Optical Communication Systems (OPT428)

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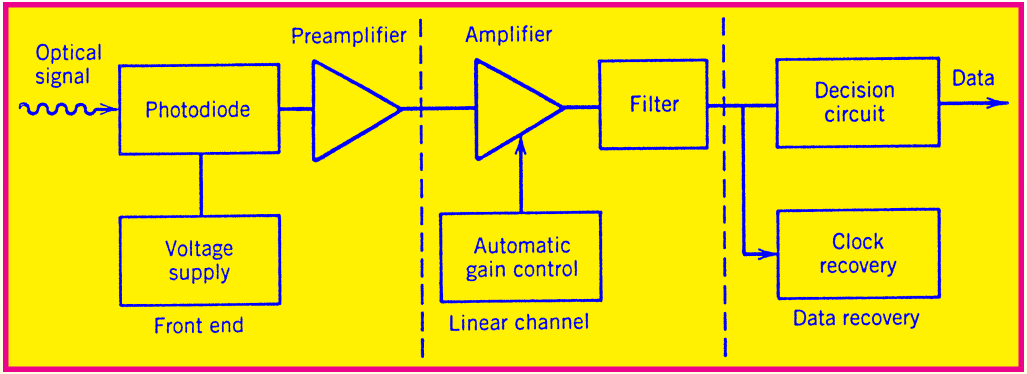
Chapter 5: Signal Recovery and Noise

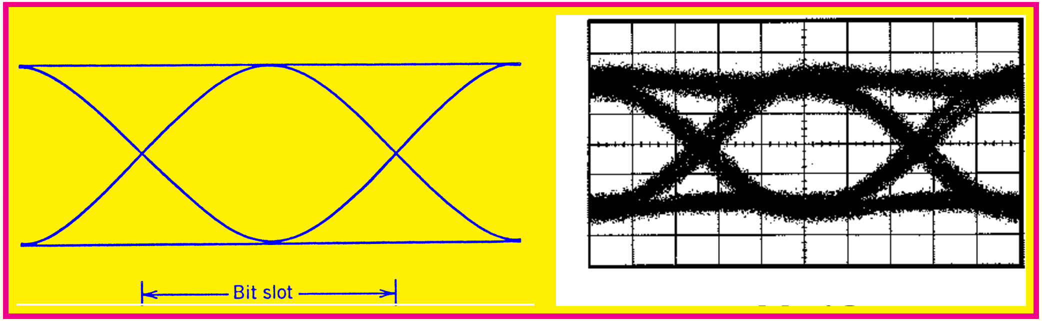
- Noise Added during Photodetection
- Signal-to-Noise Ratio (SNR)
- Bit Error Rate (BER)
- Sensitivity Degradation
- Forward Error Correction (FEC)

Optical Receivers



- Front end converts optical signal into electrical form.
- Linear channel amplifies and filters the electrical signal.
- Data recovery section creates electrical bit stream using clock-recovery and decision circuits.

Data-Recovery Section



- A clock-recovery circuit isolates the frequency $f = B$ from the received signal.
- The clock helps to synchronize the decision process.
- Decision circuit compares the output to a threshold level at sampling times set by the clock.
- Eye diagram is useful for system monitoring.
- The best sampling time corresponds to maximum eye opening.

Shot Noise

- Photocurrent, $I(t) = I_p + i_s(t)$, fluctuates because electrons are generated at random times.
- Average current $I_p = R_d P_{\text{in}}$; $\quad R_d = \eta q / h\nu_0$;
  η represents quantum efficiency of photodetector.
- Current fluctuations occur such that $\langle i_s(t) \rangle = 0$ and

$$\langle i_s(t) i_s(t+\tau) \rangle = \int_{-\infty}^{\infty} S_s(f) \exp(2\pi i f \tau)\, df.$$

- White noise: Spectral density $S_s(f)$ constant.
- Noise variance: $\sigma_s^2 = \int_{-\infty}^{\infty} S_s(f)\, df = 2qI_p \Delta f$.
- Effective noise bandwidth Δf is related to detector bandwidth.
- Adding the contribution of dark current I_d

