2006 Topical Meeting on Optical Fabrication and Testing, paper OFWA5
(Optical Society of America)

Measurement of an Optical Surface using Phase Retrieval

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Abstract: We describe the experimental measurement of a concave spherical mirror using a phase retrieval algorithm. Estimates of the resulting phases using different data sets agree to within about three thousandths of a wave RMS.

OCIS codes: (120.5050) Phase measurement; (120.3940) Metrology; (100.5070) Phase retrieval; (070.2580) Fourier optics; (050.1960) Diffraction theory

1. Introduction

Phase retrieval is a method of measuring the phase of an optical field using simple measurements of near-focus intensity distributions produced by the field. The method shows great promise to measure the figure of optical surfaces and the transmitted wavefront error of optical systems with precision and accuracy comparable to, or better than, phase-shifting interferometry techniques. The experimental arrangement required by phase retrieval is much simpler than that used in interferometry. The simplest arrangement need only consist of the optic under test and a detector array, such as a charge coupled device (CCD) array. The algorithms used to interpret the measured intensity patterns and arrive at a phase measurement are, however, more complex than those used in interferometry. Here we describe a phase retrieval algorithm and experimental arrangement that we have used to measure a concave spherical surface, and present some early laboratory results.

2. Phase retrieval algorithm

Phase retrieval algorithms are iterative algorithms that attempt to find a field in the plane of interest (in our case, a pupil plane) that, when propagated to a plane (or planes) where intensity measurements have been made, has an intensity distribution that agrees with the measurements [1]. An error metric quantifying the agreement is minimized using a nonlinear optimization algorithm operating on the phase values [2].

The algorithm uses the measured intensity distributions, $I_j(r,s)$, in one or more near-focus planes from which the field amplitude $\left|F_j(r,s)\right|$ is calculated:

$$\left|F_j(r,s)\right| = \sqrt{I_j(r,s)} \tag{1}$$

where the subscript j is the index of the measurement plane and (r, s) are sample indices. The algorithm starts by making a guess of the phase $\theta(m,n)$ in the pupil plane and combining it with $\left|g(m,n)\right|$, a known amplitude distribution (typically unity within the clear aperture) to produce a numerical field

$$g(m,n) = \left|g(m,n)\right| \exp\left[i\theta(m,n)\right]. \tag{2}$$

The phase can be represented either as point by point values or as the coefficients of a set of basis functions. In this work we represented the phase in terms of the coefficients α_k of the Zernike polynomials,

$$\theta(m,n) = \sum_{k=0}^{K} \alpha_k Z_k(m,n). \tag{3}$$

The field $g(m,n)$ is propagated to each of the J measurement planes using a diffraction propagation operator, $\mathbf{P}_j[]$:

$$G_j(r,s) = \mathbf{P}_j\left[g(m,n)\right]. \tag{4}$$

The particular propagator model we employed uses Fresnel diffraction to calculate the field in the nominal focal plane, and the angular spectrum method to propagate from there to each of the nearby measurement planes [3]. The amplitudes of the propagated fields are used with the measured amplitudes to compute an error metric,

$$E = \sum_{j=1}^{J} \sum_{r,s} W_j(r,s)\left[\left|F_j(r,s)\right| - \left|G_j(r,s)\right|\right]^2 \tag{5}$$

where $W_j(r,s)$ is a weighting function that can be used to zero out the contribution of known bad pixels in the CCD array, emphasize certain regions of the amplitude distribution, or remove the contribution of pixels where the signalto-noise ratio is unacceptably small. A nonlinear optimization algorithm, in our case the conjugate gradient algorithm [4], is used to minimize the error metric. Efficient expressions for the gradient of the error metric with respect to the parameters being varied are used to greatly speed up the algorithm [1, 2].

3. Experimental arrangement

The arrangement used for our experiments is shown in Figure 1. The illumination beam was produced by a HeNe laser that was spatially filtered using a microscope objective and pinhole. The pinhole was placed near the center of curvature of the concave spherical mirror, the optic that was being tested. The mirror used for this experiment had a radius of curvature of 500 mm and a diameter of 12.7 mm, for an f-number of 39 for the 1:1 imaging configuration used. The point source produced by the pinhole was imaged onto the CCD by tilting the mirror 3 degrees, which induces a very small amount of astigmatism. Using a small fold mirror allowed us to keep the required tilt angle small. The intensity distributions of the resulting diffraction spots were measured using an uncooled 12-bit camera, a Q-Imaging Retiga 2000R [5], mounted on a computer-controlled translation stage. The stage allows us to precisely position the camera and collect intensity patterns in a number of near-focus planes automatically.

