Direct image reconstruction from a Fourier intensity pattern using HERALDO

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We present what we believe to be the first experimental demonstration of a novel coherent lensless imaging technique: holography with extended reference by autocorrelation linear differential operator. Upon taking derivatives of the field autocorrelation this technique allows the direct reconstruction of an object complex-valued transmissivity from a measurement of its Fraunhofer diffraction pattern. We show reconstruction examples using a parallelogram, a thin slit, and a triangle as extended references. © 2008 Optical Society of America

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Current limitations in manufacturing diffraction-limited optics for high resolution imaging at x-ray wavelengths has caused an increased interest in the development of coherent lensless imaging techniques. Holography with extended reference by autocorrelation linear differential operator (HERALDO) is a lensless imaging approach that allows direct image reconstruction from a measurement of the Fraunhofer diffraction pattern (Fourier intensity pattern) by using a sharp feature on an extended reference as a holographic-like reference [1]. Although holographic approaches have increased sampling and transverse coherence requirements compared with image reconstruction by phase retrieval [2–7], their capability to reconstruct an image from a single intensity measurement in a fast, direct, noniterative computation is attractive. If needed, a reconstruction from HERALDO can be used as an initial guess for iterative phase retrieval algorithms, increasing the robustness of phase retrieval and potentially reducing the noise and improving the resolution for HERALDO.

HERALDO, a generalization of an earlier approach by Podorov *et al.* [8], provides the flexibility to tailor the reference to something that can be accurately manufactured, promising improved performance for coherent lensless imaging at x-ray wavelengths. It works at other wavelengths as well. This reconstruction approach is fundamentally different from using an extended reference and improving the resolution through deconvolution.

In this Letter we experimentally demonstrate imaging from a Fourier intensity measurement using HERALDO. In these experiments, the boundary waves from the corners of a parallelogram, from a triangle, and from the ends of a thin slit are used as holographic-like references to obtain complex-valued images.

A photograph of the "cameraman," printed on a slide, was placed within a parallelogram-shaped aperture. For "ground truth," the object amplitude shown in Fig. 1(a) was obtained by taking the square root of an image taken with a two-lens imaging setup with a resolution of $\simeq 11\ \mu$m. In the HERALDO experiment this object was illuminated with a collimated wave from a He–Ne laser ($\lambda = 632.8$ nm). The Fourier intensity pattern, shown in Fig. 1(b), was measured at the back focal plane of a lens with a 500 mm focal length using a 1200×1600, 12-bit, 7.4 μm pixel pitch, Retiga-2000R CCD. A lens was required for this experiment, since the width of the ob-