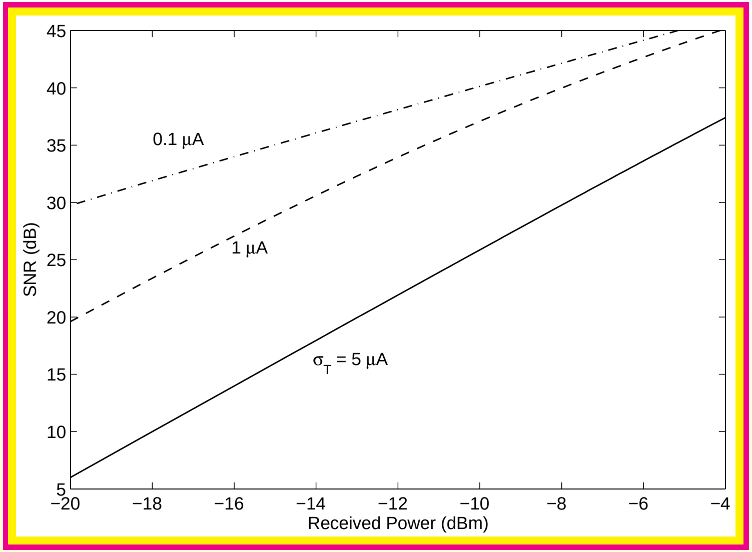
$$\sigma_s^2 = 2q(I_p + I_d)\Delta f.$$

Thermal noise

- Additional fluctuations occur at any finite temperature because of thermal motion of electrons in any resistor.
- Total current: $I(t) = I_p + i_s(t) + i_T(t)$.
- Spectral density $S_T(f) = 2k_B T/R_L$ depends on temperature and load resistor R_L.
- Noise variance: $\sigma_T^2 = \int_{-\infty}^{\infty} S_s(f)\,df = (4k_B T/R_L)\Delta f$.
- Amplifier noise: All electrical amplifiers enhance thermal noise by the amplifier noise figure F_n.
- Total thermal noise: $\sigma_T^2 = (4k_B T/R_L)F_n\Delta f$.
- Total Receiver Noise:

$$\sigma^2 = \sigma_s^2 + \sigma_T^2 = 2q(I_p + I_d)\Delta f + (4k_B T/R_L)F_n\Delta f.$$

Signal-to-Noise Ratio



$$\mathrm{SNR} = \frac{I_p^2}{\sigma^2} = \frac{R_d^2 P_{\mathrm{in}}^2}{2q(R_d P_{\mathrm{in}} + I_d)\Delta f + 4(k_B T/R_L)F_n \Delta f}.$$

Increase in SNR with received power P_{in} for three values of σ_T for a receiver bandwidth of 30 GHz.

Thermal-Noise Limit

- In the limit $\sigma_T \gg \sigma_s$, SNR becomes:

$$\mathrm{SNR} = \frac{R_L R_d^2 P_{\mathrm{in}}^2}{4k_B T F_n \Delta f}.$$

- Noise-equivalent power: Defined as the minimum optical power per unit bandwidth required to produce SNR $= 1$:

$$\mathrm{NEP} = \frac{P_{\mathrm{in}}}{\sqrt{\Delta f}} = \left(\frac{4k_B T F_n}{R_L R_d^2}\right)^{1/2} = \frac{h\nu}{\eta q}\left(\frac{4k_B T F_n}{R_L}\right)^{1/2}.$$

- NEP is often used to quantify thermal noise.
- Typical values of NEP are in the range of 1 to 10 pW/$\sqrt{\mathrm{Hz}}$.
- Optical power needed to realize a specific value of SNR obtained from $P_{\mathrm{in}} = (\mathrm{NEP}\sqrt{\Delta f})\mathrm{SNR}$.

Shot-Noise Limit

- In the opposite limit, $\sigma_s \gg \sigma_T$:

$$\mathrm{SNR} = \frac{R_d P_{\mathrm{in}}}{2q\Delta f} = \frac{\eta P_{\mathrm{in}}}{2h\nu\Delta f}.$$

- It is possible to express SNR in terms of the number of photons N_p contained in a single 1 bit.
- Pulse energy: $E_p = N_p h\nu$.
- Optical power for a bit of duration $T_B = 1/B$: $P_{\mathrm{in}} = N_p h\nu B$.
- Receiver bandwidth for NRZ bit stream: $\Delta f = B/2$.
- Putting it all together, SNR $= \eta N_p \approx N_p$.
- At 1.55-μm, $P_{\mathrm{in}} \approx 130$ nW is needed at 10 Gb/s to realize SNR = 20 dB ($N_p = 100$).

APD Receivers

- Average current larger for an APD by the gain factor M:

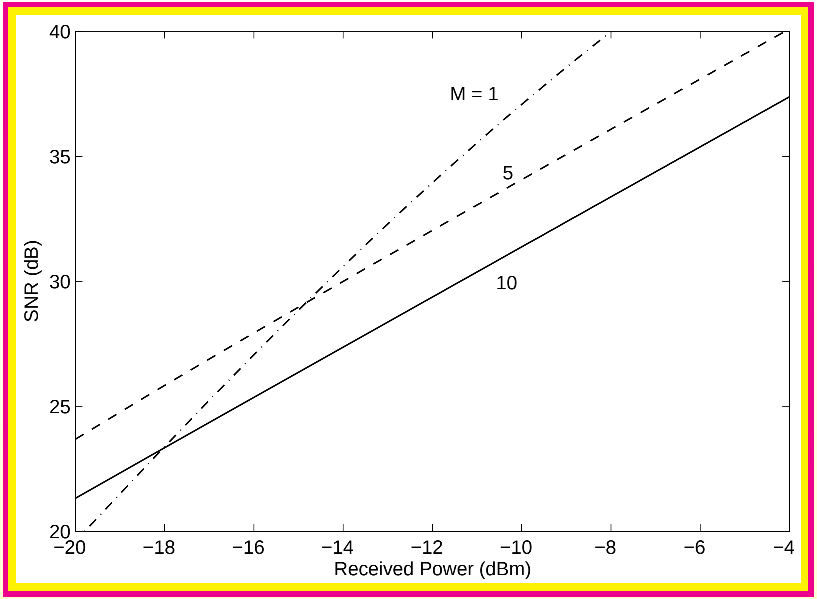
$$I_p = MR_dP_{\text{in}} = R_{\text{APD}}P_{\text{in}}.$$

- Thermal noise unchanged but shot noise enhanced by a factor F_A known as excess noise factor.
- Shot-noise variance: $\sigma_s^2 = 2qM^2F_A(R_dP_{\text{in}} + I_d)\Delta f$.
- Signal-to-Noise Ratio for an APD receiver:

$$\text{SNR} = \frac{I_p^2}{\sigma_s^2 + \sigma_T^2} = \frac{(MR_dP_{\text{in}})^2}{2qM^2F_A(R_dP_{\text{in}} + I_d)\Delta f + 4(k_BT/R_L)F_n\Delta f}.$$

- SNR is larger for APDs because thermal noise dominates in practice.