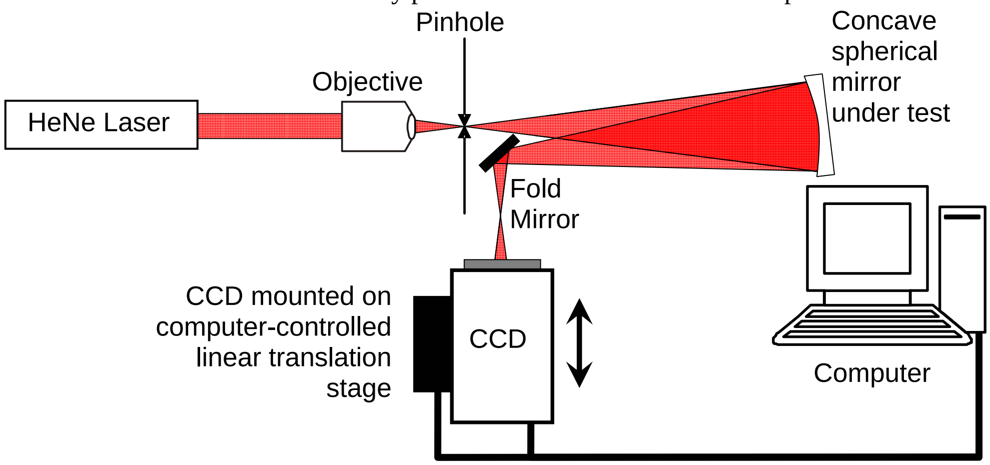

Figure 1: Diagram of experimental arrangement used for phase retrieval measurements.

4. Results

Measurements of intensity patterns in several measurement planes are shown in Figure 2. The resulting retrieved phases using different subsets of these measurements are shown in Figure 3. In cases (a) and (b) the phase retrieval algorithm represented the phase using 36 Zernike polynomials, and in case (c) by 45 Zernike polynomials. The differences between the phase maps are small: 0.0174λ PV or 0.0031λ RMS between (a) and (b), 0.0186λ PV or 0.0024λ RMS between (a) and (c) and 0.0192λ PV or 0.0018λ RMS between (b) and (c). The results indicate that the repeatability of the measurements, using disjoint sets of raw data, is about three thousandths of a wave RMS. For comparison, the magnitude of the departure from spherical of the measured wavefront in (c) is 0.1789λ PV or 0.0189λ RMS, consistent with the manufacturer's specification for the surface of the mirror of $\lambda/10$ PV. We can not speak to the accuracy of these results because, at this time, we have not had the opportunity to measure the element independently with an instrument of sufficient accuracy. Figure 4 shows the intensity patterns reconstructed using the phase retrieved in case (b) of Figure 3. The agreement between these and those shown in Figure 2 is excellent, even for the planes (at –15, 10 and 15 mm) whose intensity patterns were not used to constrain the optimization. This indicates that we have solved for the correct phase of the propagating field.

5. Conclusion

We have described a phase retrieval method that we have used to characterize a concave spherical mirror. We were able to arrive at phase values consistent with the specification of the mirror, and repeated measurements using disjoint data sets agree with one another with values to within about three thousandths of a wavelength RMS. Continuing work will attempt to compare these measurements with measurements made using other instruments, further refine our algorithms to increase their precision and accuracy, and perform investigations of the measurement range of phase retrieval by changing the alignment of the system to induce large wavefront errors.

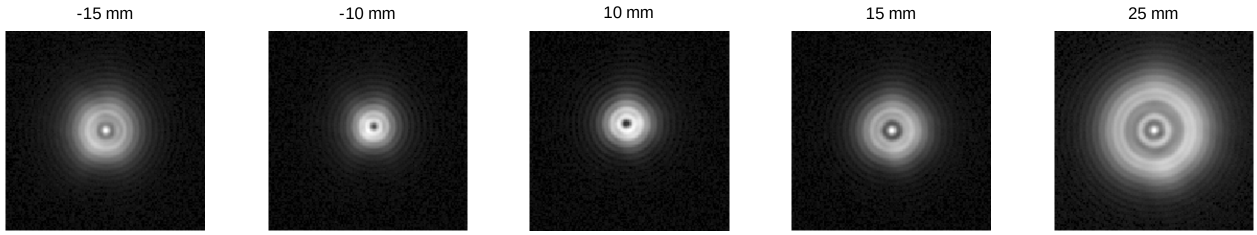

Figure 2: Intensity distributions measured in planes at the indicated distances from focal plane.

