *and blue* (457.9 nm) *light.*

effectively scaling the sampling intervals. This last scaling method can distort the image by up to one half of a sampling interval, as will be demonstrated in section 4; also, the image field sizes remain unchanged, even though the images within them are scaled, so part of the potential image space may go to waste (fig. 2).

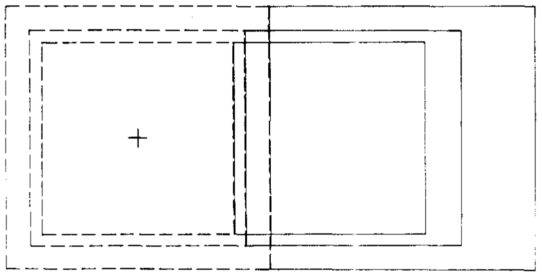

FIG. 2. — *Image fields for red (largest squares), green, and blue (smallest squares) light. Dotted lines and solid lines indicate the image fields for an-axis and first-order reconstructions, respectively.* « + » *denotes the optical axis.*

## 2. — RECONSTRUCTION GEOMETRIES

Scaling can be accomplished by the readout geometry shown in figure 3. Putting Fourier-transform holograms after, rather than before, their transforming lenses does not affect the intensity distribution in the focal plane except to change the magnification [1]. The image size in this case is proportional to *d*, the distance from the hologram to the image plane. Appropriate scaling may be attained by choosing the distances according to the wavelengths. Furthermore, no false images occur here since each hologram is illuminated only by the color appropriate for its image.

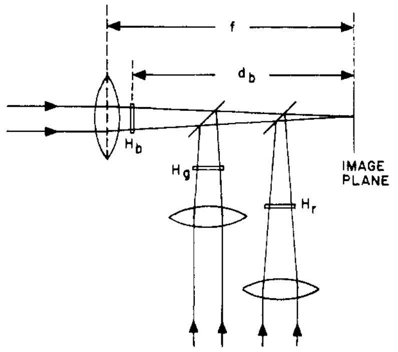


FIG. 3. — *Reconstruction geometry allowing scaling. $H_b$, $H_g$, and $H_r$ are holograms illuminated by blue, green, and red light, respectively.*

Other complicated reconstruction geometries based on spatial frequency multiplexing have also been devised [2].

Figure 4 shows a much simpler set-up in which all three plane waves have a common axis. This is particularly desirable if the same laser is used to produce more than one color. In addition, all three holograms can be made as a unit. Three color filters may be inserted (or some other optical means may be used) so that each hologram is illuminated only by light of the correct color, thus avoiding false images. Due to the shift invariance of this Fourier transform geometry, all three images will superimpose correctly, provided that the holograms have been appropriately scaled in a previous step.

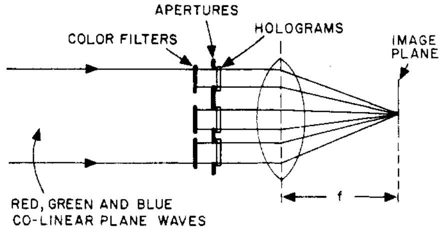


FIG. 4. — *Reconstruction geometry using three co-linear beams.*

The most desirable situation would be to have the set-up of figure 4 without needing color filters to avoid false images. The extra degrees of freedom in multilayered color film allow us to realize such a situation.

## 3. — DETOUR-PHASE HOLOGRAMS

One method of eliminating false images would be to make three binary detour-phase holograms [3] on color film, each exposed through a color filter appropriate to the color of its image. False images are avoided because only light of the desired color is transmitted by each hologram : the undesired colors are absorbed. The three holograms can be multiplexed by interlacing rows of coefficients, but this requires three times the display capability that would be required for only one color. Attempting to reduce this extra display requirement, one might simply overlay cells of different colors, as shown in figure 5.

Unfortunately, at least for Kodachrome II, the film will be thinner at places where the two apertures overlap, resulting in undesired phase variations. Thus it would be desirable to design the hologram in such a manner that apertures for different colors do not overlap.

