Retrieval of complex field using nonlinear optimization

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Abstract: Intensity measurements in two or more planes are used by a nonlinear optimization algorithm to retrieve the phase. The complex field (amplitude and phase) in the pupil or elsewhere is then computed by simple propagation.
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1. Introduction

Phase retrieval has been used in the past for determining the phase in a pupil plane from intensity measurements in one or more planes near focus and knowledge of the shape of the pupil [1-3]. However, the amplitude distribution in the pupil may not be perfectly known, and extra means may be necessary to estimate it [4]. A second approach is to use an iterative transform (Misell) type algorithm employing two or more intensity measurements near the focus to retrieve the phase in those planes, without knowledge of the amplitude distribution in the pupil [5-7]. Then, by propagation from one of these planes, one can estimate the complex field (amplitude and phase) in the pupil [7] or any other plane. As an alternative to the Misell algorithm for this second approach, we have developed an algorithm based on a nonlinear optimization method for estimating the phase in the measurement planes.

2. Description of algorithm

A nonlinear-optimization-based phase-retrieval algorithm typically retrieves the phase in a desired plane where the amplitude is assumed to be well known, usually a pupil plane. It does this by computing a numerical propagation of an estimated field in the desired plane to the planes where intensity measurements have been made, and comparing the computed field amplitudes with the measured field amplitudes (the square root of the measured intensities) using an objective function. The algorithm attempts to minimize this objective function by varying the phase in the desired plane, preferably making use of the derivative of the objective function with respect to the phase values.

In contrast, our method requires no a priori knowledge of the pupil. Instead, we initially employ one of the measured amplitudes as our desired plane instead of the pupil itself. We refer to this measurement plane as the "master" plane. We minimize an objective function iteratively by varying the phase in the master plane. The objective function is calculated by propagating a numerical field, formed from the measured amplitude and the estimated phase in the master plane, to each of the other ("slave") planes. The objective function measures the difference between the amplitudes of the numerically propagated fields and the measured amplitudes. We use a gradient-search nonlinear optimization algorithm employing the objective function and its gradient with respect to phase values in the master plane [see Ref. 1, Eq. (A4) or Ref. 3, Eq. (23), which are equivalent], to estimate the phase distribution in the master plane. Repeated iterations drive the value of the objective function toward a minimum.

However, it is unclear which of the multiple measurement planes we should use as this "master" intensity distribution. We have addressed this by using all of the planes in sequence as the master plane. To illustrate, consider the three-plane case shown in Fig. 1. To begin, we make a guess of the field in the pupil (which may be grossly in error), which is propagated to the first measurement plane (plane 1) in step A. Then, in step B, plane 1 is the master plane. The initial field guess uses the phase of the propagated field from step A and the measured amplitudes in plane 1. The nonlinear optimization described above is performed using plane 1 as the master and planes 2 and 3 as the slaves, but only for a small number of iterations. The resulting field is propagated to plane 2, where the resulting phase is used with the measured amplitude in plane 2 to form the initial guess for another set of optimization iterations for which plane 2 is the master and planes 1 and 3 are the slaves, depicted as step C. This process is repeated using plane 3 as the master and the planes 1 and 2 as the slaves in step D. We then loop through the entire process, steps B, C, and D, employing each of the planes as the master repeatedly, until we have reduced the value of the objective function to an acceptably small value. The resulting field is propagated back to the pupil plane in step E, where we now have an estimate of the pupil amplitude and phase.

Using each plane as a master helps us to avoid algorithm stagnation problems and increases the resistance of the algorithm to the noise in any particular plane.
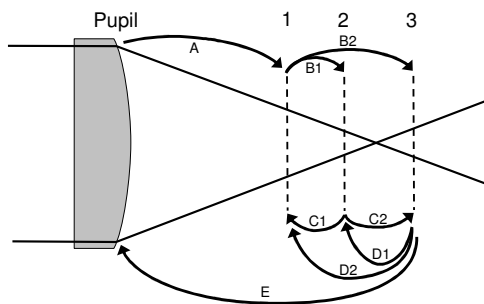
Fig. 1. Schematic diagram of algorithm used to determine the full complex field in the pupil.
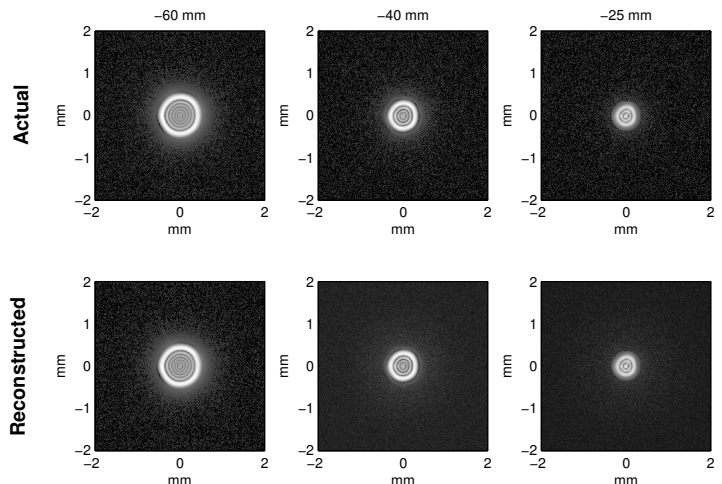