APD Receivers (continued)



- Increase in SNR with received power P_{in} for three values of APD gain M for 30-GHz bandwidth.
- Excess noise factor F_A depend on APD gain as $F_A(M) = k_A M + (1 - k_A)(2 - 1/M)$.

Optimum APD gain

- Thermal-Noise Limit ($\sigma_T \gg \sigma_s$):

$$\mathrm{SNR} = (R_L R_d^2 / 4k_B T F_n \Delta f) M^2 P_{\mathrm{in}}^2.$$

- Shot-Noise Limit ($\sigma_s \gg \sigma_T$):

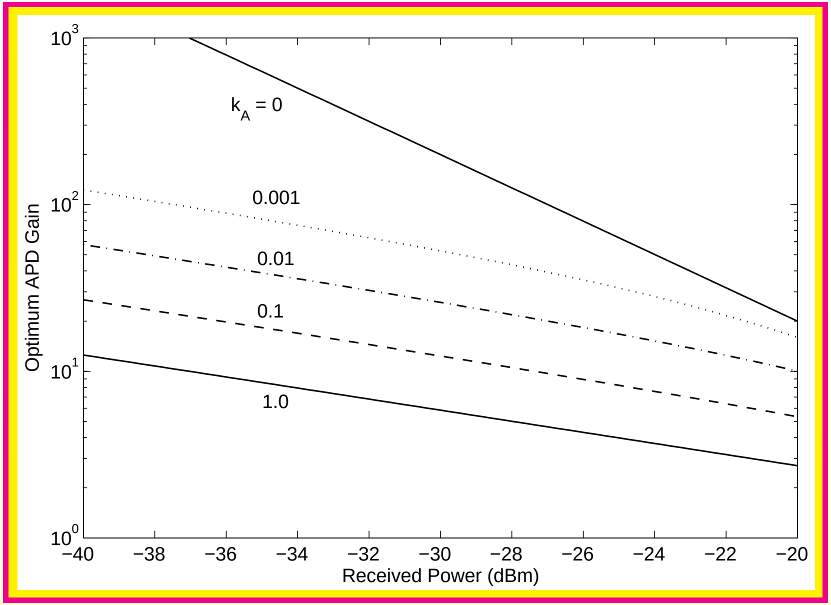
$$\mathrm{SNR} = \frac{R_d P_{\mathrm{in}}}{2qF_A \Delta f} = \frac{\eta P_{\mathrm{in}}}{2h\nu F_A \Delta f}.$$

- SNR can be maximized by optimizing the APD gain M.
- Setting $d(\mathrm{SNR})/dM = 0$, the optimum APD gain satisfies

$$k_A M_{\mathrm{opt}}^3 + (1 - k_A) M_{\mathrm{opt}} = \frac{4k_B T F_n}{qR_L(R_d P_{\mathrm{in}} + I_d)}.$$

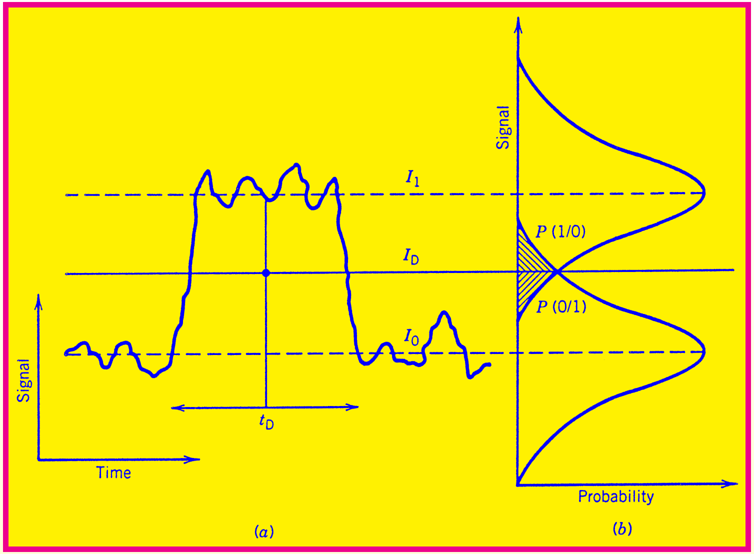
- Approximate solution: $M_{\mathrm{opt}} \approx \left[\frac{4k_B T F_n}{k_A q R_L (R_d P_{\mathrm{in}} + I_d)}\right]^{1/3}$.

Optimum APD Gain (continued)



- M_{opt} plotted as a function of P_{in} for several values of k_A.
- Parameter values correspond to a typical 1.55-μm APD receiver.
- Performance improved for APDs when $k_A \ll 1$.