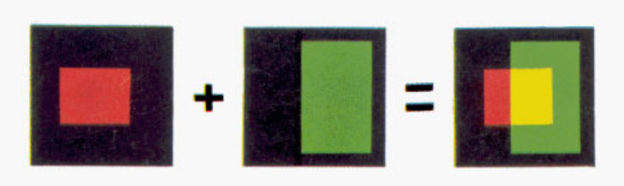

FIG. 5. — *Overlaying corresponding Lohmann-type cells of different colors. Although the equation holds for intensity transmittance, it does not hold for the complex transmittance, due to a phase error in the region of overlap, where the film is thinner.*

As demonstrated by Haskell [4], the opening of a combination of small binary apertures, or « subcells » within a cell can be used to represent a Fourier coefficient. If one uses, as an example, only a combination of five subcells along one line for each Fourier coefficient, then at a certain off-axis angle to the hologram, light coming from each of these subcells will be 1/5 wavelength out of phase with its neighbor. Any one of these apertures will contribute a unit phasor (fig. 6). By adding different combinations of these five unit phasors we can address 31 points in the complex plane. There is enough unused space in the cell to multiplex-in two more colors, without overlap and without requiring three times the display capability.

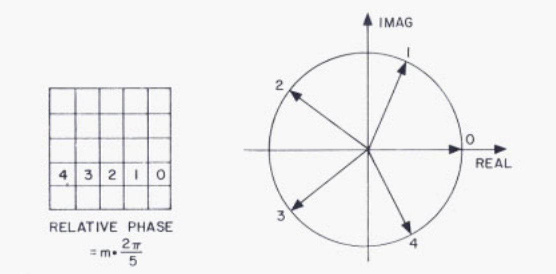


FIG. 6. — *Each open subcell in a binary detour-phase hologram contributes a unit phasor to the complex Fourier coefficient. If there are M subcells per coefficient, then each phasor is one distinct Mth root of unity.*

For multiplexed holograms, scaling is done on the digitized object. Figure 7 shows a portion of such a multiplexed hologram, which has a cell of five subcells, 40 × 50 coefficients, and was replicated four times to give the image shown in figure 8.

Two problems plagued the detour-phase holograms : their low efficiency (theoretically 10.3 % maximum, but usually 1 % or less in practice) and the large number of resolvable elements of the display device needed to encode a single Fourier coefficient. Both of these problems are solved by on-axis holograms [5].

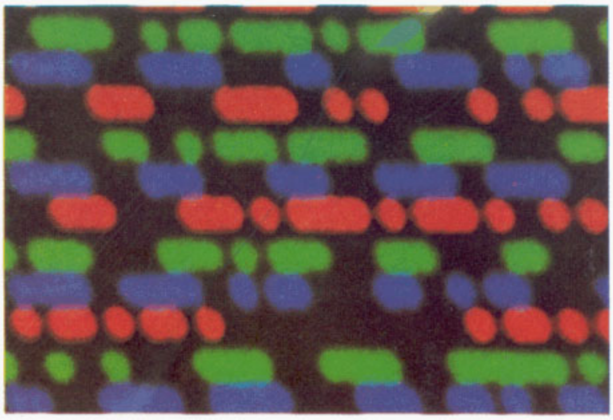

FIG. 7. — *A portion of a multiplexed binary detour-phase hologram for a color image. Since the different colors to not overlap no undesirable phase variations result.*

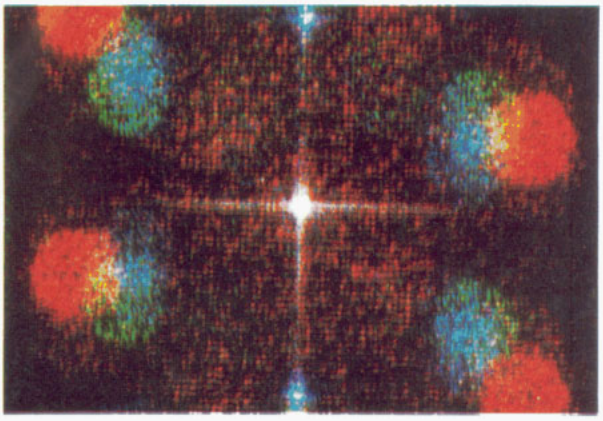

FIG. 8. — *Image from multiplexed binary detour-phase hologram. The desired and conjugate images are in the upper right and lower left corners, respectively. Also seen are zero order and other spurious images.*

