

Impact of frequency-dependent nonlinearity on soliton trajectory in microstructured optical fiber

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1. Introduction
2. Modelling of pulse propagation equation
3. Microstructured optical fiber
4. Frequency-dependent nonlinear medium
5. Comparison with analytical theory
6. Wavelength shift
7. Summary

- Optical pulses can propagate as solitons by balancing the effects of group velocity dispersion and self-phase modulation
- Dynamics of the short optical pulse are usually explained by generalized nonlinear Schrödinger equation

$$\frac{\partial \tilde{A}_\omega}{\partial z} = i\beta(\omega)\tilde{A}_\omega + i\gamma(\omega)(1 - f_R)F(A|A|^2) + if_R\gamma(\omega) \times F\left(A \int_0^\infty h_R(\tau) |A(z, t - \tau)|^2 d\tau\right)$$

$$\gamma(\omega) = \frac{n_2\omega}{cA_{eff}(\omega)}$$

$$A_{eff}(\omega_0) \longrightarrow \gamma(\omega) = \gamma_0 + \gamma_1 (\omega - \omega_0)$$

(Includes: Dispersion, SPM, Self-steepening and Raman)

- GNLSE have some limitations
- The photon number remains conserved for $\gamma_1 = \frac{\gamma_0}{\omega_0}$ i.e. $\gamma(\omega) = \frac{\gamma_0\omega}{\omega_0}$

K. J. Blow and J. Wood IEEE J. Quan. Elect. 25, 2665–2673 (1989) G. P. Agrawal, Nonlinear Fiber Opt. (Academic, 2007)

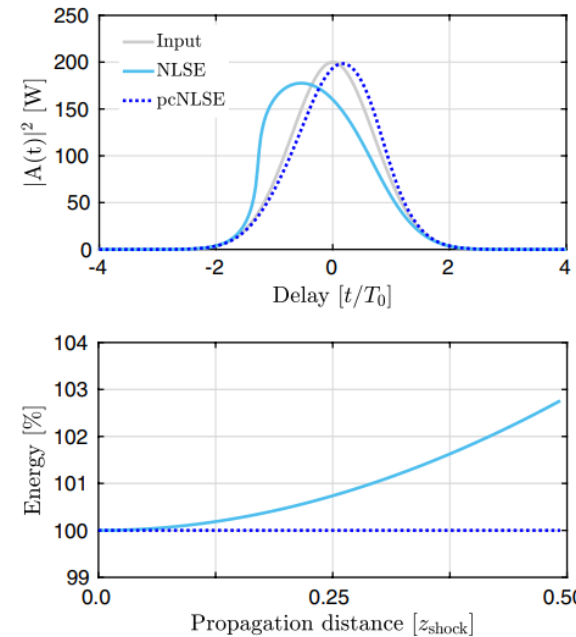
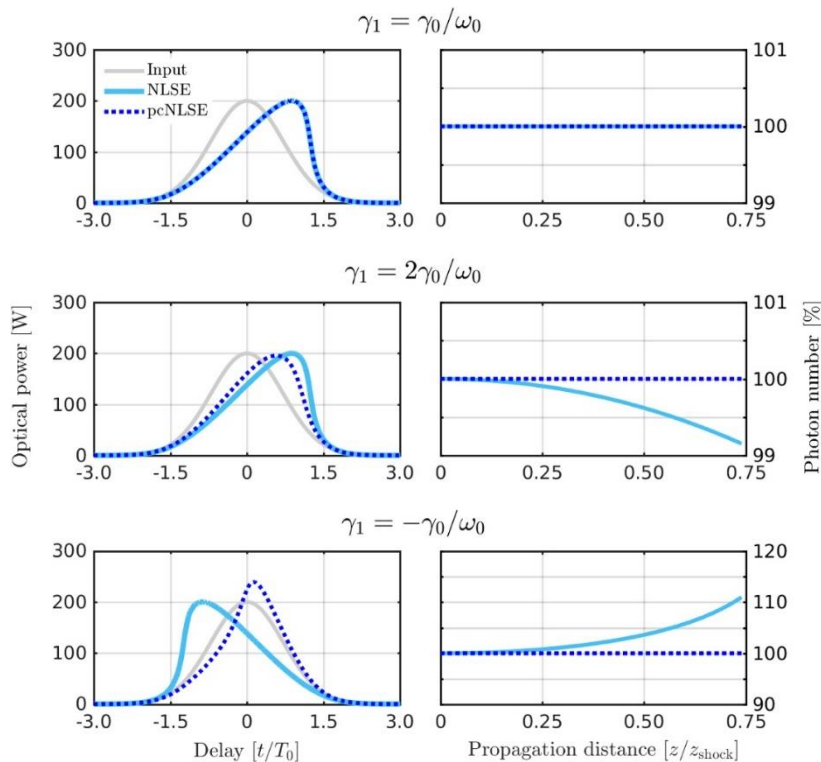
Modified Nonlinear Schrödinger equation

