Temporal Waveguiding in a Dispersive Time-Varying Medium

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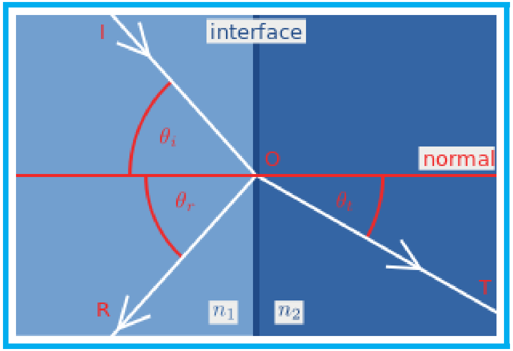
Introduction

- Wave propagation in a time-varying medium has a long history.
  F. R. Morgenthaler, IRE Trans. Microwave Theory Tech. **6**, 167 (1958);
  C. Caloz and Z. Deck-Léger, Parts I & II, IEEE Trans. Anten. Propag. **68**, 1569-1598 (2020).
- Simple case: A temporal boundary with different ε on its two side.
  T. Ruiz et al., IEEE Trans. Anten. Propag. **26**, 358 (1978);
  Y. Xiao, D. N. Maywar, and G. P. Agrawal, Opt. Lett. **39**, 574 (2014).
- A moving temporal boundary in a dispersive medium is of practical interest.
  Plansinis, Donaldson, Agrawal, PRL **115**, 183901 (2015);
  Plansinis, Donaldson, Agrawal, IEEE JQE **52**, 6100708 (2016).
- Analog of total internal reflection can be used for temporal waveguiding
  Plansinis, Donaldson, Agrawal, JOSA B **33**, 1112 (2016).

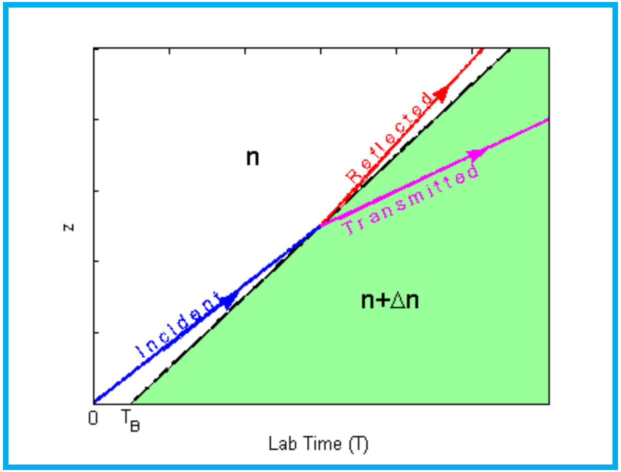
Temporal Reflection and Refraction

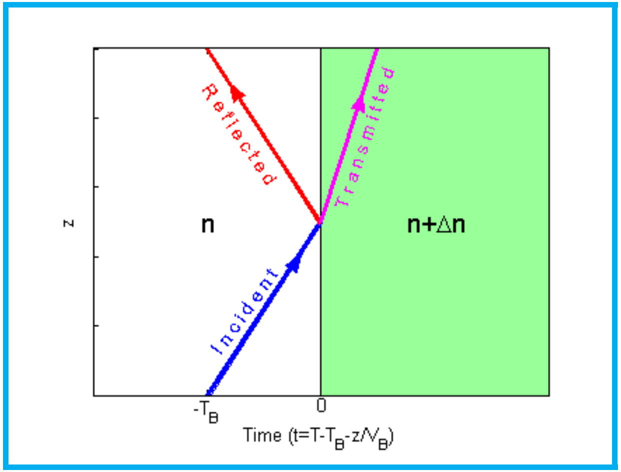
- Reflection and refraction of optical beams at a spatial boundary are well-known phenomena.
- What is the temporal analog of these two optical phenomena?



- What happens when an optical pulse arrives at a temporal boundary across which the refractive index changes from n to $n + \Delta n$?
- At a spatial boundary, frequency is preserved but momentum can change.
- At a temporal boundary, momentum is preserved but frequency can change.
- It is difficult to change n at a specific time across the entire medium. A moving temporal boundary is easier to realize.

Reflection at a Moving Boundary





- Both the pulse and temporal boundary travel forward at different speeds.
- It is convenient to work in a moving frame at which the boundary is stationary $(t = T - z/V_B)$.
- Momentum (or the wave vector) is conserved in the moving frame.