## 4. — ON-AXIS HOLOGRAMS

The referenceless on-axis complex hologram [6] (ROACH) uses multi-emulsion film, such as Kodachrome II, in which different layers can be exposed independently by light of different colors. Upon reconstruction with a given color light, one layer of the film will absorb, while the other layers will be predominantly transparent, but will cause phase shifts due to variations in film thickness and refractive index (see fig. 9). Since the complex (both amplitude and phase)

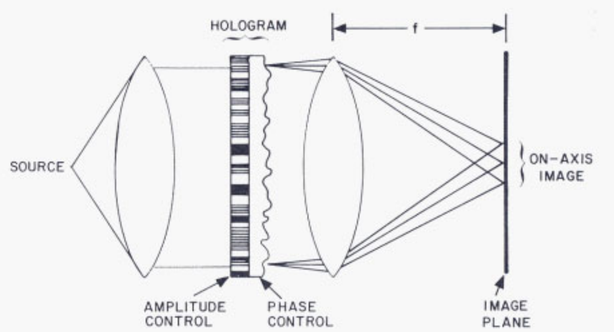


FIG. 9. — *Referenceless On-Axis Complex Hologram* (ROACH). *Phase and amplitude are controlled by different layers within the film.*

transmittance can be controlled at each point, only one resolvable element is needed to encode one complex Fourier coefficient. There are no conjugate images or central-order terms, so the efficiency is high, up to 100 % for a « perfectly diffused » object.

Figure 10 shows three ROACHs needed to produce a colored image (fig. 11). Scaling for the three wavelengths was done using the $x$ and $y$ gain controls on a CRT, our display device, giving three different sizes of

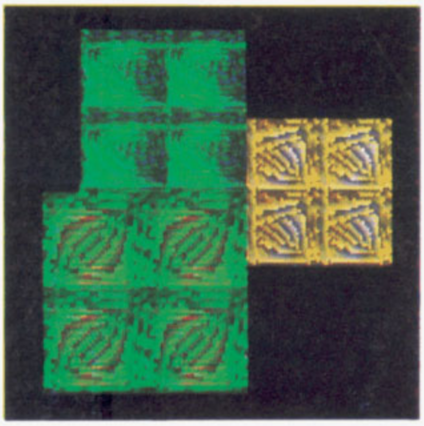

FIG. 10. — *Three* ROACHs *for red, green, and blue light, scaled according to wavelength.*

holograms. In the largest ROACH, for the red image, the red-absorbing layer controls the amplitude and both blue and green-absorbing layers control the phase. In the smaller ROACH for the green image, the green-absorbing layer controls the amplitude and the blue-absorbing layer controls the phase. In the still smaller ROACH for the blue image, the blue-absorbing layer controls the amplitude and both the green and red-absorbing layers control the phase. Each hologram is replicated 4 times, giving the image a dot structure. In its image (fig. 11) notice that the dots are equally spaced.

The parity sequence hologram [7] is also on-axis and highly efficient. In this hologram, « parity elements » are added that, together with the desired image, have a perfectly level spectrum. Thus, only a phase-controlling material is required to encode the hologram. The parity elements are chosen so that they are away from and do not interfere with the desired image. Figure 12 shows an image from three parity sequence holograms. The noise in the corners is the parity elements added to level the spectrum. In this case, scaling was done on the digitized object, in order to demonstrate the difference in sampling rates for the three different colors. This undesirable effect can be removed by de-focussing the image.

These on-axis holograms required color filters in the read-out geometry to avoid false images. It is possible to remove this requirement.

For a given hologram, one incident color produces the desired image, while the two other incident colors produce false images. Of these latter two, one color can be completely absorbed by one layer of the three-

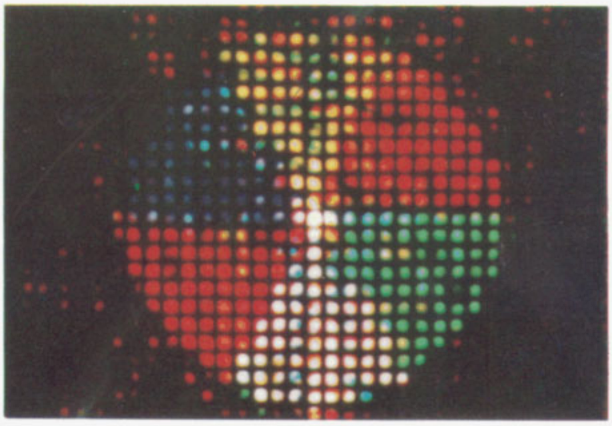

FIG. 11. — *Image from* ROACHs.