$$\frac{\partial \tilde{A}_\omega}{\partial z} = i\beta(\omega)\tilde{A}_\omega + i\frac{\bar{\gamma}(\omega)}{2}F(C^*B^2) + i\frac{\bar{\gamma}(\omega)^*}{2}F(B^*C^2)$$

$$\overline{\gamma(\omega)} = \sqrt[4]{\gamma(\omega) \times (\omega)^3}$$

$$\tilde{B}_\omega = \left(\frac{\gamma(\omega)}{\omega}\right)^{1/4} \tilde{A}_\omega \quad \tilde{C}_\omega = \left(\frac{\gamma(\omega)}{\omega}\right)^{*1/4} \tilde{A}_\omega$$

$$E = \sum_{\omega} |\tilde{A}_\omega|^2 \quad N = \sum_{\omega} \frac{(|\tilde{A}_\omega|^2)}{(\omega)}$$

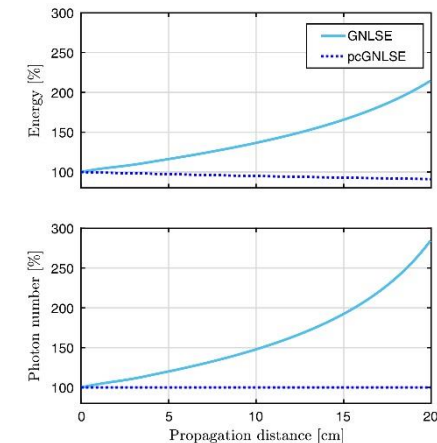
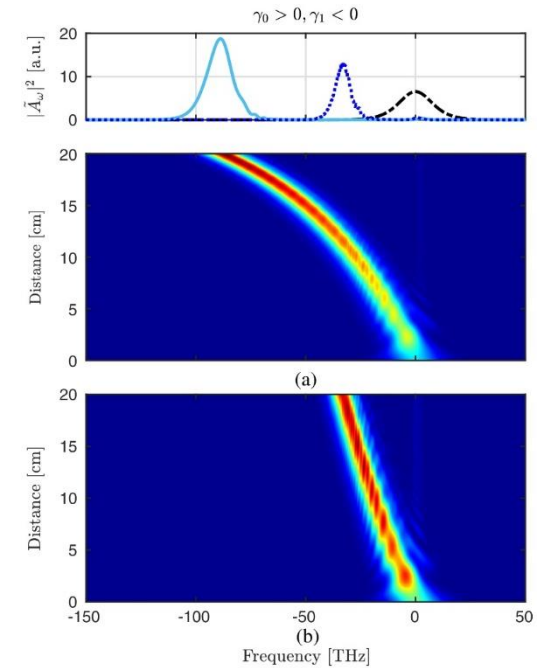
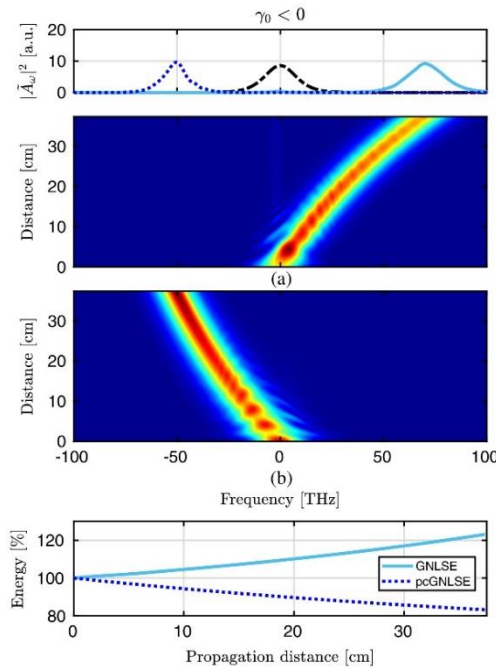
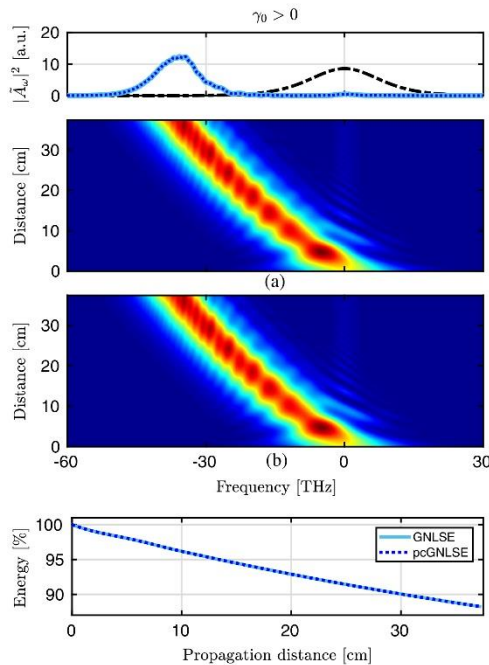