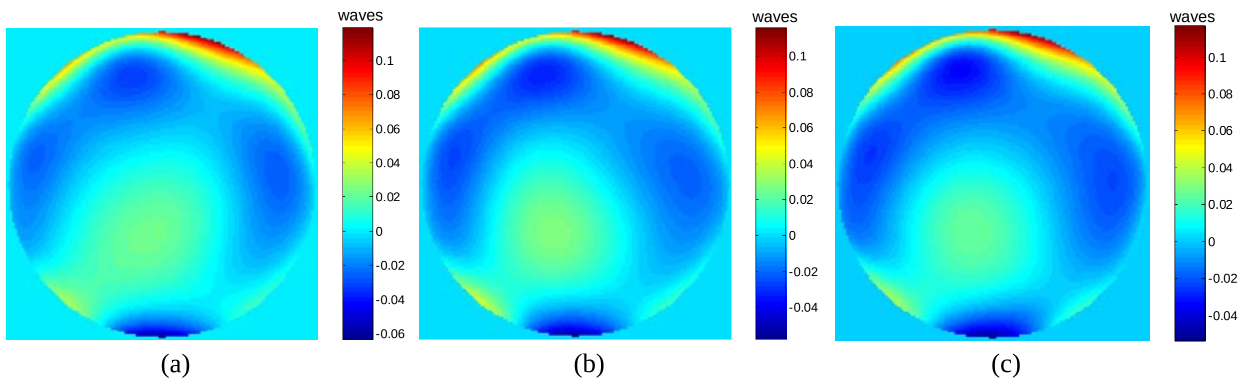

Figure 3: Recovered phases using the measurement planes at (a) –15 mm and 10 mm, (b) –10 mm and 25 mm, (c) –15 mm, –10 mm, 15 mm and 25 mm.

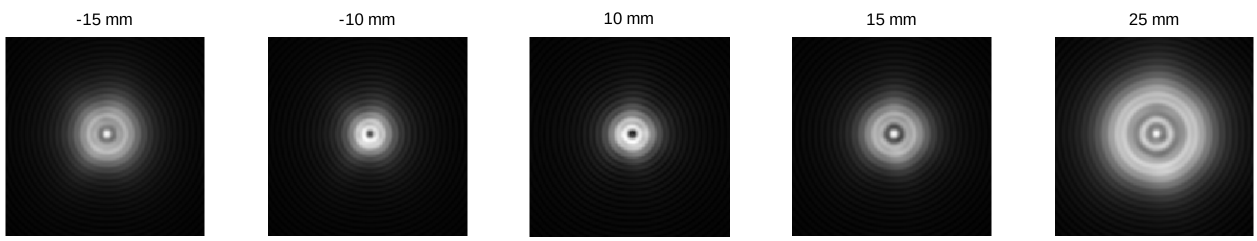

Figure 4: Intensity patterns in all of the measurement planes computed by propagating the phase result obtained using measurements in the –10 mm and 25 mm planes [case (b) of Figure 2].

6. References

[1] J.R. Fienup, "Phase Retrieval Algorithms: A Comparison," Appl. Opt. **21**, 2758-2769 (1982).

[2] J.R. Fienup, "Phase-Retrieval Algorithms for a Complicated Optical System," Appl. Opt. **32**, 1737-1746 (1993).

[3] Joseph W. Goodman, *Introduction to Fourier Optics*, 3rd Ed. (Roberts & Co., Greenwood Village, CO, 2005).

[4] W. H. Press, B. P. Flannery, S. A. Teukolsky, and W. T. Vetterling, *Numerical Recipes: The Art of Scientific Computing* (Cambridge University Press, 1986), Chap. 10.

[5] See http://www.qimaging.com/products/datasheets/Retiga2000R.pdf.