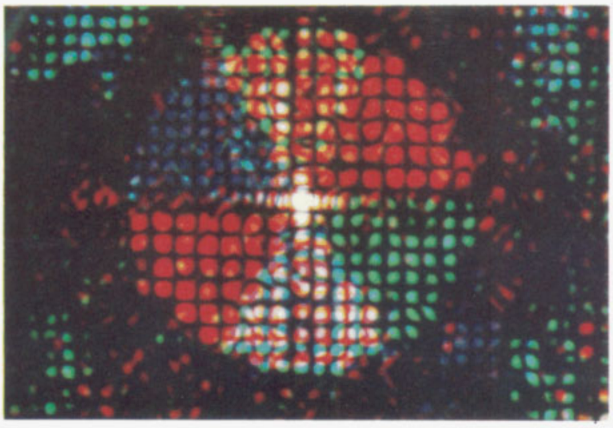

FIG. 12. — *Image from parity-sequence holograms. The difference in the spacing of the dots is a result of scaling the digital representation of the object according to wavelength. Extraneous « parity » elements in the corners were introduced to allow use of phase-only material.*

layer film, still leaving two layers for control of both amplitude and phase of the desired color. The second color leading to a false image can be diffracted away from the desired image by using the « phase-null » method described in the next section.

## 5. — PHASE-NULL METHOD TO ELIMINATE FALSE IMAGE

Consider a single Fourier coefficient cell in the blue-image hologram. The cell is divided into two halves (fig. 13), and the right-hand half is exposed to red and blue light in order to give the correct complex blue transmittance for that Fourier coefficient. The left-hand half is exposed the same to red but much less to blue light than in the right-hand side. This difference in blue exposure causes a phase difference between the two halves, and this phase difference is made to be pi ($\pi$) radians, that, is, $\frac{1}{2}$ wavelength of red light. Then, for red light, the net contribution on-axis due to this cell is zero, since the two halves cancel one another, being equal in amplitude but opposite in phase. To blue light, however, the left-hand side is opaque compared to the right-hand side, so only the

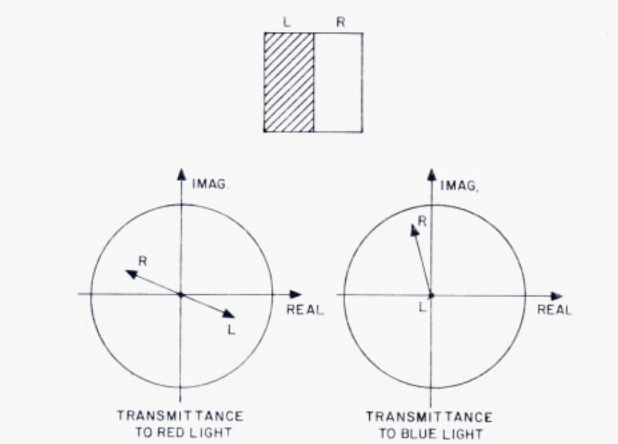


FIG. 13. — *Phase-null method. To the desired color (blue), the right half has the proper complex transmittance for that Fourier coefficient and the left half is relatively opaque. To the undesired color (red), the right and left halves have equal amplitudes, but opposite phases, so the net on-axis contribution is zero.*

right-hand side makes a significant contribution to the phasor ; and since the right-hand half was exposed to give the proper complex transmittance, then the blue image reconstructs as it should. Thus, while one potential false image color (green) is absorbed by the film, the remaining false image (red) is diffracted off to the sides without disturbing the desired image (blue). Similarly, the phase-null method can be used to eliminate a false image from a red or green-image hologram. A more detailed mathematical treatment of the phase-null effect is given in the Appendix.

Figure 14*a* shows such a phase-null ROACH for a blue image illuminated with blue light. Notice the alternating vertical bands of transmitting and opaque blue half-cells. Figure 14*b* shows the same phase-null ROACH illuminated with red light. Both halves of each square cell transmit equally, but due to the difference in exposure to blue, the two halves are out of phase with one another. Figure 15 shows the reconstruction of such a phase-null ROACH illuminated with both red and blue light. The blue image reconstructs on-axis as expected, while the false red image is diffracted to both sides, away from the desired blue image.

