Recent Approaches To Computer – Generated Holograms*

D.C. Chu

Hewlett-Packard Co., 5301 Stevens Creek Blvd., Santa Clara, California 95050

and

J.R. Fienup

Information Systems Laboratory, Stanford University, Stanford, California 94305

(Received February 4, 1974)

ABSTRACT

Computer holography is a subject of interest to a small but increasing number of researchers. This paper briefly describes the concept and history of computer holography, how it works, and its possible applications. Also presented are the recent approaches to computer holography developed at Stanford University, and a procedure for synthesizing this new class of computer holograms. These new on-axis holograms are very efficient both in their use of reconstruction light and in their use of display resolution elements during synthesis. They keep the advantages of the kinoform, a previous approach by IBM, but do not have its limitations. One approach makes use of color reversal film, such as Kodachrome II. Another approach only requires phase-transmittance material, but preserves both amplitude and phase information. Experimental results are presented.

INTRODUCTION

In ordinary interferometric holography, the hologram is made by recording the interference pattern of an object complex wave and a phase-coherent reference wave. Upon illumination of the hologram, an image of the original object can be reconstructed.

In synthesizing holograms by a computer, no physical object exists. Instead, one starts with a mathematical description of an object, usually in the form of an array of phase-coherent point sources. The complex wavefront from such an array on a fixed plane some distance away can be computed. If a transparency can be synthesized whose complex transmittance is equal to the computed wavefront on that plane, then upon illumination of such a transparency (or hologram) by a plane wave, light will be modulated by the hologram so that it will emerge exactly as though it came from the object. With this technique, therefore, a visible image can be constructed, in three dimensions if desired, of an object which does not exist physically. The problem, of course, is the realization of the transparency whose transmittance is proportional to a computed complex function.

Black and white film can be used to control the amplitude, or modulus, of a light wave and a bleached film can be used to control the phase of light, due to changes in refractive index and surface relief effects. A transparency of a given complex transmittance can be synthesized by making a "sandwich" of two transparencies, one controlling amplitude and the other the phase. In practice, however, this is seldom done because of the difficulty in obtaining proper registration between the two transparencies. To overcome this problem, many ingenious approaches were devised to represent a complex function by using a recording medium which controls only amplitude or only phase. Some of these are described later on in this paper.

These holograms of non-physical objects are called "synthetic holograms". Since a digital computer is often used in their synthesis they are sometimes also called "computer-generated holograms".

FOURIER TRANSFORM HOLOGRAMS

One particularly important class of synthetic holograms is the Fourier transform hologram which essentially contains

This work was sponsored by National Science Foundation and the Hewlett-Packard Company.

the Fourier transform† of the desired image. Upon illumination of the hologram in front of a lens, the desired image is found one focal length behind the lens. Fourier transform holograms have many superior properties; for example, lateral translation of the hologram produces no change in the intensity distribution of the image, which remains stationary. This property is a direct result of a property of the Fourier transform where translation in one domain is accompanied only by a linear phase shift in the other, without affecting the modulus. Another desirable property of the Fourier transform is its ability to distribute localized disturbances in one domain more or less uniformly in the other. As a result, localized defects of the hologram due to dust particles and scratches do not result in localized blemishes on the image, but rather result in a slight general loss of fidelity of the entire image. By simply repeating the Fourier transform a number of times side by side on the hologram, the image is not repeated, but approaches a sampled version and appears in a dot structure. This last property is particularly useful if the desired image is that of an array of binary data. This discrete dot structure facilitates detection by discrete electronic detection devices such as photo-diodes, and reduces interference between adjacent data elements. The repeating of the transform increases the redundancy, suppresses the speckle, and increases the signal to noise ratio. Notice also that the increased redundancy is accomplished without an accompanying increase in the number of detection devices in the image domain.

APPLICATIONS OF COMPUTER HOLOGRAMS

The word "hologram" immediately brings to mind three-dimensional images; but that is just one of many possible uses for computer-generated holograms. A general wavefront modulator such as a hologram can also be a coherent optical spatial filter, an optical memory, a component in the testing of optical elements, or an optical element itself.

A computer hologram can be made of any one, two or three-dimensional object which can be described mathematically.[1] The description can be a set of equations or a sequence or array of numbers obtained from such things as a scanned photograph or an acoustic microscope. Color images may be obtained by making three color-separation holograms for the red, green and blue information and illuminating them with the respective colors of coherent light.[2] The principal drawback is that the large number of resolvable elements in a high quality image, typically 10^5 or 10^6 for a two-dimensional picture and many times that for a three-dimensional one, combined with the extra redundancy desirable in a hologram, requires time-consuming and expensive computation and display.