Pulse Propagation in a Dispersive Medium

- Consider an optical pulse propagating inside a medium with the dispersion relation $\beta(\omega)$.
- Temporal discontinuity at $t = T_B$ can be incorporated through a step function. Its use provides the dispersion relation:

$$\beta(\omega) = \beta_0 + \Delta\beta_1(\omega - \omega_0) + \frac{\beta_2}{2}(\omega - \omega_0)^2 + \beta_B H(t - T_B).$$

- $\Delta\beta_1 = \beta_1 - 1/V_B$ is pulse's speed relative to the temporal boundary located at $t = T_B$; $H(t - T_B)$ is the Heaviside step function.
- $\beta_B = k_0 \Delta n$ when refractive index changes by Δn for $t > T_B$ $(k_0 = 2\pi/\lambda)$.
- The dispersion relation can be used to investigate changes in pulse's shape and spectrum occurring when the pulse arrives at the boundary.

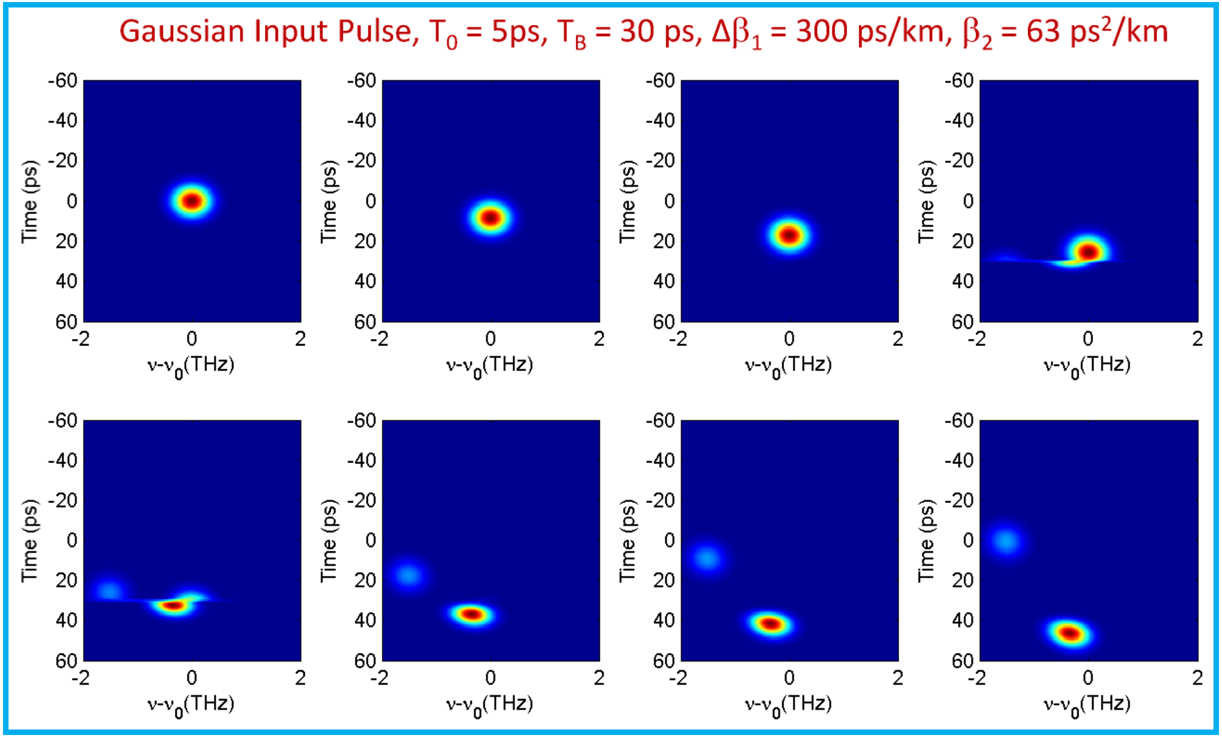
Equation for the Pulse Envelope

- Slowly varying envelope of the pulse satisfies

$$\frac{\partial A}{\partial z} + \Delta\beta_1 \frac{\partial A}{\partial t} + \frac{i\beta_2}{2}\frac{\partial^2 A}{\partial t^2} = i\beta_B H(t - T_B)A.$$

- This linear equation can be solved with the split-step Fourier method to study what happens at a moving temporal boundary.
- Numerical results how that the pulse splits into two parts with the spectra shifted in frequency by different amounts.
- Two parts propagate at different speeds because of chromatic dispersion.
- Reflected part moves slower and never crosses the temporal boundary.
- Fraction of energy in the reflected pulse depends on the index change Δn. Total reflection is possible for large Δn.