Due to non-ideal absorption by the dyes in this film, exposure compensation is sometimes necessary in the left half of the cell to allow for absorption cross-talk. Implementation of the phase-null effect requires very accurate exposure control during synthesis of the hologram. Since there are no disturbing false images, the three holograms could be spatially multiplexed, if desired.

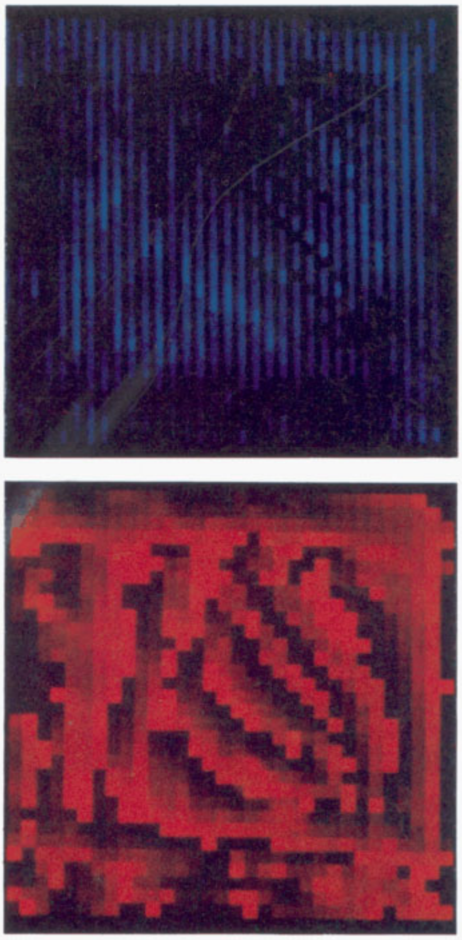

FIG. 14. — (a) *A* ROACH *for blue light using the phase-null effect, illuminated by blue light.* (b) *the same* ROACH *illuminated by red light.*

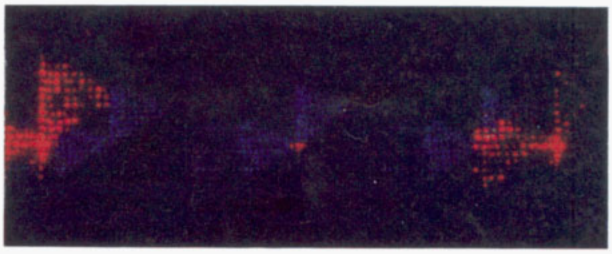

FIG. 15. — *Reconstruction from a phase-null* ROACH. *The desired central blue image is undisturbed while the undesired red image is diffracted off to the sides.*

## 6. — *CONCLUSION*

We have shown three methods for eliminating false images : by the optics of the reconstruction set-up, by wavelength-dependent absorption of color film, and by the phase-null effect. We have also shown ways to take care of the scaling problem : by a digital step, during synthesis, and during reconstruction.

Table 1 compares the different types of computer-generated color holograms discussed. If ease of synthesis is the prime consideration, then binary detour-phase holograms are the most attractive. But if displaying the maximum possible number of Fourier coefficients on a CRT and high diffraction efficiencies are more important, then the on-axis holograms are superior.

TABLE I. — *Comparison of types of holograms discussed*

| Hologram type (a) | Material required | Plotter points per coefficient | Maximum diffraction efficiency (%) | Inherent noise |
|---|---|---|---|---|
| Binary Detour-phase | Black/white (b) | Many (c) | 10.3 | Quantization (c) and detour-phase approx. |
| Parity Sequence | Phase only | 2 | 100 | Parity noise |
| ROACH | Multi-emulsion | 1 | 100 | None |

(a) Holograms using the phase-null method require multi-emulsion film, require double the number of plotter points per Fourier coefficient, and have a quarter of the diffraction efficiency listed.
(b) Multi-emulsion film is required for color multiplexing.
(c) Quantization noise is decreased by increasing the number of plotter points per Fourier coefficient.

## APPENDIX

Here we use the notation

$$\text{(1)} \qquad \operatorname{comb}(x/d) = \sum_{n=-\infty}^{\infty} \delta(x/d - n)$$

$$\text{(2)} \qquad \operatorname{rect}(u/d) = \begin{cases} 0, & |u/d| > \frac{1}{2} \\ \frac{1}{2}, & |u/d| = \frac{1}{2} \\ 1, & |u/d| < \frac{1}{2} \end{cases}$$

$$\text{(3)} \qquad \operatorname{sinc}(x/d) = \frac{\sin(\pi x/d)}{\pi x/d} .$$

The symbol $\supset$ denotes the Fourier transform relationship and $*$ denotes convolution [8].