Quadratic
term to the
nonlinearity

[J. Bonneti et al., J. Opt. Soc. Am. B 36, 3139–3144 (2019)]

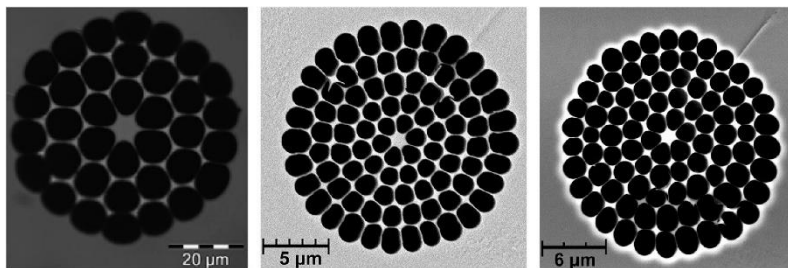
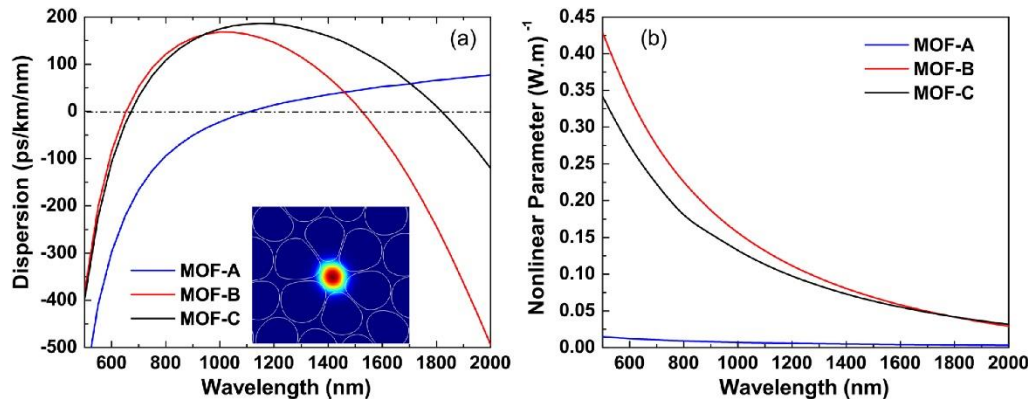
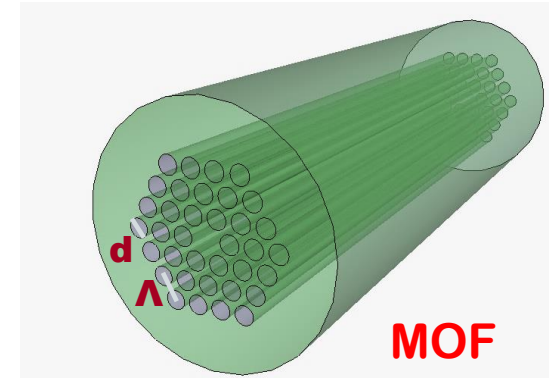
Photon conserving GNLSE (pcGNLSE)

$$\frac{\partial \tilde{A}_\omega}{\partial z} = i\beta(\omega)\tilde{A}_\omega + i\frac{\bar{\gamma}(\omega)}{2}F(C^*B^2) + i\frac{\bar{\gamma}(\omega)^*}{2}F(B^*C^2) + if_R\bar{\gamma}(\omega)^*F\left(B\int_0^\infty h_R(\tau)|B(t-\tau)|^2d\tau - B|B|^2\right)$$