Simulations for Gaussian pulses

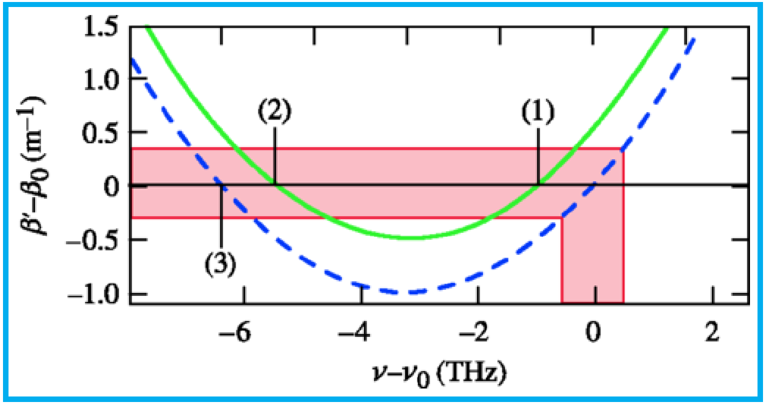


Spectrograms at different distances showing pulse's splitting at the TB.

Momentum Conservation

Conservation of momentum explains all results:

- Blue curve for $t < T_B$
- Green curve for $t > T_B$
- Red region: pulse spectrum



- Reflection corresponds to solution (3) on the blue curve.
- Refraction corresponds to solution (1) on the green curve.
- Solution (2) is not physical because its slope is opposite to that of (1).
- Both reflection and refraction manifest as red-shifted pulses; blue shifts occur if β_2 or Δn is negative.

Spectral Shifts at the Temporal Boundary

- Momentum conservation provides analytical expressions for frequency shifts:

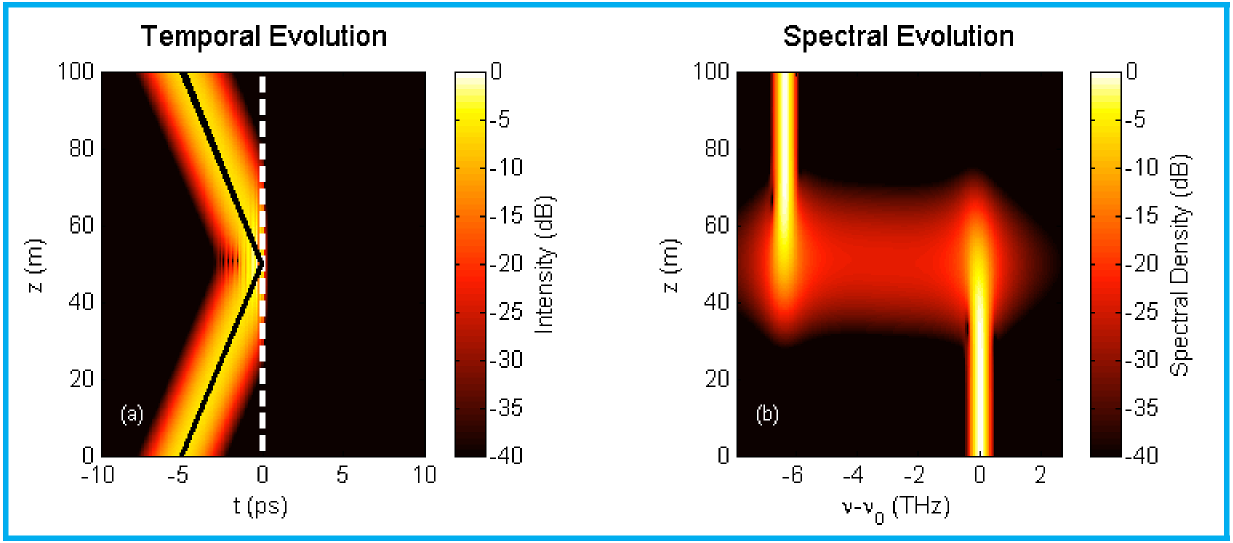
$$\omega_r = \omega_0 - 2(\Delta\beta_1/\beta_2),$$

$$\omega_t = \omega_0 + \frac{\Delta\beta_1}{\beta_2}\left(\sqrt{1 - \frac{2\beta_B\beta_2}{(\Delta\beta_1)^2}} - 1\right).$$

Total Internal Reflection

- For large β_B, ω_t becomes a complex quantity. This situation corresponds to temporal analog of total internal reflection (TIR).
- The TIR condition is $(2\beta_B\beta_2) > (\Delta\beta_1)^2/$. Temporal TIR occurs only if β_B and β_2 have the same signs.
- In the case of $\beta_2 > 0$, n increases after the temporal boundary. The opposite scenario occurs for $\beta_2 < 0$.