Bit Error Rate



- $\text{BER} = p(1)P(0/1) + p(0)P(1/0) = \frac{1}{2}[P(0/1) + P(1/0)]$.
- $P(0/1)$ = conditional probability of deciding 0 when 1 is sent.
- Since $p(1) = p(0) = 1/2$, $\text{BER} = \frac{1}{2}[P(0/1) + P(1/0)]$.
- Common to assume Gaussian statistics for the current.

Bit Error Rate (continued)

- $P(0/1)$ = Area below the decision level I_D

$$P(0/1) = \frac{1}{\sigma_1\sqrt{2\pi}} \int_{-\infty}^{I_D} \exp\left(-\frac{(I-I_1)^2}{2\sigma_1^2}\right) dI = \frac{1}{2}\operatorname{erfc}\left(\frac{I_1 - I_D}{\sigma_1\sqrt{2}}\right).$$

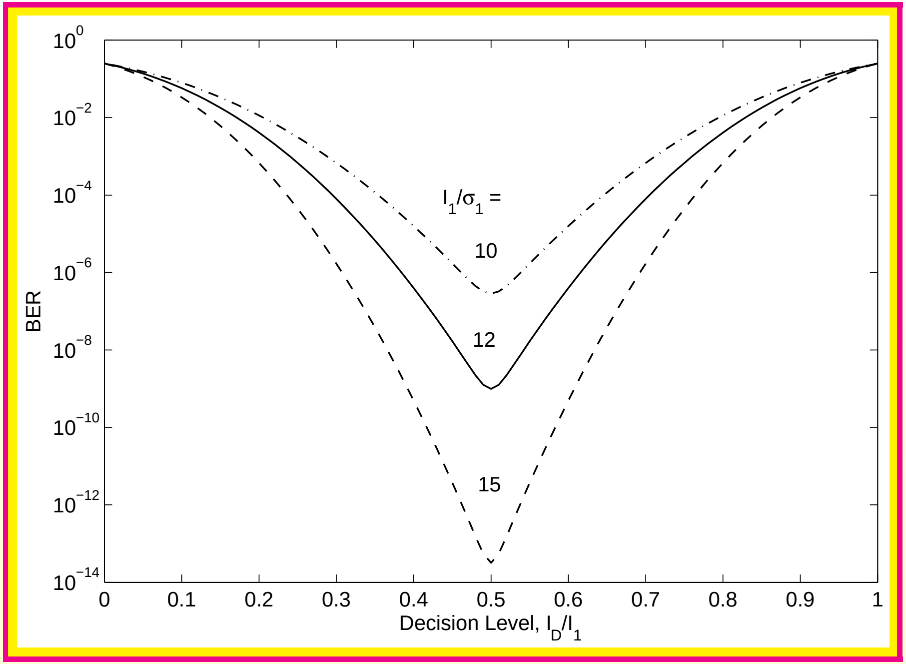
- $P(1/0)$ = Area above the decision level I_D

$$P(1/0) = \frac{1}{\sigma_0\sqrt{2\pi}} \int_{I_D}^{\infty} \exp\left(-\frac{(I-I_0)^2}{2\sigma_0^2}\right) dI = \frac{1}{2}\operatorname{erfc}\left(\frac{I_D - I_0}{\sigma_0\sqrt{2}}\right).$$

- Complementary error function $\operatorname{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^{\infty} \exp(-y^2)\, dy$.
- Final Answer

$$\text{BER} = \frac{1}{4}\left[\operatorname{erfc}\left(\frac{I_1 - I_D}{\sigma_1\sqrt{2}}\right) + \operatorname{erfc}\left(\frac{I_D - I_0}{\sigma_0\sqrt{2}}\right)\right].$$

Role of Decision Level



- BER depends on the decision threshold I_D.
- I_D is optimized in practice to reduce the BER.

Minimum Bit Error Rate

- Minimize BER by setting $d(BER)/dI_D = 0$.
- Minimum BER occurs when I_D is chosen such that

$$\frac{(I_D - I_0)^2}{2\sigma_0^2} = \frac{(I_1 - I_D)^2}{2\sigma_1^2} + \ln\left(\frac{\sigma_1}{\sigma_0}\right).$$

- Last term is negligible in most cases, and

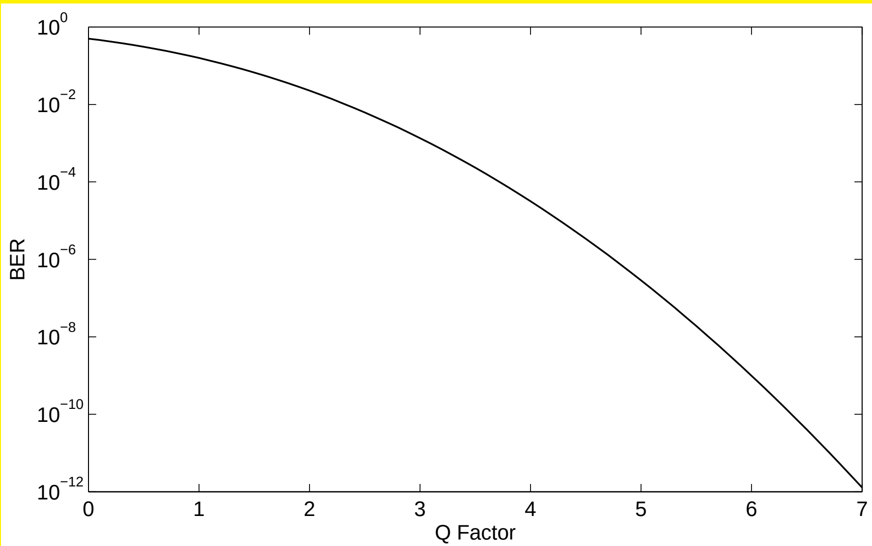
$$(I_D - I_0)/\sigma_0 = (I_1 - I_D)/\sigma_1 \equiv Q.$$