A promising use of computer holograms is for archival storage of data from computer outputs. The data to be stored may exist in core, and in this case the digital Fourier transformation step would represent the only extra step in comparison with writing the data directly on film. The benefits of translation invariance, burst noise resistance and redundancy mentioned earlier make this extra step worthwhile, particularly if the memory is to be machine-read later. The outstanding features of holographic memory are high storage density, quick accessibility due to parallel processing and low cost.

†Strictly speaking it should be the inverse transform. The distinction, however, depends only on the definition of spatial coordinates and is conceptually unimportant.

The construction of a memory based on computer holography has recently been completed at Radiation, Inc., in Michigan.

Another area in which computer holography has been put to practical use is in the testing of optical elements.[3] Manufacturing errors in an optical element can be detected by observing the difference between a wavefront from the optical element and a reference wave in an interferometer. Computer-generated holograms can be used to generate the reference wavefront standard. This is particularly useful in the testing of aspheric surfaces, where the complexity of the wave makes it difficult to synthesize by any other means.

In addition to their use in testing optical elements, computer generated holograms can be the optical elements themselves. Examples of possible uses include computer-generated Fresnel lenses, diffraction gratings, zone plates, and diffusers. With the ability to control amplitude as well as phase transmittance, a whole new class of optical elements is possible, including an imaging device which resolves beyond the diffraction limit.[4]

Still another application of computer-generated holograms is their use as optical spatial filters. Although many types of coherent optical spatial filters can be optically generated[5], computer generation is often the most convenient – and sometimes the only way to make them. Spatial filters find use in image enhancement[6], restoration of degraded images[7], matched filtering[8], and code translation.[9] With computer-generated holograms, matched filtering with incoherent illumination is also possible.[10]

PREVIOUS APPROACHES

Most display instruments used for synthesizing holograms, such as digital plotters or computer controlled CRT's, have an upper limit to the number of addressable locations, or resolution elements, restricting the ultimate complexity of the hologram as expressed by the space-bandwidth product. A very important parameter in the synthesis of holograms is therefore the number of such resolution elements required for encoding each Fourier coefficient in a Fourier transform hologram.

The early attempts at computer generation of holograms were merely simulations of the process of the optically generated Leith and Upatnieks hologram.[11,12,13] The complexity of the computation involving a reference beam, and the large number of display resolution elements needed to encode the resulting high frequency interference fringes made these approaches unattractive.

Perhaps the best known computer-generated hologram, one which departs from a mere simulation of the interferometric hologram, is the detour-phase binary hologram of Brown and Lohmann.[14] In this technique exposure control is not critical since only binary amplitude transmittance is required. The method divides the hologram into cells, each one representing a complex Fourier coefficient. Within each cell is an aperture, with its size related to the amplitude and its position within the cell to the phase of the coefficient. When illuminated, the desired image is obtained off-axis in the first diffraction order. In this method, a large number of display resolution elements are used for each Fourier coefficient. Any attempt to reduce the number results in increased amplitude and phase quantization error. There are other variations of the detour-phase binary holograms by Haskell[15], as well as gray-level detour-phase holograms by Lee[16] and Burkhardt.[17] However, they are all off-axis techniques and the light diffracted to the desired image is typically about 1% of that incident on the hologram.

Another well-known wave-front reconstruction device is the kinoform[18] which is not a hologram in the sense that the totality (holo) of an amplitude and phase information is not preserved. For the kinoform, the Fourier (or Fresnel) transform amplitude information is discarded under the assumption that for a "well-diffused" object, the transform amplitude can be approximated by a constant. Accordingly, the kinoform is a transparency whose phase transmittance is the transform phase and whose amplitude transmittance is unity. Since there is no general method to diffuse any given object perfectly, significant loss of fidelity often results from the amplitude quantization.

There are other computer-generated holograms described in the literature. The interested reader is referred to two survey papers on the subject[19,20] as well as a chapter of the text *Optical Holography*.[21]

RECENT APPROACHES

Despite its fidelity problem, the kinoform has many advantages. It is efficient in its use of display resolution elements, each one of which can generate a Fourier coefficient (without amplitude). Also, all incident light on the kinoform is diverted to one useful image, and the efficiency is ideally 100%. Our work at Stanford University has been directed at making holograms which retain most of the advantages of the kinoform, but remove its limitations.