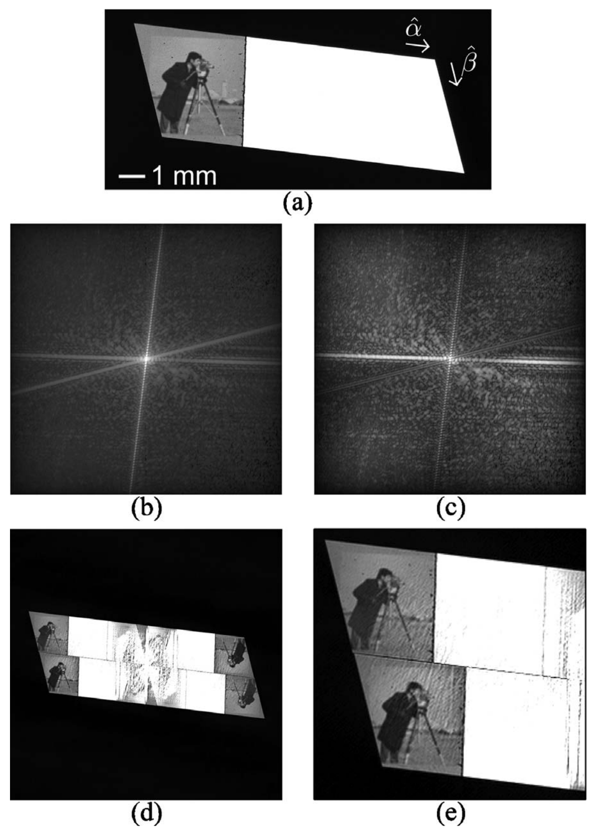

Fig. 1. (a) Amplitude of object within a parallelogram aperture. (b) 1024×1024 measured intensity pattern. (c) Product of (b) and $(\mathbf{u}\cdot\hat{\alpha})(\mathbf{u}\cdot\hat{\beta})$. (d) IFT of (c). (e) 250 $\times 250$ pixel inset of (d). Owing to their high dynamic range, the tenth and fifth roots are shown in (b) and (c), respectively. For better visualization we show in (d) a 700×700 inset of the full 1024×1024 array and saturate the brighter contributions near the center of the array.

ject ($\simeq$1 cm) would require a propagation distance impractical in our laboratory. For x rays, however, the sample is typically much smaller and the far-field intensity pattern is accessible after free-space propagation.

To capture the large dynamic range of the Fourier intensity patterns ($\sim 10^7$), we stitched frames with eight different exposure times (ranging from 30 μs to 4 s). For each exposure we averaged several frames of data to reduce the noise, compensated for patterned detector artifacts, and subtracted dark frames. After taking the 1024×1024 central portion of the array, unsaturated pixels were identified and combined with appropriate scaling. To further reduce the noise and sidelobe artifacts in the images, without sacrificing much resolution, the outer 256 pixels of the array were tapered with a raised cosine function.

Reconstruction by HERALDO requires computation of directional derivatives of the field autocorrelation, which is the inverse Fourier transform (IFT) of the measured intensity pattern. In practice we apply the directional derivatives by taking the product of the measured intensity and the corresponding polynomial [5] and then computing the IFT. For reconstruction from a corner we compute

$$-\frac{4\pi^2}{|\sin(\beta-\alpha)|}\mathcal{F}^{-1}\{(\mathbf{u}\cdot\hat{\alpha})(\mathbf{u}\cdot\hat{\beta})I(\mathbf{u})\}, \qquad (1)$$

where $I(\mathbf{u})=|F(\mathbf{u})|^2$ is the measured Fourier intensity, $\hat{\alpha}$ and $\hat{\beta}$ are unit vectors pointing along the edges of the corner (with angles α and β, respectively, with respect to the x axis), $\mathbf{u}=(u,v)$ are the set of Cartesian coordinates in Fourier space, and $\mathcal{F}^{-1}\{\,\}$ denotes an IFT.

The result of the polynomial product on the measured intensity pattern is shown in Fig. 1(c). The angles $\alpha\simeq-6.9°$ and $\beta\simeq-75.7°\pm0.3°$ were directly estimated from the streaks in the diffraction pattern. For a reconstruction using HERALDO we do not need to estimate the support of the extended reference. The IFT of the processed diffraction pattern [computed with an inverse fast Fourier transform (FFT)] is shown in Figs. 1(d) and 1(e). Because all corners of the parallelogram share the direction of their edges, we would expect four independent reconstructions upon computation of Eq. (1). However, because only the two rightmost corners of the parallelogram satisfy the required HERALDO separation conditions [1], only two of the reconstructions (and twin images) can be separated from other autocorrelation terms. The reconstructions are in good agreement with the object shown in Fig. 1(a) to within the maximum resolution ($\simeq$41.75 μm, limited by the diameter of the 1024×1024 array of pixels on the detector) that can be achieved with our HERALDO imaging setup.

Figure 2(a) shows the amplitude of the cameraman with a thin slit reference. The measured intensity pattern is shown in Fig. 2(b). Reconstruction with a vertical thin slit is obtained by computing the derivative of the autocorrelation with respect to y [1]. The