$$I_D = \frac{\sigma_0 I_1 + \sigma_1 I_0}{\sigma_0 + \sigma_1}, \qquad Q = \frac{I_1 - I_0}{\sigma_1 + \sigma_0}.$$

- Final Expression

$$\mathrm{BER} = \frac{1}{2}\mathrm{erfc}\left(\frac{Q}{\sqrt{2}}\right) \approx \frac{\exp(-Q^2/2)}{Q\sqrt{2\pi}}.$$

Q Parameter



- $Q = \frac{I_1 - I_0}{\sigma_1 + \sigma_0}$ is a measure of SNR.
- $Q > 6$ required for a BER of $< 10^{-9}$.
- $Q = 7$ provides a BER of $< 10^{-12}$.

Minimum Average Power

- Receiver sensitivity = Minimum average power needed to keep the BER below a certain value ($< 10^{-9}$).
- We need to relate Q parameter to incident optical power.
- Assume 0 bits carry no optical power so that $P_0 = I_0 = 0$.
- $I_1 = MR_dP_1 = 2MR_d\bar{P}_{\text{rec}}$, where $\bar{P}_{\text{rec}} = (P_1 + P_0)/2$.
- Including both shot and thermal noise,

$$\sigma_1 = (\sigma_s^2 + \sigma_T^2)^{1/2} \quad \text{and} \quad \sigma_0 = \sigma_T,$$

$$\sigma_s^2 = 2qM^2F_AR_d(2\bar{P}_{\text{rec}})\Delta f, \qquad \sigma_T^2 = (4k_BT/R_L)F_n\Delta f.$$

- Using these results

$$Q = \frac{I_1}{\sigma_1 + \sigma_0} = \frac{2MR_d\bar{P}_{\text{rec}}}{(\sigma_s^2 + \sigma_T^2)^{1/2} + \sigma_T}.$$

Back
Close

Receiver Sensitivity

- Solving for received power, we obtain

$$\bar{P}_{\mathrm{rec}} = \frac{Q}{R_d}\left(qF_A Q\Delta f + \frac{\sigma_T}{M}\right).$$

- For a *p–i–n* receiver, we set $M = 1$.
- Since thermal noise dominates for such a receiver,

$$\bar{P}_{\mathrm{rec}} \approx Q\sigma_T/R_d.$$

- Using $R \approx 1$ A/W near 1.55 μm, $\bar{P}_{\mathrm{rec}} = Q\sigma_T$.
- As an example, if we use $R_d = 1$ A/W, $\sigma_T = 100$ nA, and $Q = 6$, we obtain $\bar{P}_{\mathrm{rec}} = 0.6$ μW or -32.2 dBm.

APD Receiver Sensitivity

- Receiver sensitivity improves for APD receivers.
- If thermal noise dominates, $\bar{P}_{\rm rec}$ is reduced by a factor of M.
- When shot and thermal noise are comparable, receiver sensitivity can be optimized by adjusting the APD gain M.
- $\bar{P}_{\rm rec}$ is minimum for an optimum value of M:

$$M_{\rm opt} = k_A^{-1/2}\left(\frac{\sigma_T}{Qq\Delta f} + k_A - 1\right)^{1/2} \approx \left(\frac{\sigma_T}{k_A Qq\Delta f}\right)^{1/2}.$$

- Best APD responsivity

$$\bar{P}_{\rm rec} = (2q\Delta f/R)Q^2(k_A M_{\rm opt} + 1 - k_A).$$

Number of Photons/Bit

- Receiver sensitivity can be expressed in terms of number of photons N_p contained within a single 1 bit.
- In the shot-noise limit, $I_0 = 0$ and $\sigma_0 = 0$ when 0 bits carry no power, and $Q = I_1/\sigma_1 = (\text{SNR})^{1/2}$.
- SNR related to N_p as SNR $\approx \eta N_p$, or

$$\text{BER} = \frac{1}{2}\text{erfc}\left(\sqrt{\eta N_p/2}\right).$$

- For $\eta = 1$, BER $= 1\times10^{-9}$, $N_p = 36$. Thus, 36 photons are sufficient in the shot-noise limit.
- In practice, most optical receivers require $N_p > 1000$ because of thermal noise.