ROACH

One of our approaches is to seek a hologram material which not only controls the phase, as with bleached film, but adds amplitude control without having to align two transparencies. The referenceless on-axis complex†† hologram[22] (ROACH) uses multi-emulsion film, such as Kodachrome II, in which different layers can be exposed independently by light of different colors. Upon illuminating with light of a given color, one layer will absorb, while the other two layers will be predominantly transparent, but will cause phase shifts due to variations in film thickness and refractive index (Figure 1). Since the complex transmittance, both amplitude

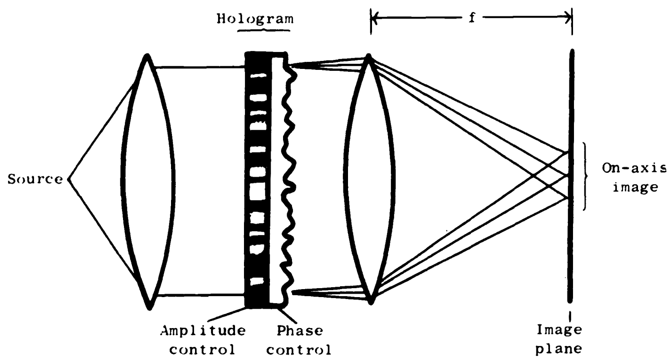

Figure 1. A schematic diagram showing the reconstruction of a ROACH. The amplitude and phase are independently controlled by different emulsion layers.

††The word "complex" here refers to the transmittance of film which simultaneously controls amplitude by absorption and phase through film thickness variation.

and phase, can be controlled at each point, only one resolvable element is needed to encode one complex Fourier coefficient. In the reconstruction, there are no conjugate images or central-order spot, so the efficiency is high.

To make a ROACH for reconstruction with, for example, the red 632.8 nm light from a helium-neon laser, we simply expose the amplitude pattern with red light since only the red-sensitive layer will absorb red after processing, and expose the phase pattern with blue-green light to generate the relief pattern. The red exposure is calculated to give a final amplitude transmittance to red proportional to the amplitude of the transform. The blue-green exposure is computed to give a final density to blue-green light (which we assume to be linearly related to the phase of red light) proportional to the transform phase, subtracting an appropriate amount to compensate for the undesirable relief due to the amplitude pattern. Detailed experimental procedure is given later in this paper.

The hologram constructed with this method is simply a kinoform with amplitude control. The improvement due to amplitude control can be seen by comparing the kinoform and ROACH reconstruction in Figure 2. The non-uniformity of the dot intensities in the kinoform reconstruction shown in Figure 2a is due primarily to the loss of the hologram amplitude information.

SPECTRUM SHAPING METHODS

Another approach for improving the kinoform is by encoding or modifying the computed Fourier transform mathematically in such a way that each element has the same amplitude, with various phases. Thus, a hologram with effective amplitude and phase control can be synthesized with a phase-only transparency. Unlike the kinoform, the hologram phase is now no longer equal to the transform phase, but modulated in accordance with the transform amplitude. The reconstruction is the optical Fourier (inverse) transform of this modified array. This procedure is illustrated in the block diagram of Figure 3. Here f represents the data array. After a fast-Fourier transform (FFT) digital processing step, the complex transform array F is obtained which is then encoded to give the modified array H. H has uniform modulus and the hologram $\hat{H}$ is synthesized whose phase transmittance is made equal to the phase of H. The array $\hat{h}$ represents the field reconstructed from the hologram although only the magnitude image $|\hat{h}|$ is observable.

Immediately, the question is raised that since H is different from F, then $\hat{h}$ is different from f; in other words, the reconstructed image is other than the one desired. The key to success, however, lies in the fact that $\hat{h}$ contains all the elements of f, uncorrupted, together with some extraneous "parity" elements introduced by the modification of the Fourier spectrum. Their locations are clearly separated from those of the data and can be ignored. Also, these parity elements act as a light ballast, drawing light into or away from the data in such a manner that the intensity of each data bit is constant, regardless of the number of "on" bits in the array. Alternatively, if constant intensity is not important, one can design for maximum efficiency.

There are many ways of encoding the Fourier transform to achieve data and parity separation. We shall describe two methods used in our experiments. The first is simply called the parity sequence method.[23] If $\{f_{n,q}\}$ is an $N \times Q$ data array whose complex Fourier transform is $\{F_{m,p}\}$, also $N \times Q$ in size, then we encode $\{F\}$ to generate a $2N \times 2Q$ hologram