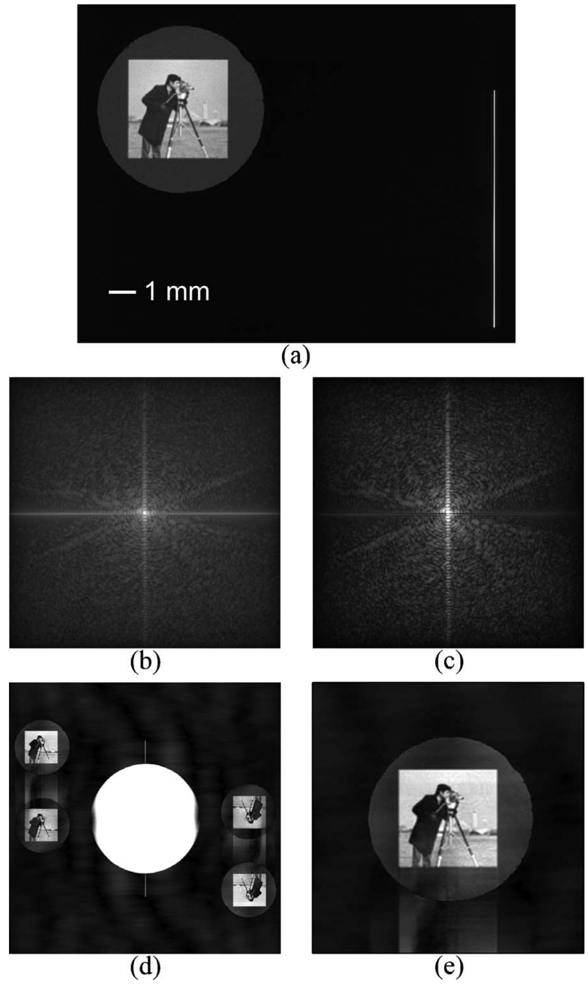

Fig. 2. (a) Amplitude of object with a thin slit. (b) Measured intensity pattern. (c) Product of (b) and $i2\pi v$. (d) IFT of (c). (e) Inset of (d) showing upper reconstruction.

slit orientation was determined by its corresponding streak in Fig. 2(b). The result of multiplying the measured diffraction pattern by $i2\pi v$ is shown in Fig. 2(c). The reconstruction, obtained by an IFT of the processed pattern, is shown in Figs. 2(d) and 2(e). Because the two ends of the slit satisfy HERALDO separation conditions [1] we expect two independent reconstructions (and twin images).

An object with a triangle reference and the corresponding measured Fourier intensity pattern are shown in Figs. 3(a) and 3(b), respectively. For this reference the three corners satisfy HERALDO separation conditions [1], but because the corners do not share the direction of their edges, the measured intensity needs to be processed differently to get a reconstruction from each corner. The angles of the triangle sides ($\alpha\simeq12.6°$, $\beta\simeq87.71°$, and $\gamma\simeq-51.57°$) were estimated from the streaks in the diffraction pattern. Figure 3(c) shows the result of applying the polynomial product on the measured intensity pattern to obtain a reconstruction from the lower-left corner of the triangle ($\hat{\alpha}$ and $\hat{\beta}$). The reconstruction, obtained after computing the IFT of the processed pattern, is shown in Figs. 3(d) and 3(e).

The same intensity measurement [Fig. 3(b)] can be used to obtain reconstructions from the remaining