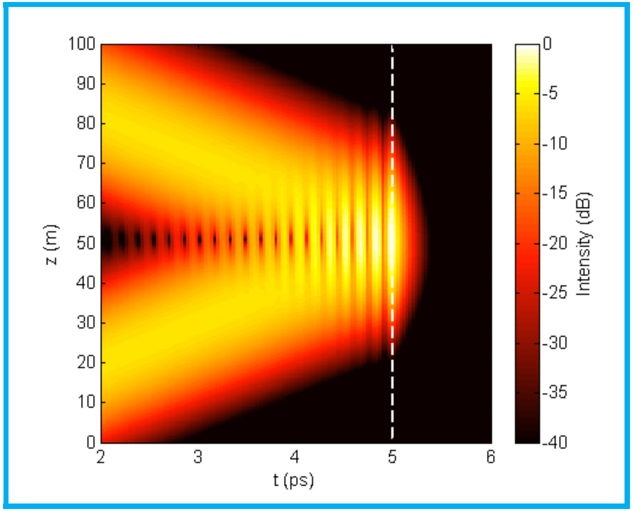
TIR of Gaussian Pulses



Plansinis, Donaldson, Agrawal, PRL **115**, 183901 (2015).

- Index change was large enough ($\Delta n \sim 10^{-6}$) to satisfy the TIR condition.
- Entire pulse energy gets reflected at the temporal boundary.
- Spectrum shifts by > 6 THz after TIR of the pulse at the boundary.

TIR of Gaussian Pulses (cont.)

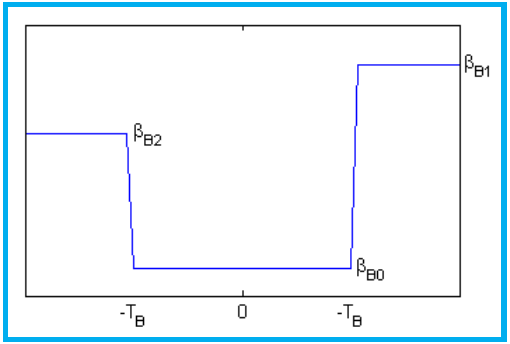


Magnified view of the temporal TIR and the evanescent wave associated with it.

- Fringe patten is a consequence of interference between the incident and reflected pulses: Fringe spacing $= \Delta t = (\Delta \nu)^{-1}$.
- Temporal analog of the Goos-Hänchen shift also exists.
  Ponomarenko, Zhang, Agrawal, Phys. Rev. A **106** L061501 (2022).

Temporal Waveguides

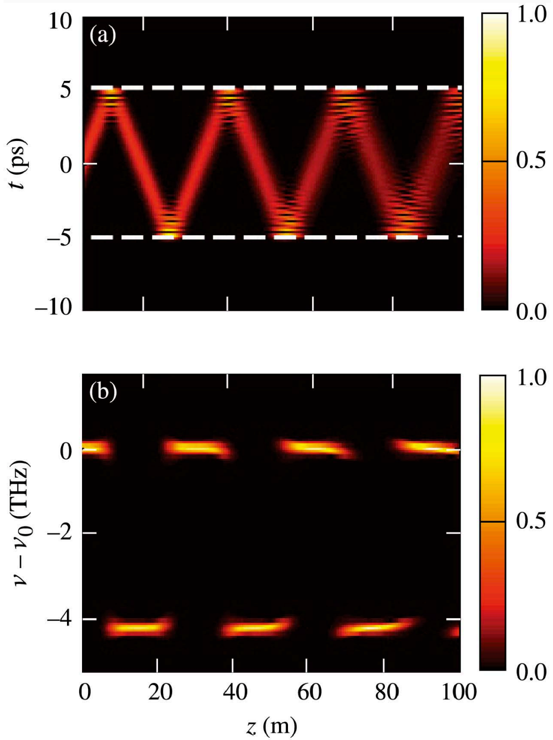
- TIR can be used to make temporal waveguides that trap optical pulses.
- Two temporal boundaries are needed.
- Refractive index of the central region can be lower or higher.