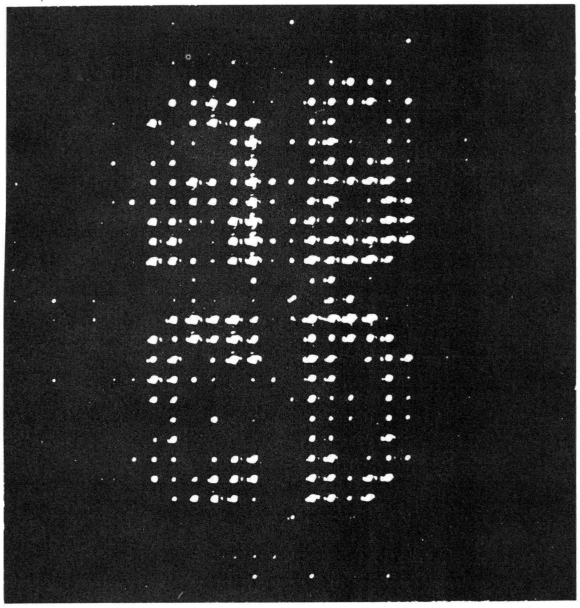

Figure 2a. Image reconstructed from a kinoform (only spectrum phase is controlled). The nonuniformity of the dot intensities is due to the loss of the spectrum modulus information.

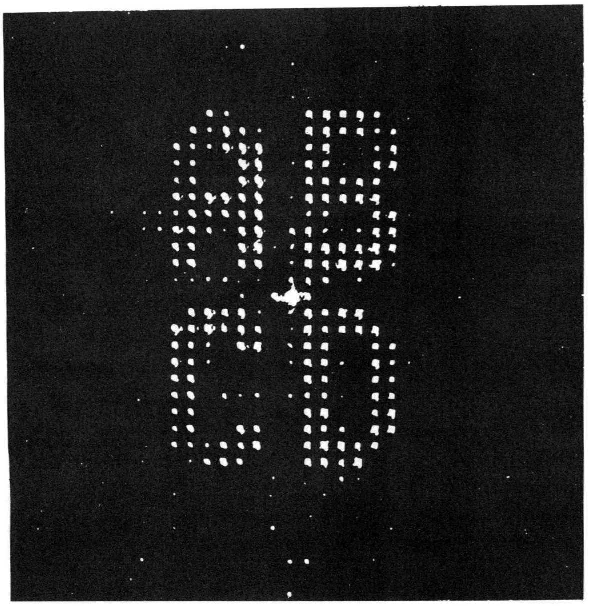

Figure 2b. The same image reconstructed from a ROACH (both spectrum modulus and phase are controlled).

array H whose phase transmittances are as follows:

$$\hat{H}_{m,p} = \hat{H}_{m+N,p+Q} = \exp\left\{ i\left[\theta_{m,p} + (-1)^{m+p}\, a_{m,p}\right]\right\}, \quad (1)$$

$$\hat{H}_{m+N,p} = \hat{H}_{m,p+Q} = \exp\left\{ i\left[\theta_{m,p} - (-1)^{m+p}\, a_{m,p}\right]\right\}, \quad (2)$$

where

$$F_{m,p} \triangleq |F_{m,p}|\left[\exp\ i\,\theta_{m,p}\right] \quad (3)$$

$$a_{m,p} = \cos^{-1}\frac{|F_{m,p}|}{A}, \text{ if } A \geqslant |F_{m,p}|$$

$$= \quad 0, \text{ if } A < |F_{m,p}| \ . \quad (4)$$

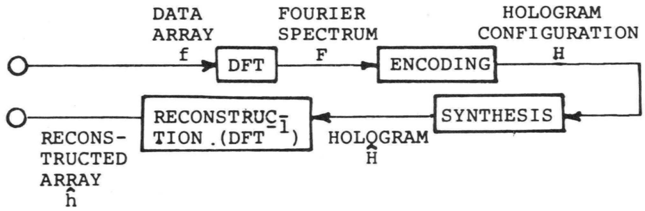

Figure 3. Block diagram representation of the synthesis and reconstruction of a computer-generated hologram involving coding of the Fourier spectrum.

The constant A is a parameter which can be chosen to tradeoff fidelity and efficiency. For $A = |F_{m,p}|_{max}$, one can ideally obtain 100% fidelity of data, but a significant proportion of light energy may go toward the parity sequences as compared with the energy in the data. As A is lowered, one obtains more light to the data at the expense of parity elements, but increased loss of data fidelity can be observed as A is lowered too much. For constant intensity per bit, however, A should be a constant for all arrays. As A approaches zero, the resulting hologram is simply the kinoform, which is a limiting case of maximum efficiency for minimum fidelity in the trade-off.

In the reconstruction, the image is represented by a 2N×2Q array $h_{n,q}$. The original data $\{f_{n,q}\}$ can be found in locations given by

$$h_{2n,2q} = f_{n,q} \ . \quad (5)$$

The parity elements are located at

$$h_{2n+1,2q+1} = p_{n,q} \ , \quad (6)$$

and can also be easily computed in advance, if desired. In addition, it can be shown using the inverse Fourier transform that

$$h_{2n+1,p} = h_{p,2q+1} = 0 \ . \quad (7)$$

Figures 4a and 4b show the pattern for a 3×3 data array and the reconstructed (6×6) array. The alternating sign of $a_{m,p}$ in Equations 2 and 3 effectively insures that the parity elements immediately next to the data elements are weak.

