

# Range of Phase Retrieval in Optical Metrology

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**Abstract:** The range of phase retrieval for optical metrology or wave-front sensing is limited by detector sampling. We describe limits on f-number and wavefront asphericity and strategies for extending these limits.

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**OCIS codes:** (100.5070) Phase retrieval; (120.5050) Phase measurement; (120.3940) Metrology; (220.4840) Optical testing

## 1. Introduction

Phase retrieval is a method of measuring an optical wavefront or optical surface using only the information contained in the intensity distribution produced by the field near its focus. The measurement arrangement is very simple, consisting of just an illuminating field, the optic under test, and a detector array. These arrangements typically allow for the measurement of aspheric wavefronts without having to account for retrace errors. Adequately sampling the optical fields or intensity patterns sets limits on the allowed range of these measurements. We discuss these limits in relation to f-number and the departure of a wavefront from spherical.

## 2. Limits on F-number

The limit on f-number can be shown to be  $f/\# \geq Sd_\xi/\lambda$ , where  $d_\xi$  is the sample spacing in the detector plane and  $S$  is the sampling ratio. To Nyquist sample the measured intensity pattern,  $S = 2$ . To Nyquist sample the underlying complex field,  $S = 1$ , and  $S < 1$  results in undersampling even of the field. For a detector with a  $7.5\mu\text{m}$  pixel size at a wavelength of  $632.8\text{nm}$ , assuming  $S = 2$ , the minimum  $f/\#$  that can be tested is 23.7. We can reduce the measurable  $f/\#$  by increasing the magnification of the pattern on the detector array (using some additional optics) or by reducing  $S$  (by undersampling) [1]. We can also contrive a set of conjugates that increases the  $f/\#$  of the beam from the surface that ordinarily would have a smaller  $f/\#$ . This will effect the aberrations measured, but in a predictable manner.

## 3. Limits on wavefront departure from spherical

The maximum deviation of the wavefront slope is limited by the ability of the detector to collect the light from the surface. For a distance  $z$  from an exit pupil of diameter  $D$ , this requirement corresponds to

$$Nd_\xi \geq \frac{2z}{D} \left[ \left( \frac{\partial W}{\partial \rho} \right)_{\max} - \left( \frac{\partial W}{\partial \rho} \right)_{\min} \right] \quad (1)$$

where  $N \times N$  is the number of detector pixels,  $W$  is the wavefront in units of length,  $\rho$  is the lateral coordinate normalized to unity at the outer radius of the pupil, and  $z/D = f/\#$ . As an example, consider the case where we have primary spherical aberration with some compensating focus so that the RMS spot size is minimum [2],

$W = W_{040} \left[ \rho^4 - (4/3)\rho^2 \right]$ . Then the maximum measurable spherical aberration is  $W_{040} \leq (3/16)Nd_\xi/f/\#$ .

Combining this with the requirement that  $f/\# \geq Sd_\xi/\lambda$ , we have  $W_{040} \leq (3/16)N\lambda/S$ , independent of any magnification factor to the detector. For Nyquist sampling of the intensity and a detector with  $N = 512$  elements, the maximum spherical aberration that can be measured is 48 waves when  $S = 2$ . Greater range of  $W_{040}$  is possible with smaller  $S$ . We will show the effect on the accuracy of the retrieved wavefront as the sampling ratio  $S$  is decreased.

## 4. Conclusion

Phase retrieval as an optical metrology tool is limited in range by detector sampling considerations. Undersampling the intensity measurements can extend that range.

## 5. References

- [1] J.R. Fienup, "Phase Retrieval for Undersampled Broadband Images," *J. Opt. Soc. Am. A* **16**, 1831-1839 (1999).
- [2] Warren J. Smith, *Modern Optical Engineering 3<sup>rd</sup> Ed.* (McGraw-Hill, 2000), Chap. 11.