If the sampled version of an object is $f(x, y)$, and if $f(x, y) \supset F(u, v)$, then our hologram, $H$, is a sampled version of $F(u, v)$, with sampling interval $d$, and

$$\text{(4)} \quad H = \operatorname{rect}\left(\frac{u}{d}, \frac{v}{d}\right) * \left[F(u, v) . \operatorname{comb}\left(\frac{u}{d}, \frac{v}{d}\right)\right] .$$

That is, our hologram is made up of an array of square cells of side $d$. Each cell has uniform transmittance over its area.

Since $f(x, y)$ is sampled, $F(u, v)$ is periodic. Wheter $H$ contains many or only one period of $F(u, v)$ does not matter for the purpose of this analysis, since that will affect only the microstructure of the image.

If a lens of focal length $f$ is used in reconstruction, then the reconstructed image, $R$, from this computer-generated sampled hologram can be computed by taking the inverse Fourier transform :

$$\text{(5)} \quad \operatorname{rect}\left(\frac{u}{d}, \frac{v}{d}\right) * \left[F(u, v) . \operatorname{comb}\left(\frac{u}{d}, \frac{v}{d}\right)\right] = H \subset R =$$
$$= \operatorname{sinc}\left(\frac{dx}{\lambda f}\right) . \operatorname{sinc}\left(\frac{dy}{\lambda f}\right) . \left[f(x, y) * \operatorname{comb}\left(\frac{dx}{\lambda f}, \frac{dy}{\lambda f}\right)\right] .$$

The convolution of $f(x, y)$ with $\operatorname{comb}(dx/\lambda f, dy/\lambda f)$ causes the image $f(x, y)$ to be repeated, a result of sampling in the Fourier domain ; but the sinc factors insure that the central-order image is strong and the repeated images are weak.

Now suppose that each hologram cell is divided into two halves along the $u$-direction. The right half has the same complex transmittance as before, and the left half has that complex transmittance multiplied by $A$, a complex constant (fig. 16*b*). Then we have

$$\text{(6)} \quad H = \left[\operatorname{rect}\left(\frac{2u}{d} + \frac{1}{2}, \frac{v}{d}\right) + A . \operatorname{rect}\left(\frac{2u}{d} - \frac{1}{2}, \frac{v}{d}\right)\right] *$$
$$* \left[F(u, v) . \operatorname{comb}\left(\frac{u}{d}, \frac{v}{d}\right)\right] \subset R = \frac{1}{2}\left[\exp\left(i 2 \pi \frac{dx}{4 \lambda f}\right)\right.$$
$$\left. + A . \exp\left(- i 2 \pi \frac{dx}{4 \lambda f}\right)\right] \times \operatorname{sinc}\left(\frac{dx}{2 \lambda f}\right) . \operatorname{sinc}\left(\frac{dy}{\lambda f}\right) .$$
$$. \left[f(x, y) * \operatorname{comb}\left(\frac{dx}{\lambda f}, \frac{dy}{\lambda f}\right)\right] .$$

To the desired color, the left half of the cell is opaque, that is, $|A| \cong 0$. Then the reconstruction becomes

$$\text{(7)} \quad R \cong \frac{1}{2} \exp\left(i 2 \pi \frac{dx}{4 \lambda f}\right) . \operatorname{sinc}\left(\frac{dx}{2 \lambda f}\right) . \operatorname{sinc}\left(\frac{dy}{\lambda f}\right) .$$
$$. \left[f(x, y) * \operatorname{comb}\left(\frac{dx}{\lambda f}, \frac{dy}{\lambda f}\right)\right] .$$

This is the same as our original reconstruction (5) except that the sinc factor fall-off is more gentle in the $x$-direction and the overall image is less intense. The phase factor is the result of shifting the centers of the hologram cells by $\frac{1}{4} d$ (fig. 16*a*, *b*).