Fig. 2. Simulated measured intensity patterns and the resulting reconstructed intensity patterns using 3 planes.

3. Simulation Results

We demonstrate this new approach with computer simulations. We computed three intensity patterns, shown in the top row of Fig. 2, located 60 mm, 40 mm and 25 mm inside the paraxial focal plane of the system; hence these positions are given a negative sign in our convention. We included Poisson noise such that the brightest pixel in each image contained 32,000 photoelectrons. We also included Gaussian read noise of 16 photoelectrons, and quantized the images to 12 bits. These values are consistent with a moderate-quality CCD camera. The lens simulated had a 500 mm focal length and a 10 mm diameter aperture (f/50). The lens was given 0.30 waves RMS of smoothly varying phase aberrations (mostly spherical aberration of various orders, but also some small asymmetric aberrations) that we wish to recover. The aberrations are shown in the leftmost column of the bottom row of Fig. 3.

 The bottom row in Fig. 2 shows some representative reconstructed intensity patterns in a case where the optimization was run including all three planes. The agreement in the intensity patterns is excellent, indicating that the algorithm converged to a propagating field in good agreement with all three intensity measurements.
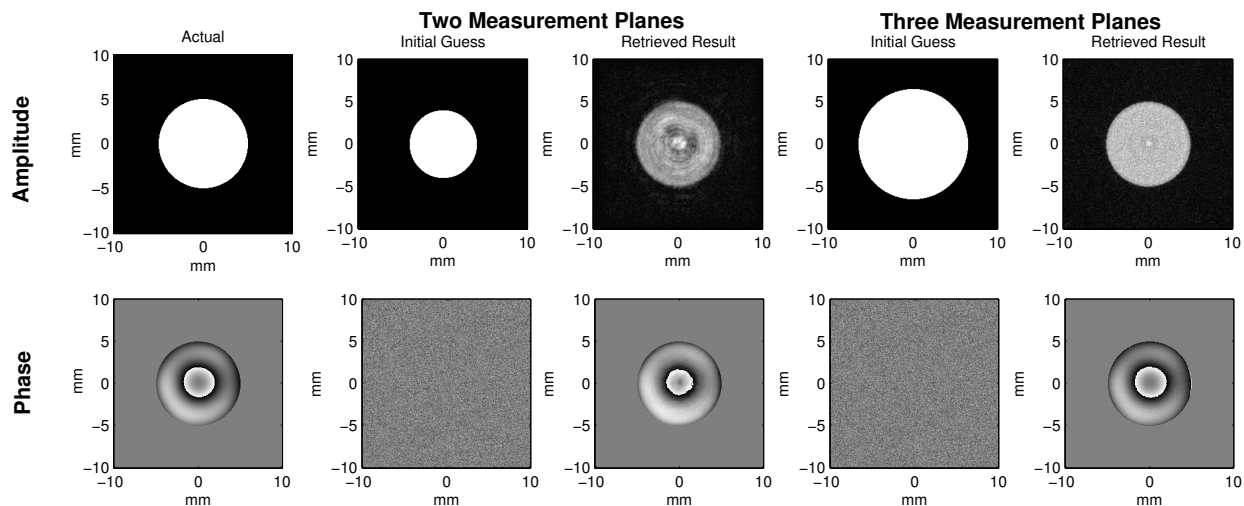

Fig. 3. Example pupil plane reconstructions using two and three planes, showing the initial guess of the phase and amplitude in the pupil plane and the resulting recovered (2π wrapped) phase maps.