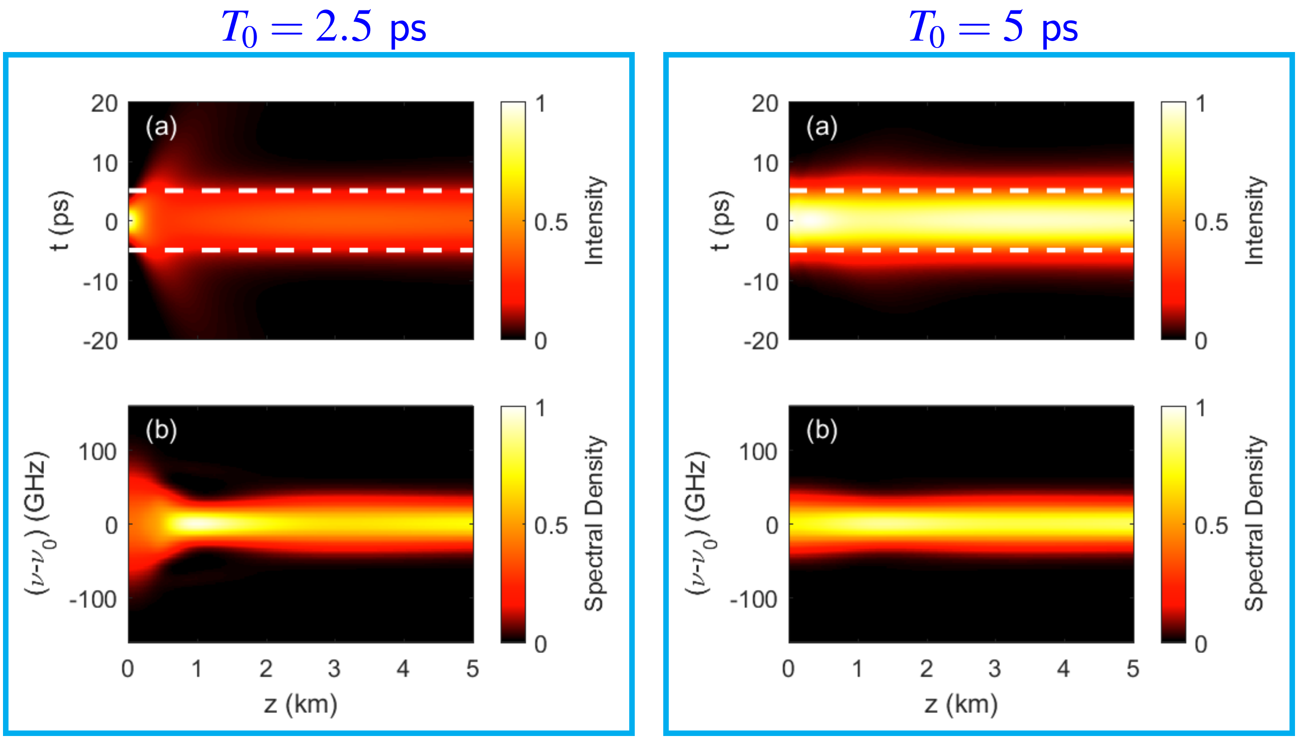
- A pulse is trapped inside the waveguide if it undergoes TIR at both temporal boundaries.
- Temporal waveguide support modes just as spatial waveguides do.
- The number of modes depends on the parameter $V = 2\sqrt{k_0 \Delta n T_B^2/\beta_2}$.
  Plansinis, Donaldson, Agrawal, JOSA B **33**, 1112 (2016).

Multimode Waveguides

- A 1-ps Gaussian pulse trapped inside a 10-ps wide waveguide: $\beta_B = 5.6\ \text{m}^{-1}$, $\Delta\beta_1 = 66.7$ ps/km, $\beta_2 = 50$ ps^2/km
- Pulse undergoes TIR and its spectrum shifts after each reflection.
- Pulse broadening eventually creates distortions and pulse excites multiple waveguide modes ($V = 26.8$).
- This approach can work when the propagation length is a fraction of the dispersion length ($z < L_D$).



Single-Mode Waveguides



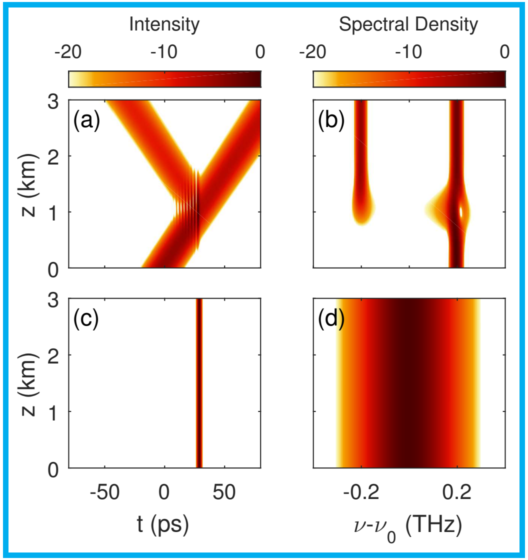
Plansinis, Donaldson, Agrawal, JOSA B **33**, 1112 (2016)

Experimental Considerations

- Experimental realization of temporal waveguides requires two moving boundaries. Two techniques can be used for this purpose.
- A high-index temporal window can be created through the Pockels effect in an electro-optic medium using microwaves.
- An intense pump pulse can create a high-index region over its width in a nonlinear medium through the optical Kerr effect.
- Spreading of pump pulse can be controlled if its wavelength lies near the zero-dispersion wavelength of an optical fiber.
- This case has been studied numerically: Plansinis et al., J. Opt. Soc. Am. B **35**, 436 (2018).
- Simulations show that probe pulses can be confined to a temporal window created using super-Gaussian pump pulses.