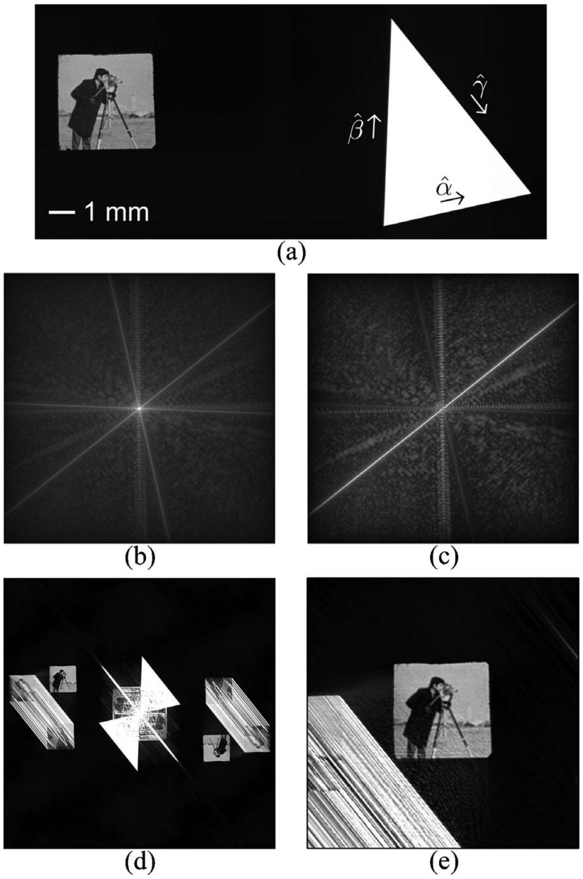

Fig. 3. (a) Amplitude of object with a triangle reference. (b) Measured intensity pattern. (c) Product of (b) and $(\mathbf{u}\cdot\hat{\alpha})(\mathbf{u}\cdot\hat{\beta})$ to obtain a reconstruction from the lower-left corner of the triangle. (d) IFT of (c). (e) Inset of (d).

two corners. Figures 4(a) and 4(b) show the results of applying the polynomial product to the measured intensity to obtain reconstructions from the upper ($\hat{\beta}$ and $\hat{\gamma}$) and lower-right ($\hat{\alpha}$ and $\hat{\gamma}$) triangle corners, respectively. The IFT of the processed patterns shown in Figs. 4(a) and 4(b) are shown in Figs. 4(c) and 4(d), respectively.

We experimentally demonstrated imaging from a measurement of the Fourier intensity pattern of an object using HERALDO [1]. This proof-of-concept experiment at optical wavelengths is a key intermediate step toward implementation for high-resolution coherent lensless imaging at the x-ray regime. Reconstructions were obtained, in a noniterative computation, for parallelogram, thin slit, and triangle references. For these objects the boundary waves diffracted from the corners (or slit ends) serve as holographic-like references. We did not make use of any *a priori* quantitative knowledge about the object. Although knowledge of the shape of the reference was used to determine the appropriate linear differential operators, the angles were estimated from the streaks in the measured diffraction patterns.

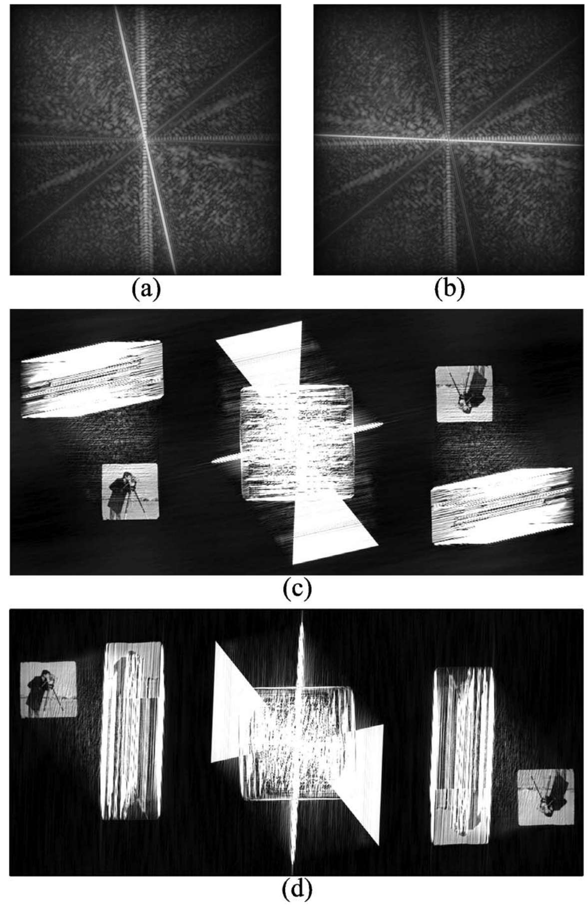

Fig. 4. Product of intensity pattern [Fig. 3(b)] and polynomial to obtain a reconstruction from the (a) upper and (b) lower-right corners of the triangle. (c) and (d) show 420 $\times$900 insets of the IFTs of (a) and (b), respectively.

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