Figure 5 shows a reconstruction from such a hologram. Note that the data pattern and the parity elements are offset from each other by one space and can clearly be distinguished.

Another coding method is the synthetic coefficient method[24], where we code as follows:

$$H_{2m,2p} = H_{2m+1,2p+1} = \exp\left\{ i\left[\theta_{m,p} + a_{m,p}\right]\right\} \quad (8)$$

$$H_{2m+1,2p} = H_{2m,2p+1} = \exp\left\{ i\left[\theta_{m,p} - a_{m,p}\right]\right\} \quad (9)$$

where $\theta_{m,p}$ and $a_{m,p}$ are the same as in Equation 1.

It can be shown mathematically that the reconstructed array is given by

$$h_{n,q} = \exp\left[i\pi\left(\frac{n}{N}+\frac{q}{Q}\right)\right]\left[f_{n,q}\cos\frac{\pi n}{2N}\cos\frac{\pi q}{2Q} + p_{n,q}\sin\frac{\pi n}{2N}\sin\frac{\pi q}{2Q}\right], \qquad (10)$$

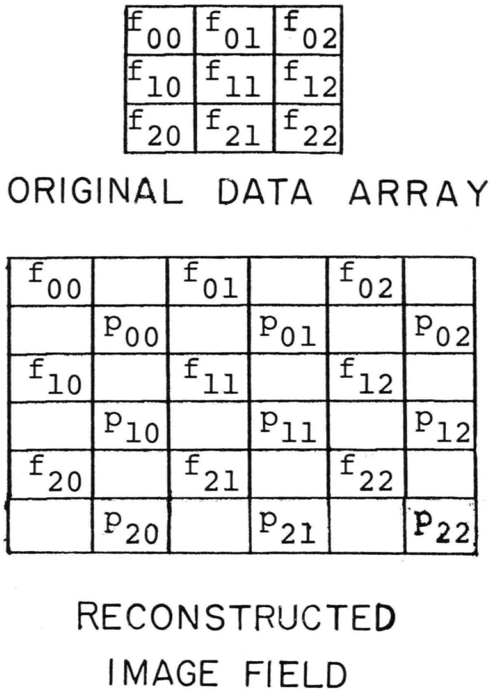

Figure 4. The original data array shown together with the expected reconstruction array from a parity sequence hologram. The original data is preserved, interlaced diagonally with extraneous parity elements.

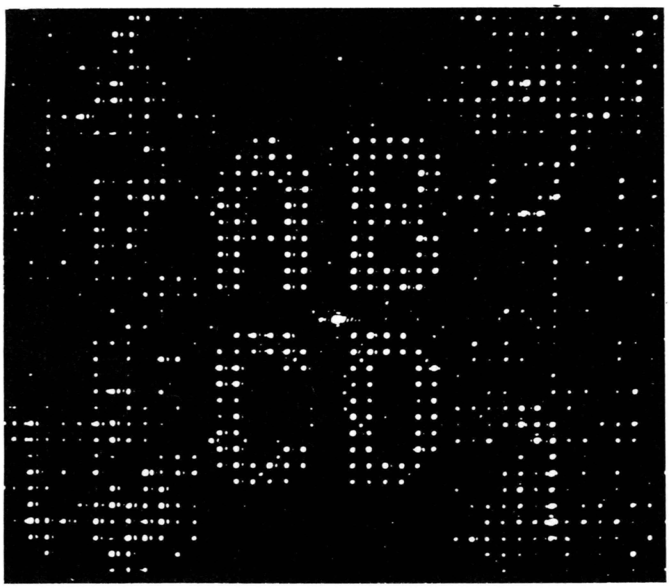

Figure 5. Image reconstruction from a parity sequence hologram. Only phase transmitting material is needed. Note that the parity noise and the data are offset in both the x and the y directions.

where f and p are the data and parity elements, respectively. Because of the sine type coefficient of the parity elements $\{p\}$, they are suppressed near the origin, while the data is multiplied by a cosine type function and is strongest in this region. Hence, the data is reconstructed with good fidelity near the origin and surrounded by parity elements in the outer field region. This is confirmed in Figure 6, which shows an actual reconstruction of a hologram synthesized by this method.