Quantum Limit of Photodetection

- The BER obtained in the shot-noise limit not totally accurate.
- Its derivation based on the Gaussian approximation for noise.
- Poisson statistics should be used for small number of photons.
- For an ideal detector (no thermal noise, no dark current, and $\eta = 1$), 0 bits produce no photons, and $\sigma_0 = 0$.
- Error occurs only if 1 bit fails to produce even one electron.
- Probability of generating m electrons: $P_m = \exp(-N_p)N_p^m/m!$.
- Since $P(0/1) = \exp(-N_p)$, $\mathrm{BER} = \exp(-N_p)/2$.
- $N_p = 20$ for BER $= 1 \times 10^{-9}$ (10 photons/bit on average).
- $\bar{P}_{\rm rec} = N_p h\nu B/2 = \bar{N}_p h\nu B = 13$ nW or -48.9 dBm at $B = 10$ Gb/s.

Sensitivity Degradation

- Real receivers need more power than $\bar{P}_{\text{rec}}$.
- Increase in power is referred to as power penalty.
- In decibel units, power penalty is defined as

$$\text{Power Penalty} = 10\log_{10}\left(\frac{\text{Increased Power}}{\text{Original Power}}\right).$$

- Several mechanisms degrade the receiver sensitivity.
  - ⋆ Finite Extinction ratio ($P_0 \neq 0$)
  - ⋆ Intensity Noise of received optical signal
  - ⋆ Pulse broadening induced by fiber dispersion
  - ⋆ Timing Jitter of electronic circuits

Finite Extinction Ratio

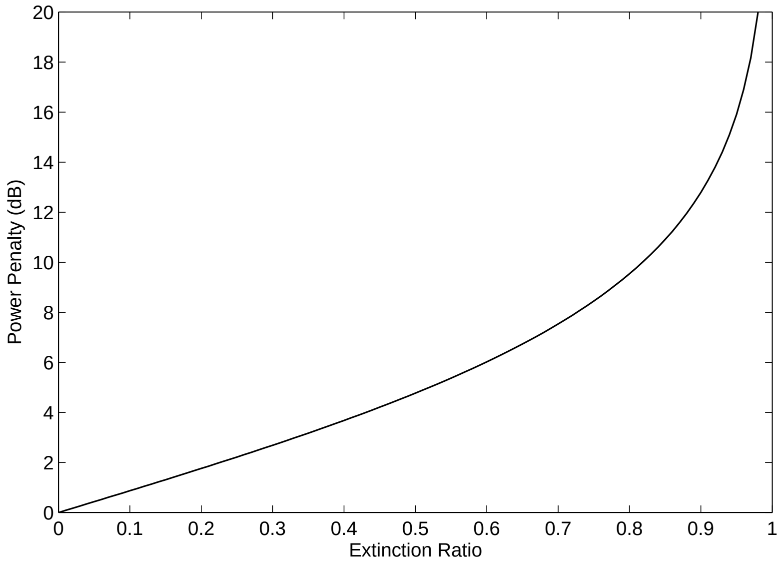
- Extinction ratio is defined as $r_{\rm ex} = P_0/P_1$.
- Power penalty can be obtained by calculating Q parameter.
- For a *p–i–n* receiver $I_1 = R_d P_1$ and $I_0 = R_d P_0$.
- Using $\bar{P}_{\rm rec} = (P_1 + P_0)/2$,

$$Q = \left(\frac{1 - r_{\rm ex}}{1 + r_{\rm ex}}\right) \frac{2R_d \bar{P}_{\rm rec}}{\sigma_1 + \sigma_0}.$$

- In thermal noise limit, $\sigma_1 \approx \sigma_0 \approx \sigma_T$.
- Received power for a finite extinction ratio

$$\bar{P}_{\rm rec}(r_{\rm ex}) = \left(\frac{1 + r_{\rm ex}}{1 - r_{\rm ex}}\right) \frac{\sigma_T Q}{R_d}.$$

Extinction Ratio (continued)



$$\delta_{\text{ex}} = 10\log_{10}\left(\frac{\bar{P}_{\text{rec}}(r_{\text{ex}})}{\bar{P}_{\text{rec}}(0)}\right) = 10\log_{10}\left(\frac{1+r_{\text{ex}}}{1-r_{\text{ex}}}\right).$$

1-dB penalty occurs for $r_{\text{ex}} = 0.12$; increases to 4.8 dB for $r_{\text{ex}} = 0.5$.

Intensity Noise of Lasers

- So far, incident optical power is assumed to be constant.
- In practice, all lasers exhibit intensity noise.
- Optical amplifiers add additional power fluctuations.
- Receiver converts power fluctuations into current fluctuations, which add to those resulting from shot and thermal noise.
- Total noise variance can be written as

$$\sigma^2 = \sigma_s^2 + \sigma_T^2 + \sigma_I^2.$$

- Intensity noise $\sigma_I = R_d \langle (\Delta P_{\text{in}}^2) \rangle^{1/2} = R_d P_{\text{in}} r_I, \quad r_I = \langle (\Delta P_{\text{in}}^2) \rangle^{1/2} / P_{\text{in}}.$
- Parameter r_I related to the RIN of a laser as

$$r_I^2 = \frac{1}{2\pi} \int_{-\infty}^{\infty} \text{RIN}(\omega)\, d\omega.$$

Intensity Noise (continued)

- Consider a *p–i–n* receiver with $I_1 = R_d P_1$ and $I_0 = 0$.
- Using $\bar{P}_{\text{rec}} = (P_1 + P_0)/2$

$$Q = \frac{2R_d \bar{P}_{\text{rec}}}{(\sigma_T^2 + \sigma_s^2 + \sigma_I^2)^{1/2} + \sigma_T}.$$