[J. Bonneti et al., J. Opt. Soc. Am. B **37**, 445-450 (2020)

- Dispersion properties can be tailored
- Enhanced nonlinearity due to tight modal confinement in the core.
- Effective area becomes frequency dependent

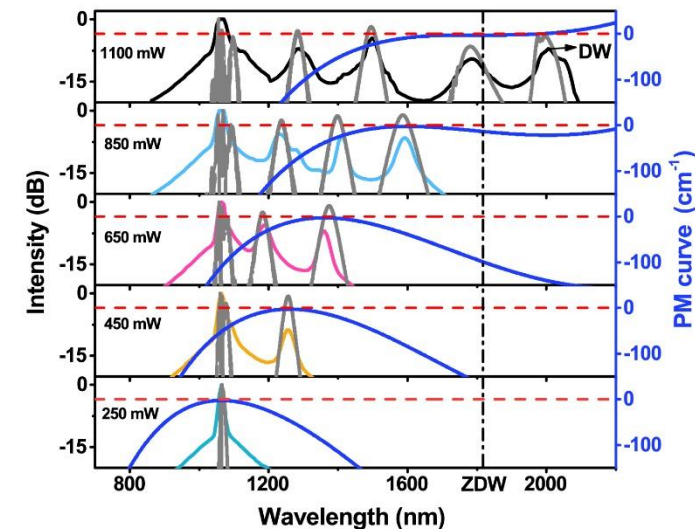


50μm

15μm

20μm

Manipulation of dispersive wave

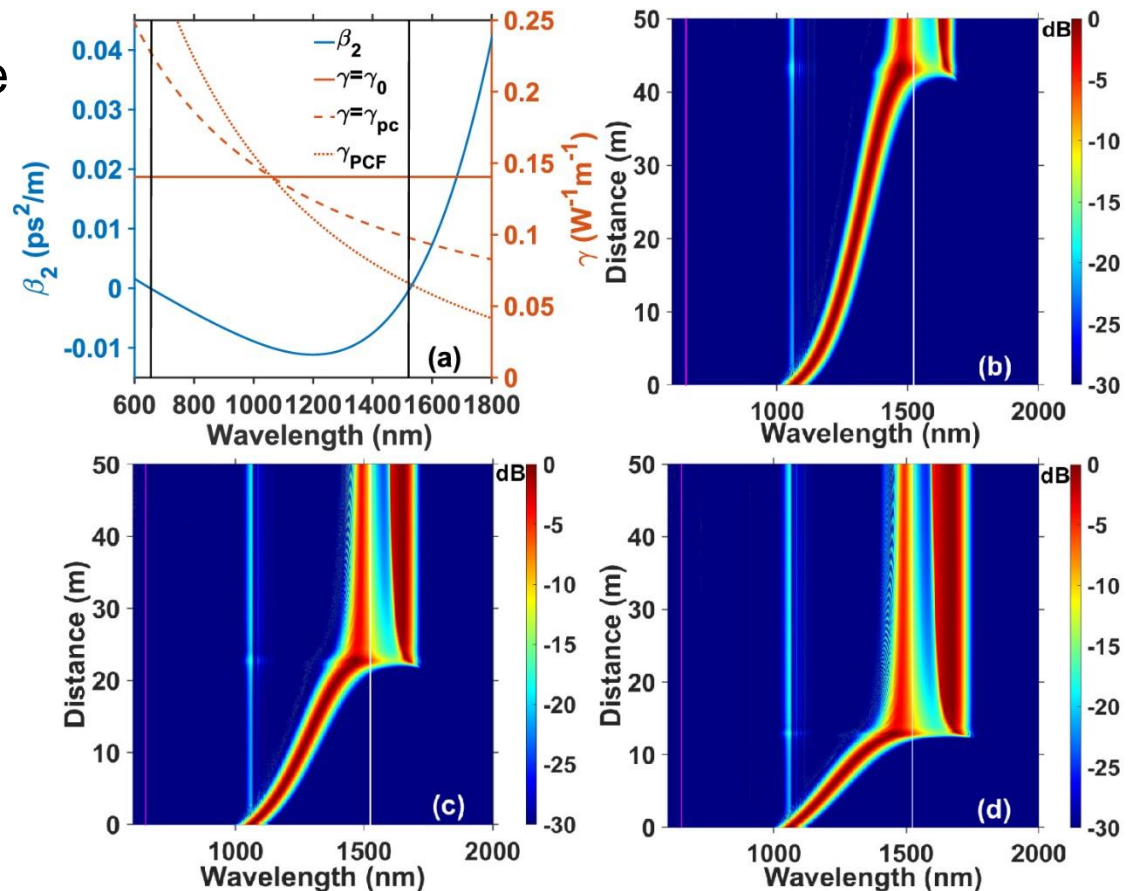


S. Bose et al., Appl. Opt. **59**, 9015-9022 (2020)

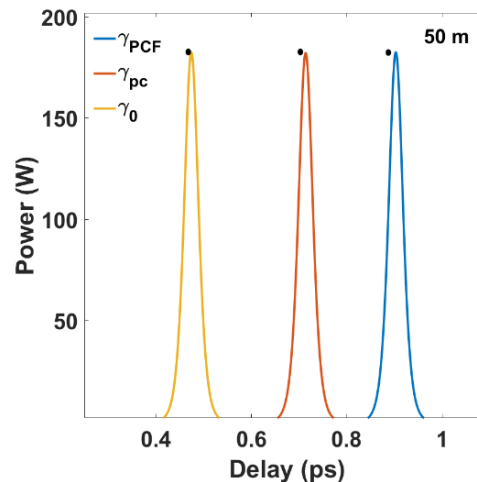
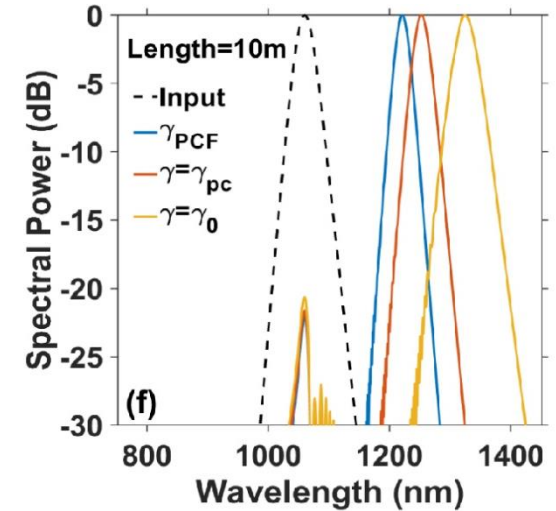
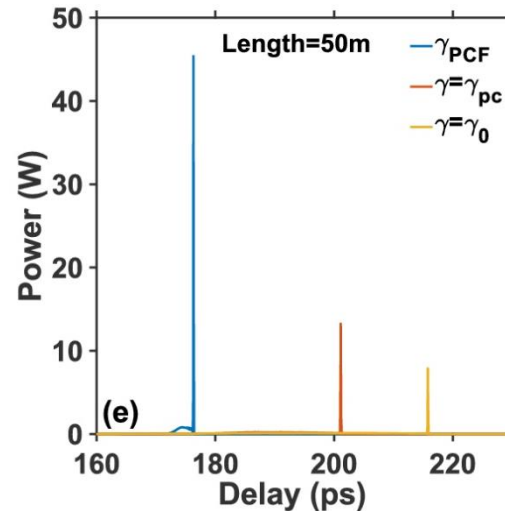
Spectral evolution of the fundamental soliton

- Dynamic interplay between the higher order nonlinear and dispersion terms on the shaping of the Raman soliton
- Raman induced frequency shift (RIFS) suppression through spectral recoil
- Non-solitonic radiation (NSR)
- Standard GNLSE overestimates the RIFS for most PCFs

$$A(0, t) = \sqrt{P_0} \text{sech}(T/t_0) \quad t_0 = 20 \text{ fs}; \quad P_0 = 180 \text{ W}$$



- The temporal and spectral shift are recorded for three different nonlinear profile
- Variation of peak power of the soliton are observed
- Compare with an analytical expression