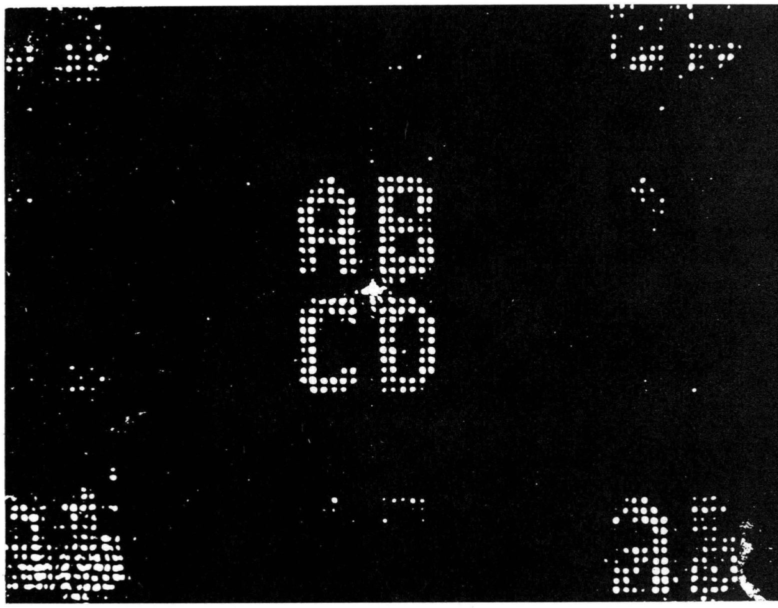

Figure 6. Image reconstruction from a synthetic coefficient hologram. Again, only phase-transmitting material is needed.

For all three holograms discussed, it is advantageous to introduce arbitrary phases to the object array to improve efficiency by lowering the spectrum dynamic range. This is analogous to diffusion with ground-glass in ordinary interferometric holography. The phases may be selected randomly, iteratively through numerical methods[25], or deterministically.[26]

SYNTHESIS PROCEDURE

To synthesize phase-only holograms as well as ROACHs, we expose Kodachrome II directly from a computer-controlled CRT and have the film processed by Eastman Kodak. The combination of one-step synthesis and dependable commercial processing is a convenient way to make computer holograms. Since we use red 632.8 nm helium-neon laser illumination to reconstruct the image, the phase information is displayed on the CRT and photographed through a blue-green filter. In a second exposure, the amplitude information is displayed and photographed through a red filter. For phase-only holograms, in which the amplitude is constant, this re-exposure step is more economically accomplished by photographing a uniformly illuminated surface through a red filter. Determination of the correct exposure to achieve a desired complex transmittance requires a calibration procedure described here in detail.

INITIAL PREPARATION

We control exposure by the length of time that the CRT beam is turned on at each point. To minimize exposure times, the highest intensity that will not damage the phosphor is determined. To get consistent results, we measure the spot brightness directly with a light meter after the CRT is fully warmed up.

The ideal display is a grid of adjoining squares with uniform illumination over each of the squares and no gaps between the squares; thus, the X and Y scales on the CRT are adjusted so that adjoining lines just overlap. One of two additional methods may be employed to approximate the ideal: the electron beam may be slightly defocused so the spot attains a square shape; or four adjoining positions (in a 2X2 square) may be used to represent one coefficient. The latter method, however, reduces the number of available display positions by a factor of four.

The distance from the camera to the CRT is determined so that the finest detail is recorded at spatial frequencies lower than 50 lines/mm, in order not to exceed the resolution of the film. We have used only 20 lines/mm. To minimize exposure time, the widest lens opening should be chosen so that the finest detail can still be resolved. The CRT is shielded from room light with a cardboard tube.

Since both the blue-sensitive and the green-sensitive layers contribute after processing to the phase information, it is desirable to choose a color "phase" filter that exposes both layers about equally. Thus, both layers can be simultaneously exposed in the linear parts of their density vs. log of exposure (H&D) curves. In addition, ROACHs require the red-sensitive layer to be relatively unexposed by the phase-controlling exposure in order to avoid exposure cross talk. For type P31 phosphor, Wratten filter No. 38A satisfies these requirements. Similarly, a red filter for the amplitude-controlling exposure is required that leaves the blue and green-sensitive layers unexposed. Wratten filters No. 26 and 92 are found to be satisfactory. If the available CRT phosphor were to emit predominantly blue or green, then just the one layer can be used to control the phase. A "white" phosphor is most desirable. The relative exposures of the three layers can be determined by numerical integration over the product of the CRT phosphor's spectral curve, filter spectral transmittance curve, and film spectral sensitivity curve. A visual idea of the effect of a given filter can also be obtained simply by looking at the light from the CRT through it. Naturally, if the hologram is to be illuminated by a color other than red, then a different set of filters is required.