Temporal Reflection from a Soliton

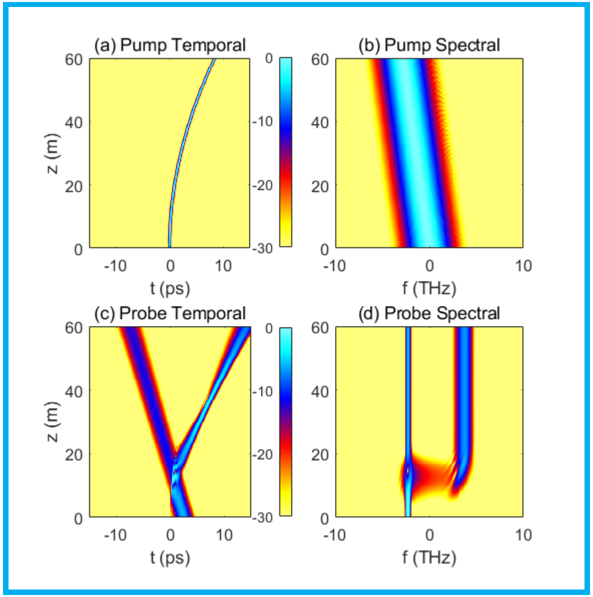
- Solitons are ideal for producing a moving temporal boundary.
- They require a medium with anomalous GVD ($\beta_2 < 0$).
- Figure shows reflection of a probe pulse from a soliton: Plansinis et al., JOSA B **35**, 436 (2018).



- Pump pulse (3-ps wide) travels as a soliton inside the fiber ($\beta_2 < 0$).
- Probe pulse (16-ps wide) arrives near soliton at 1 km ($\beta_2 = 25$ ps^2/km).
- It is partly reflected by the soliton with a large spectral shift.

Impact of Raman scattering

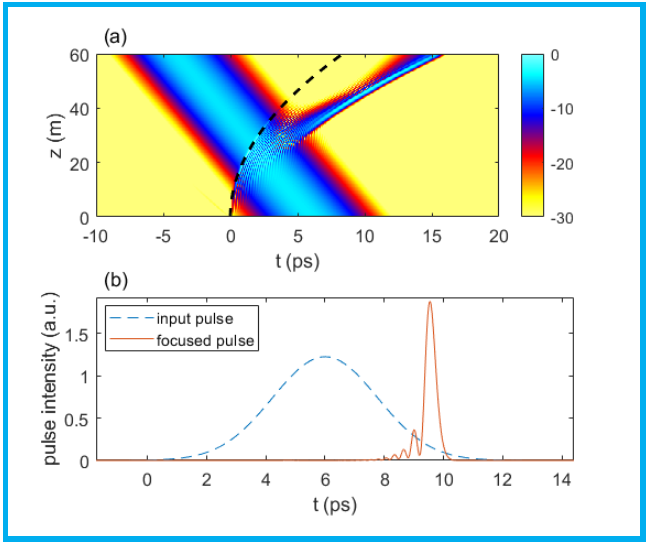
- Intrapulse Raman scattering (IRS) becomes relevant for short solitons.
- Solitons slow down as their spectrum red-shifts through IRS.
- Reflection of probe occurs from a curved temporal boundary.



- Reflection of a 4-ps probe pulse from a 100-fs soliton is shown.
- Soliton's trajectory is bent because of Raman-induced spectral changes.
- Zhang, Donaldson, Agrawal, JOSA B **39**, 1950 (2022).

Temporal Focusing

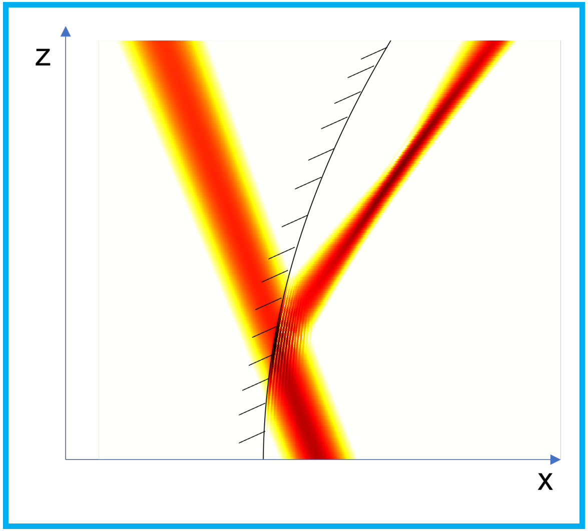
- Probe pulse exhibits temporal focusing after reflection.
- Its compression is induced by the curved soliton's trajectory acting as a temporal boundary.
- Zhang, Donaldson, and Agrawal, JOSA B **39**, 1950 (2022).