 Fig. 3 shows examples of the initial guess of the phase and amplitude in the pupil and the resulting reconstructions for cases where two and three measurement planes were used in the reconstruction. The initial guess for the pupil amplitude was a circular aperture of a diameter markedly different from the actual. In our work, we are typically dealing with circular apertures, however the exact diameter of the aperture may not be known with adequate precision and there may be vignetting. The initial guess for the phase array was zero-mean Gaussian-

distributed random phases with a standard deviation of 0.1 waves. The examples shown here were results after 25 cycles of the algorithm were performed, with 10 optimization iterations performed with each plane as the master for each cycle. The agreement between the actual and retrieved pupil phase is excellent: 0.029 waves RMS in the two-plane case shown and 0.017 waves RMS in the three-plane case shown. The reconstructed pupil amplitude distributions do not agree as well, being more sensitive than the retrieved phase to measurement noise. Increasing the SNR of the measurement or the number of measurement planes improves these pupil amplitude reconstructions. However, the pupil reconstructions are certainly clear enough to determine the shape of the aperture to within a pixel. If one can assume that the amplitude within the aperture is uniform, this result and the already excellent phase estimate can be used as the input to a more conventional nonlinear optimization to determine the phase more precisely.

The convergence of the algorithm is shown in Fig. 4. The case where we have used three measurement planes converges most quickly and to the lowest error value, 0.019 waves RMS, after only 60 optimization iterations (2 cycles, with 10 iterations per plane per cycle). All three combinations of two planes were tried. The best two-plane case had about twice the error of the three-plane case, and the second-best two-plane case had twice that error, or about 1/16 wave RMS. The third two-plane case did not converge and stagnated at a result worse even than the initial guess. This illustrates the desirability of using additional measurement planes, both from the standpoint of convergence and absolute accuracy of the results, and the importance of carefully selecting the locations of the planes.
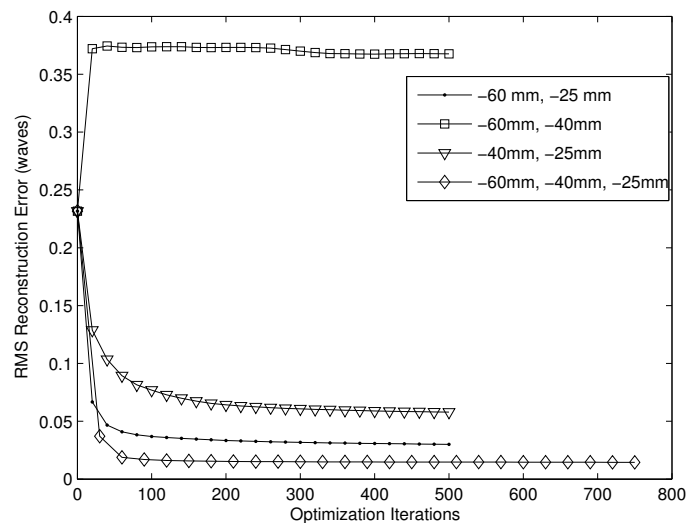

Fig. 4. Reconstruction error versus number of iterations for cases where two
planes and threes planes have been used.

4. Conclusion

We have shown that it is possible to get good reconstructions of the full field in the pupil plane of an optical system using a nonlinear-optimization-based method and intensity measurements from just a few planes. The algorithm converges to a result that allows for an excellent estimate of the phase and a good estimate of the amplitude in the pupil after a small number of optimization iterations.

5. References

[1] J. R. Fienup, "Phase retrieval algorithms: a comparison," Appl. Opt. **21**, 2758-2769 (1982).
[2] J.N. Cederquist, J.R. Fienup, C.C. Wackerman, S.R. Robinson and D. Kryskowski, "Wave-Front Phase Estimation from Fourier Intensity Measurements," J. Opt. Soc. Am. A **6**, 1020-1026 (1989).
[3] J. R. Fienup, "Phase-retrieval algorithms for a complicated optical system," Appl. Opt. **32**, 1737-1746 (1993).
[4] J. R. Fienup, J. C. Marron, T. J. Schulz, and J. H. Seldin, "Hubble Space Telescope characterized by using phase-retrieval algorithms," Appl. Opt. **32**, 1747-1767 (1993).
[5] D. L. Misell, "A method for the solution of the phase problem in electron microscopy," J. Phys. D: Appl. Phys. **6**, L6-L9 (1973).
[6] R. A. Gonsalves, "Phase retrieval and diversity in adaptive optics," Opt. Eng. **21**, 829-832 (1982).
[7] C. Roddier and F. Roddier, "Combined Approach to Hubble Space Telescope Wave-Front Distortion Analysis," Appl. Opt. **32**, 2992-3008 (1993).