For phase-only holograms, the processed red-absorbing layer should be as transparent as possible, implying that the red-sensitive layer must be heavily exposed selectively. We accomplish this by photographing a uniformly illuminated white piece of paper through a deep red (Wratten No. 92) filter, using eight times the exposure indicated by an unfiltered meter reading from the white paper.

ESTABLISHING THE PHASE-CONTROLLING EXPOSURE

A gray scale is displayed and photographed using the parameters and phase filter determined previously. Then the film is rewound in the camera and re-exposed to red light. The resulting densities to blue and green light are measured, and H&D curves are plotted to determine E_{min}, the minimum exposure at which both the blue and green H&D curves start to become linear.

Assuming that phase excursions are linear with density[27], we expose over the linear part of the H&D curve. The more heavily exposed parts of Kodachrome II are thinner, giving positive phase. If an exposure range E_{min} to E_{max} gives 2π phase, then the exposure E for a phase ϕ is given by

$$\log E = \log E_{min} + \log (E_{max}/E_{min}) \cdot \phi/2\pi \qquad (11)$$

or

$$E = E_{min} \cdot (E_{max}/E_{min})^{\phi/2\pi} \qquad (12)$$

Determination of the value of E_{max} corresponding to 2π phase, the "phase matching" condition, can be accomplished by displaying and photographing a series of patterns through the phase filter, with exposure ranges E_{min} to various E_{max}; the film then is re-exposed to red light. The pattern displayed should be one with a known diffraction pattern, such as a sawtooth or sine wave pattern or a phase-only hologram. E_{max} is determined by observing which hologram's image is nearest to the desired result. For a sawtooth pattern, which is nothing more than a prism modulo 2π, phase-matching occurs when the maximum intensity is diffracted into the first order. For phase-matching, $\log(E_{max}/E_{min})$ is found to be approximately 0.7 if both the blue and the green-absorbing layers are used. After this step, phase holograms can be made using the filters and the values of E_{min} and E_{max} determined in the calibration procedure.

ADDING AMPLITUDE CONTROL FOR ROACHs

The desired amplitude transmittance t_r is obtained by dividing the calculated amplitude A by a normalizing constant A_{max} so that t_r is less than 1.0 everywhere. By displaying and photographing gray scales through the red filter, the H&D curve for red light can be obtained. The exposure E_r needed to get a desired amplitude transmittance t_r is determined from the H&D curve using the relation

$$D_r - D_{rmin} = -\log t_r^2 = -2 \log t_r , \qquad (13)$$

where D_{rmin} is the density to red for the maximum exposure E_{rmax}. If, for simplicity, only the linear part of the H&D curve is used, then

$$\log E_r = \log E_{rmax} + B \cdot \log t_r \qquad (14)$$

or

$$E_r = E_{rmax}\, t_r^B \qquad (15)$$

where $B = 2(\log E_{rmax} - \log E_{rmin})/(D_{rmax} - D_{rmin})$ and D_{rmax} is the density corresponding to the minimum exposure, E_{rmin}.

Unfortunately, by adding amplitude information, we also add some undesired phase effects. If we assume that the phase contributed by the red-absorbing layer is also linear with its density, we can compensate for this phase by subtracting an appropriate amount from the computed phase exposure by replacing Equation 11 with

$$\log E = \log E_{min} + (\phi/2\pi) \cdot \log(E_{max}/E_{min}) - \beta \log (E_r/E_{rmin}). \qquad (16)$$

If computed log E is less than log E_{min}, then $\log(E_{max}/E_{min})$ is added to log E in order to preserve the phase modulo 2π. The factor β is determined experimentally and was found to be about 0.25.

Finally, since the calibration procedure we used is only approximate, one parameter at a time was varied to converge on the best set of exposure parameters. In particular, ROACHs should be made with half and double the exposures indicated by Equation 14. Once a good set of exposure parameters is found, ROACHs can be made dependably since highly repeatable commercial processing is available.

CONCLUSION

In this paper we have discussed new ways to generate computer holograms that are highly efficient, both in the use of the display device in synthesis and in the use of light in reconstruction. We have also described our synthesis procedure and practical advice for making these computer holograms.

ACKNOWLEDGMENT

We are grateful for the guidance of J.W. Goodman of Stanford University where this work was performed.