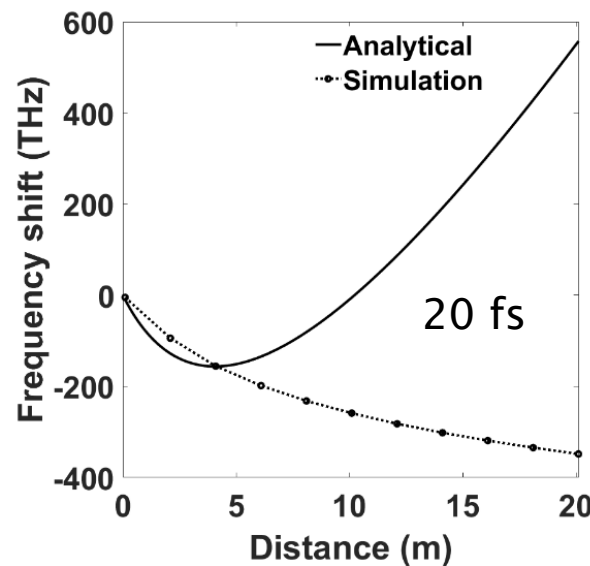
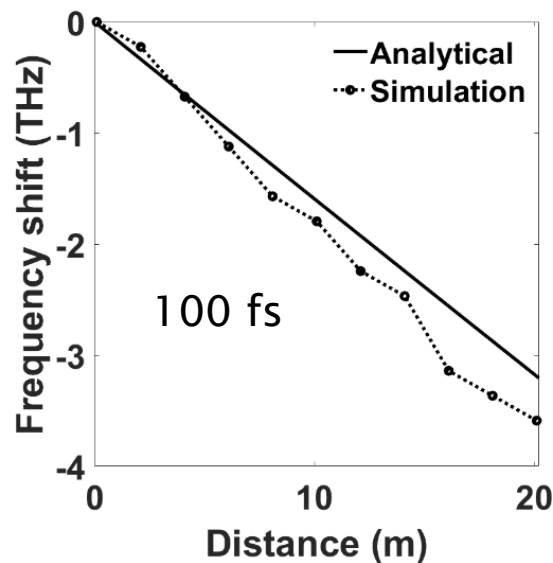
In absence of HOD and Raman terms

$$T = \frac{a + 2}{3} \frac{\beta_2 z}{\omega_0 t_0^2} \quad a = \frac{\gamma_1 \omega_0}{\gamma_0}$$

Linale et al., Opt. Lett. **45**, 4535-4538 (2020)

- The analytical expression is based on the moment method
- RIFS in the presence of propagation losses, self-steepening, and dispersion slope

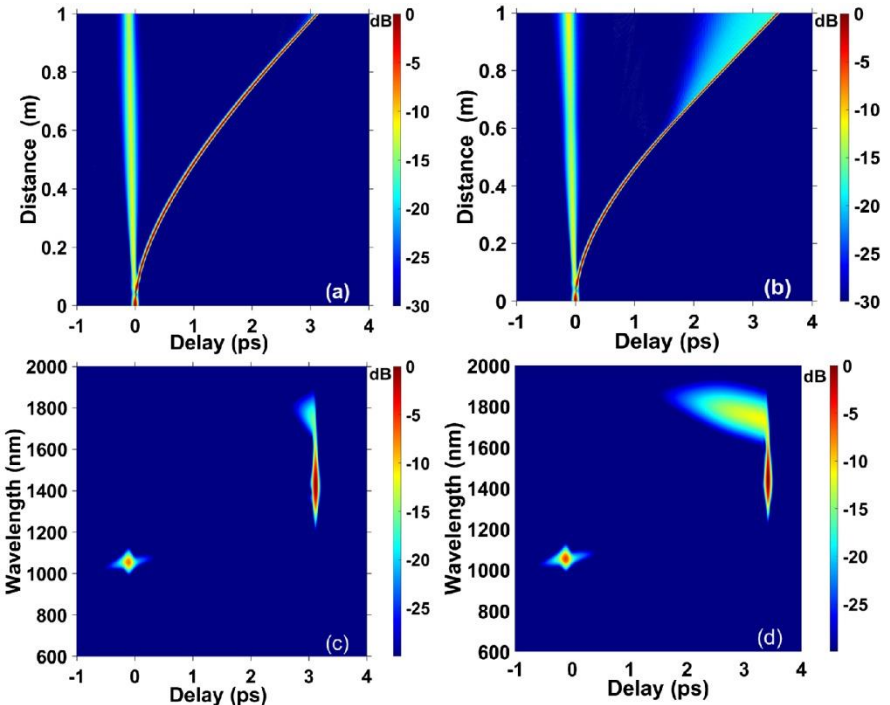
$$\Omega_{tot} = -\omega_0 \left[1 - \left(\sqrt[3]{\frac{5\omega_0 T_0^4}{8T_R |\beta_2| z_{eff} + 5\omega_0 T_0^4}} \right) - \frac{|\beta_2|}{\beta_3} \left[\sqrt[4]{\left(1 + \frac{32T_R \beta_3}{15T_0^4} z_{eff} \right)} - 1 \right] + \frac{8T_R |\beta_2|}{15T_0^4} z_{eff} \right]$$