- Figure shows reflection of 4-ps probe pulse from 100-fs solitons.
- Reflected pulse is compressed by a factor of 10 at the focal point.
- The focusing is a consequence of the Raman-induced red-shift of the pump pulse traveling as a soliton.

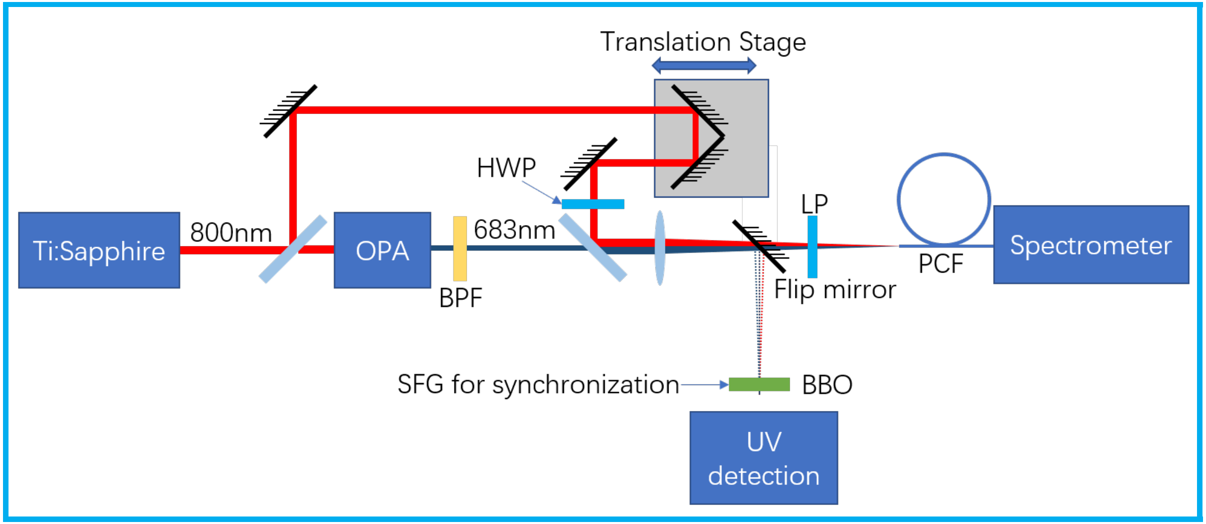
Space-Time Duality for Focusing

- A parabolic mirror curves the phase front on reflection.
- This is the reason why the reflected light comes to a focus.
- Any optical Beam is compressed by a parabolic mirror at its focal point.



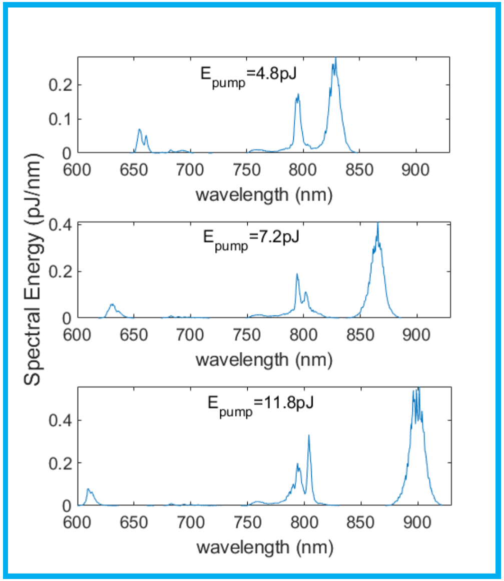
- Raman-induced temporal focusing inside a dispersive nonlinear medium is the temporal analog of a parabolic mirror.
- Curved trajectory of the soliton, resulting from frequency shifts induced by Raman scattering, mimics a parabolic mirror.