REFERENCES

1. L.B. Lesem, P.M. Hirsch, and J.A. Jordan, Jr., "Computer Synthesis of Holograms for 3-D Display", *Commun. ACM,* Vol. 11, pp. 661-674 (1968).
2. J.R. Fienup and J.W. Goodman, "Three New Ways to Make Computer Generated Color Holograms", *J. Opt. Soc. Am.,* Vol. 63, p. 1325A (1973).
3. K.G. Birch and Frances J. Green, "The Application of Computer-Generated Holograms to Testing Optical Elements", *J. Phys. D. Applied Physics,* Vol. 5, pp. 1982-1992 (1972).
4. B.R. Frieden, "On Arbitrarily Perfect Imagery with a Finite Aperture", *Optica Acta,* Vol. 16, pp. 795-807 (1969); B.R. Frieden, "The Extrapolating Pupil, Image Synthesis, and Some Thought Applications", *Appl. Opt.,* Vol. 9, pp. 2489-2496 (1970).
5. A. Vander Lugt, "Signal Detection by Complex Spatial Filtering", *IEEE Trans. Inform. Theory,* Vol. IT-10, pp. 139-145 (1964).
6. A.W. Lohmann and D.P. Paris, "Computer Generated Spatial Filters for Coherent Optical Data Processing", *Appl. Opt.,* Vol. 7, pp. 651-655 (1968).
7. M. Severcan, "Computer Generation of Coherent Optical Filters with High Light Efficiency and Large Dynamic Range", Information Systems Lab. Tech. Report No. 6415-6, Stanford University (1973).
8. B.R. Brown and A.W. Lohmann, "Complex Spatial Filtering with Binary Masks", *Appl. Opt.,* Vol. 5, pp. 967-969 (1966).
9. A.W. Lohmann, D.P. Paris and H.W. Werlich, "A Computer Generated Spatial Filter, Applied to Code Translation", *Appl. Opt.,* Vol. 6, pp. 1139-1140 (1967).
10. A.W. Lohmann and H.W. Werlich, "Incoherent Matched Filtering with Fourier Holograms", *Appl. Opt.,* Vol. 10, pp. 670-672 (1971).
11. E.N. Leith and J. Upatnieks, "Reconstructed Wavefronts and Communication Theory", *J. Opt. Soc. Am.,* Vol. 52, pp. 1123-1130 (1962).
12. J.J. Burch, "A Computer Algorithm for the Synthesis of Spatial Frequency Filters", *Proc. IEEE,* Vol. 55, pp. 599-601 (1967).
13. L.B. Lesem, P.M. Hirsch, and J.A. Jordan, "Holographic Display of Digital Images", *AFIPS Conference Proceedings, Fall Joint Computer Conference,* Vol. 41, (Spartan Books, Inc., New York, 1967), pp. 41-49.
14. B.R. Brown and A.W. Lohmann, "Computer-Generated Binary Holograms", *IBM J. Res. Develop.* Vol. 13, pp. 160-168 (1969).
15. R.E. Haskell, "Computer-Generated Holograms with Minimum Quantization", *J. Opt. Soc. Am.,* Vol. 63, p. 504, (1973).
16. W.H. Lee, "Sampled Fourier Transform Hologram Generated by Computer", *Appl. Opt.,* Vol. 9, pp. 639-643 (1970).
17. C.B. Burckhardt, "A Simplification of Lee's Method of Generating Holograms by Computer", *Appl. Opt.,* Vol. 9, p. 1949 (1970).
18. L.B. Lesem, P.M. Hirsch, and J.A. Jordan, Jr., "The Kinoform: A New Wavefront Reconstruction Device", *IBM J. Res. Develop.,* Vol. 13, pp. 150-155 (1969).
19. T.S. Huang, "Digital Holography", *Proc. IEEE,* Vol. 59, pp. 1335-1346 (1971).
20. P.L. Ransom, "Synthesis of Complex Optical Wavefronts", *Appl. Opt.,* Vol. 11, pp. 2554-2561 (1972).
21. R.J. Collier, C.B. Burckhardt, and L.H. Lin, *Optical Holography* (Academic Press, New York, 1971), pp. 542-563.
22. D.C. Chu, J.R. Fienup, and J.W. Goodman, "Multiemulsion On-Axis Computer Generated Hologram", *Appl. Opt.,* Vol. 12, pp. 1386-1388 (1973).
23. D.C. Chu and J.W. Goodman, "Spectrum Shaping with Parity Sequences", *Appl. Opt.,* Vol. 11, pp. 1716-1724 (1972).
24. D.C. Chu, "Three Ways to Obsolete the Kinoform", *J. Opt. Soc. Am.,* Vol. 63, p. 1325B (1973).
25. N.C. Gallagher and B. Liu, "Method for Computing Kinoforms that Reduces Reconstruction Error", *Appl. Opt.,* Vol. 12, pp. 2328-2335 (1973).
26. W.J. Dallas, "Deterministic Diffusers for Holography", *Appl. Opt.,* Vol. 12, pp. 1179-1187 (1973).
27. R.L. Lamberts, "Characterization of a Bleached Photographic Material", *Appl. Opt.,* Vol. 11, pp. 33-41 (1972).