It matches well
with long pulses

Pulse evolution of the higher order soliton

- Second-order soliton undergoes fission and splits into two fundamental solitons of different widths
- Significant changes in the spectrogram



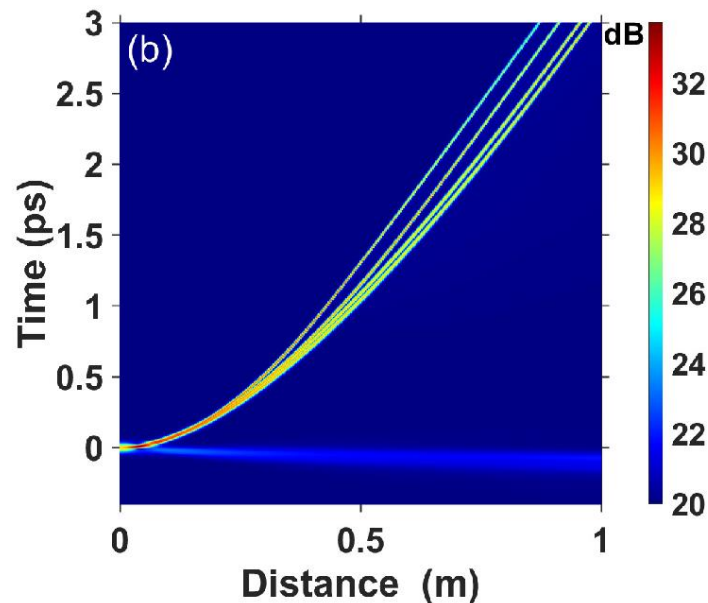
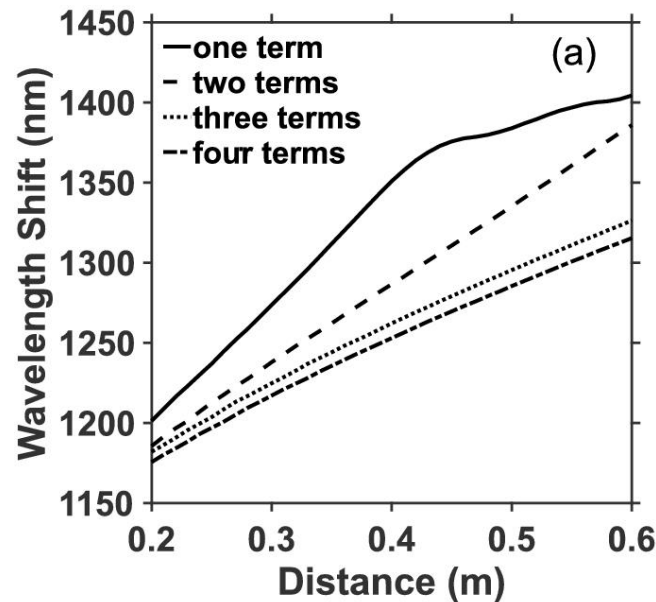
$$N = 2; P_0 = 715W$$

Additional spectral feature at 1620 nm due to the trapping of the NSR

$$N = 3; P_0 = 1610 W$$

$$\gamma(\omega) = \gamma_0 + \gamma_1(\omega - \omega_0) + \gamma_2/2! (\omega - \omega_0)^2 + \gamma_3/3! (\omega - \omega_0)^3 + \dots$$

An important question is how many terms should be retained in the Taylor expansion of $\gamma(\omega)$?