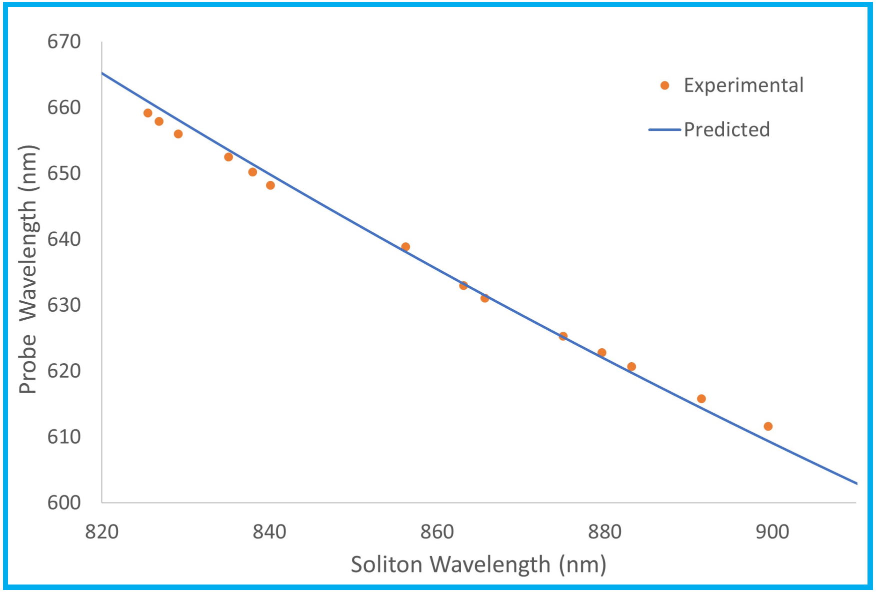
Recent Experiment on Temporal Waveguides



- A photonic crystal fiber (3.75-m-long) used as a dispersive nonlinear medium.
- Pump pulses were only 30-fs wide at the 800-nm wavelength.
- Probe pulses at 683-nm were > 1 ps wide after the bandpass filter.
- Output spectra recorded for different energies of pump pulses,

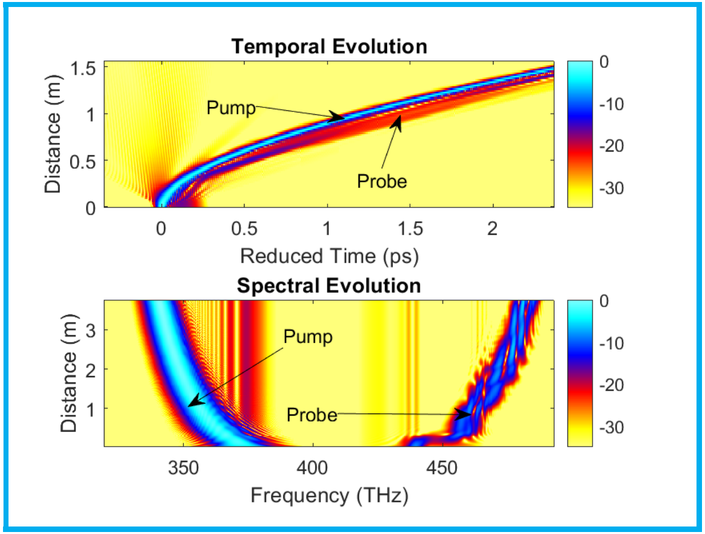
Measured Output Spectra

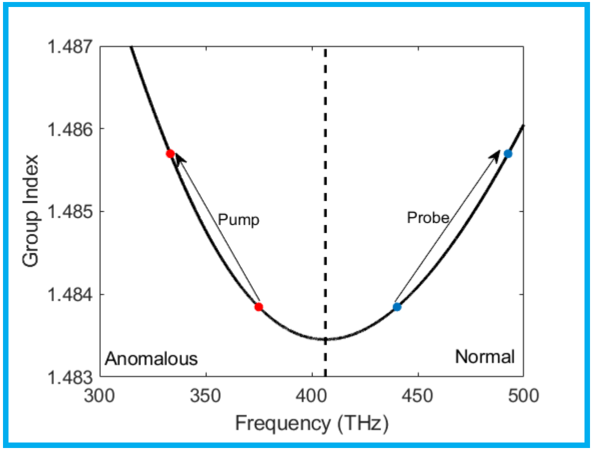




- Pump wavelength shifts toward red because of Raman scattering.
- Probe wavelength shifts toward blue because of temporal waveguiding.
- These shifts agree with predictions based on dispersion of the fiber.

Simulations and Dispersion Data





- Probe follows pump's trajectory because of multiple temporal reflections.
- The two pulses shift their spectra on opposite sides to ensure that they move at the same speed.
- Measured spectral shifts agree fully with the dispersion data.

Conclusions

- Inclusion of chromatic dispersion is relevant for pulses propagating inside a time-varying medium.
- Kerr nonlinearity of a dispersive medium can be exploited to create moving temporal boundaries using intense pump pulses.
- Temporal equivalent of total internal reflection occurs for optical pulses at a moving temporal boundary.
- A waveguide is formed by two such temporal boundaries.
- Such waveguides are temporal analog of planar optical waveguides.
- Raman scattering leads to new effects when femtosecond pulses are propagated as solitons.
- An experiment has shown that such short solitons, forming inside a photonic crystal fiber, can guide probe pulses along their trajectories.