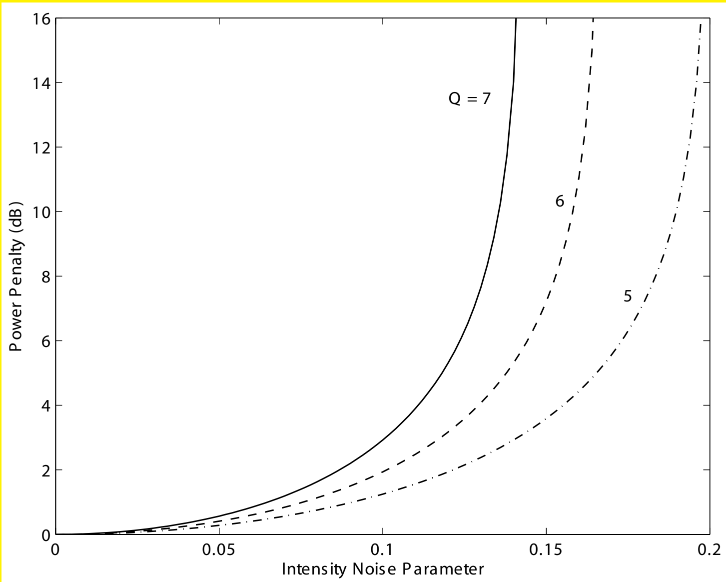
- Optical power required for a finite intensity noise

$$\bar{P}_{\text{rec}}(r_I) = \frac{Q\sigma_T + Q^2 q\Delta f}{R_d(1 - r_I^2 Q^2)}.$$

- Power penalty is found to be

$$\delta_I = -10\log_{10}(1 - r_I^2 Q^2).$$

Intensity Noise (continued)



- A 2-dB penalty occurs for $r_I = 0.1$.
- Penalty becomes infinite when $r_I > Q^{-1}$ (BER floor).
- In practice, $r_I < 0.01$ (power penalty negligible).

Dispersive Pulse Broadening

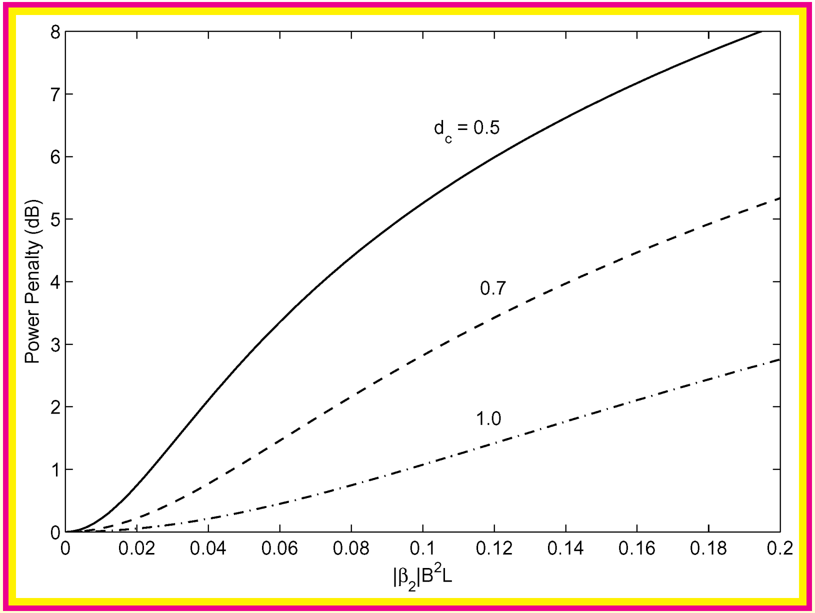
- Pulse energy in the bit slot decreases with pulse broadening.
- Receiver requires more average power to maintain SNR.
- For Gaussian pulses, peak power is reduced by the pulse broadening factor f_b found in Chapter 3.
- $f_b^2 = 1 + (DL\sigma_\lambda/\sigma_0)^2$ when source bandwidth dominates.
- σ_0 is related to duty cycle d_c as $4\sigma_0 = d_c T_b$.
- Using $\sigma = (4B)^{-1}$, power penalty is given by

$$\delta_d = 10 \log_{10} f_b = 5 \log_{10}[1 + (4BLD\sigma_\lambda/d_c)^2].$$

- For a narrowband source and unchirped Gaussian pulses

$$\delta_d = 5 \log_{10}[1 + (8\beta_2 B^2 L/d_c^2)^2].$$

Dispersive Pulse Broadening



- Power penalty negligible for $\mu = |\beta_2|B^2L < 0.05$ and $d_c > 0.5$.
- Increases rapidly as μ increases and exceeds 5 dB for $\mu = 0.1$.
- At 10-Gb/s, L <50 km when standard fibers are used.