RIFS as a function of distance for $N=2$

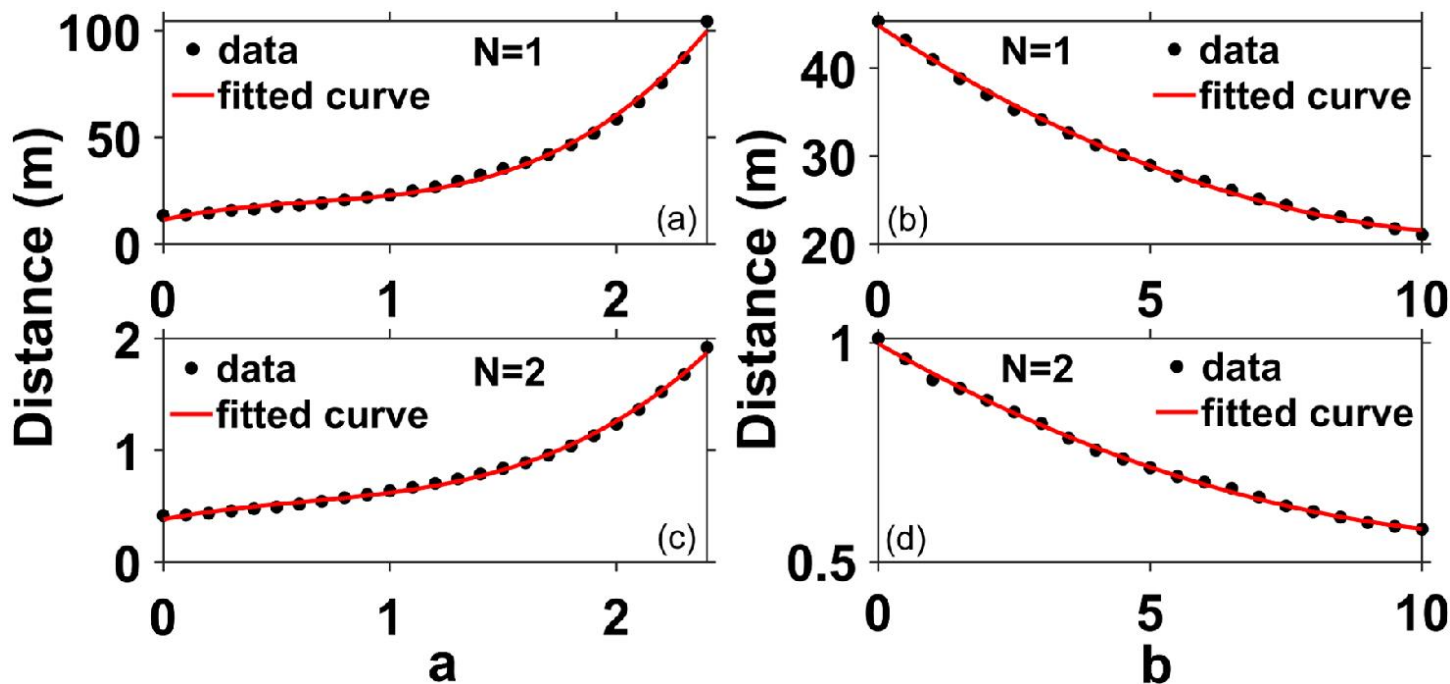
Effect of higher order nonlinear terms

$$\gamma(\omega) = \gamma_0 \left(1 + a(\omega - \omega_0)/\omega_0 + b(\omega - \omega_0)^2/2\omega_0^2 \right)$$

dimensionless

$$a = \frac{\gamma_1 \omega_0}{\gamma_0}$$

$$b = \frac{\gamma_2 \omega_0}{\gamma_0}$$



The length scale is reduced by a factor of 50 when we switch from $N=1$ to $N=2$ soliton

- Results are true for any high-confinement optical waveguide, including tapered fibers and suspended-core fibers
- The frequency dependence of $\gamma(\omega)$ in such fibers cannot be replicated using the conventional GNLSE based on the self-steepening effect
- To ensure the conservation of the photon number, we used the recently proposed modified GNLSE to reveal the importance of few-cycle solitons in a nonlinear waveguide
- Numerical simulations reveal that the rate of RIFS (along the fiber's length) is influenced by higher-order nonlinear terms
- Solitons are used to design a wavelength-tunable optical source



Thank you very much for your attention!