To the undesired color, the left half of the cell is $\pi$ radians out of phase with the right half, that is, $A = -1$. Then the reconstruction is

$$\text{(8)} \quad R = i . \sin\left(\frac{\pi dx}{2 \lambda f}\right) . \operatorname{sinc}\left(\frac{dx}{2 \lambda f}\right) . \operatorname{sinc}\left(\frac{dy}{\lambda f}\right) .$$
$$. \left[f(x, y) * \operatorname{comb}\left(\frac{dx}{\lambda f}, \frac{dy}{\lambda f}\right)\right] .$$

Due to the $\sin(\pi dx/2 \lambda f)$ factor, this undesired image is attenuated near the center.

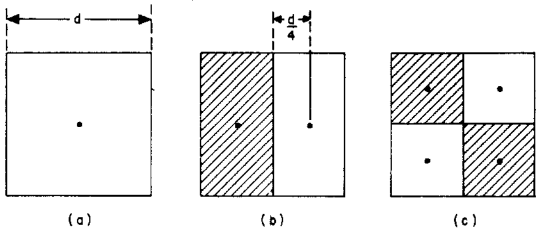


FIG. 16. — *Dividing the cell for the phase-null method. Shaded areas have a complex transmittance equal to A times the transmittance of the unshaded areas* (a) : *regular cell* ; (b) : *phase-null method in one dimension* ; (c) : *phase-null method in both dimensions.*

Suppose $N$ samples were taken in the $x$-direction in the image at positions

$$x_n = n\,\Delta x\,, \quad n = -\tfrac{1}{2}N\,, \quad -\tfrac{1}{2}N + 1, \ldots, \tfrac{1}{2}N\,.$$

The relation between the width of the image, $N\,\Delta x$, and the spacing of the hologram samples, $d$, is given by

$$N\,\Delta x = \frac{\lambda f}{d}\,. \tag{9}$$

Thus, the $n$th element in the image is located at $x = n\,\Delta x = n\lambda f/(Nd)$. Rewriting,

$$\frac{\mathrm{d}x}{\lambda f} = \frac{n}{N}\,. \tag{10}$$

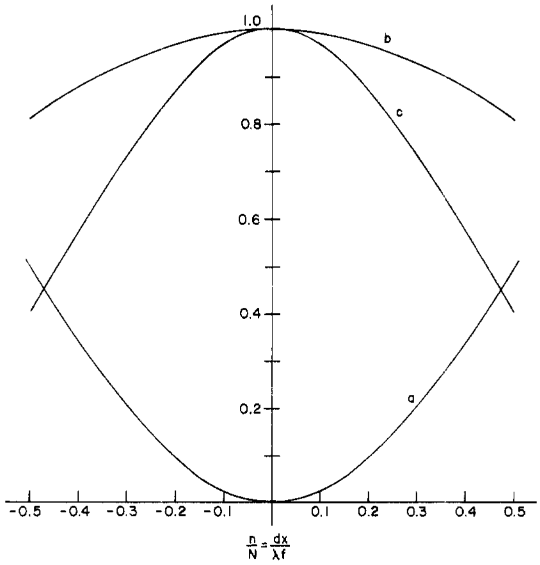


FIG. 17. — *Phase-null effect.* (a) : $\sin^2(\pi n/2N)$ *factor attenuating the undesired color* ; (b) : $\mathrm{sinc}^2(n/2N)$ *factor attenuating the desired color* ; (c) : $\mathrm{sinc}^2(n/N)$ *factor that would attenuate the image if the phase-null method were not used.*

Figure 17 shows the squares of the factors attenuating the various images, the intensities of which are actually observed. We see that even with the phase-null effect, the false image will reconstruct with one half its previous intensity at the edge of the pattern, but that the degree of attenuation increases toward the center. Thus, by imbedding the image in a sufficiently large field of zeros, the false image can be attenuated to any degree desired. A much greater attenuation of the false image can be achieved by dividing the cell still further, as in figure 16*c*, for example, in which the phase-null effect is used in both the $x$ and $y$-directions. Then the attenuating factor is

$$\sin^2\left(\frac{\pi\,\mathrm{d}x}{2\,\lambda f}\right).\sin^2\left(\frac{\pi\,\mathrm{d}y}{2\,\lambda f}\right).$$

In this case, if the image is imbedded to half the the size in both the $x$ and $y$-directions, then the false image position in the corner will reconstruct with only 0.021 5 times its original intensity, and the interior positions will be attenuated still further. Thus the phase-null method can be very effective in eliminating an undesired color.

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