Frequency Chirping

- Chirping of optical pulses affects pulse broadening.
- For chirped Gaussian pulses pulse broadening factor is

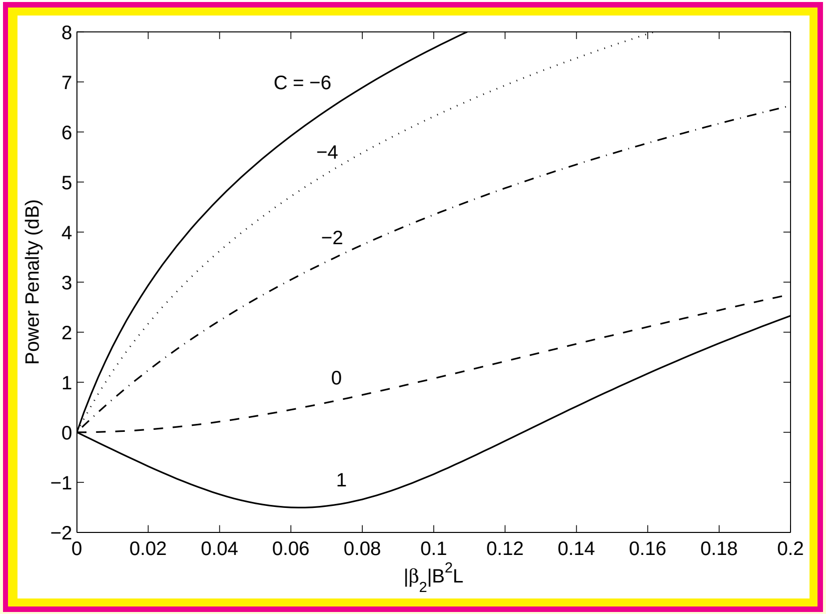
$$f_b^2 = 1 + 8C\beta_2 B^2 L/d_c^2)^2 + (8\beta_2 B^2 L/d_c^2)^2.$$

- Power penalty then becomes

$$\delta_c = 5\log_{10}[(1 + 8C\beta_2 B^2 L/d_c^2)^2 + (8\beta_2 B^2 L/d_c^2)^2].$$

- Penalty can be quite large when $\beta_2 C > 0$.
- This is the case for directly nodulated DFB lasers ($C > -4$) operating near 1.55 μm ($\beta_2 < 0$).

Frequency Chirping



- To keep penalty below 0.1 dB, $|\beta_2|B^2L < 0.002$ is required.
- For standard fibers B^2L is limited to 100 (Gb/s)^2-km.
- System performance can be improved by ensuring that $\beta_2 C < 0$.

Timing Jitter

- Signal must be sampled at the peak of the current pulse.
- Decision instant determined by the clock-recovery circuit.
- In practice, sampling time fluctuates from bit to bit.
- If bit is not sampled at the bit center, sampled value is reduced by an amount that depends on timing jitter Δt.
- Since Δt is a random variable, signal becomes more noisy.
- SNR reduced as a result of such additional fluctuations.
- SNR can be maintained by increasing received power (power penalty).

Timing Jitter (continued)

- Q parameter in the presence of timing jitter

$$Q = \frac{I_1 - \langle \Delta i_j \rangle}{(\sigma_T^2 + \sigma_j^2)^{1/2} + \sigma_T}.$$

- If $S_p(t)$ governs the shape of current pulse, $\Delta i_j = I_1[S_p(0) - S_p(\Delta t)]$.
- Approximating S_p as $S_p(t) = 1 - \frac{1}{2}(c_p B t)^2$, $\Delta i_j = (c_p B \Delta t)^2 I_1$.
- Probability density of timing jitter Δt

$$p(\Delta t) = \frac{1}{\tau_j \sqrt{2\pi}} \exp\left(-\frac{\Delta t^2}{2\tau_j^2}\right).$$

- Find $p(\Delta i_j)$ and use it to calculate $\langle \Delta i_j \rangle$ and σ_j.

Timing Jitter (continued)

- Probability density of current fluctuation Δi_j

$$p(\Delta i_j) = \frac{1}{\sqrt{\pi b \Delta i_j I_1}} \exp\left(-\frac{\Delta i_j}{b I_1}\right), \qquad b = (c_p B \sigma_t)^2.$$

- Average and standard deviation are found to be

$$\langle \Delta i_j \rangle = b I_1/2, \qquad \sigma_j = b I_1/\sqrt{2}.$$

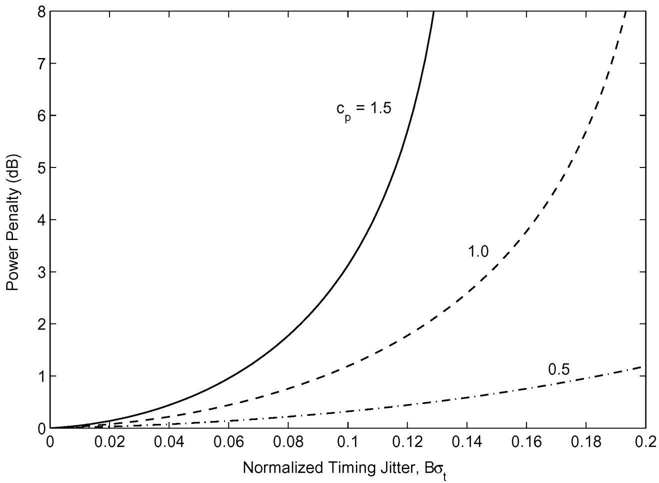
- Receiver sensitivity

$$\bar{P}_{\text{rec}}(b) = \left(\frac{\sigma_T Q}{R_d}\right) \frac{1 - b/2}{(1-b/2)^2 - b^2 Q^2/2}.$$

- Power penalty is found to be

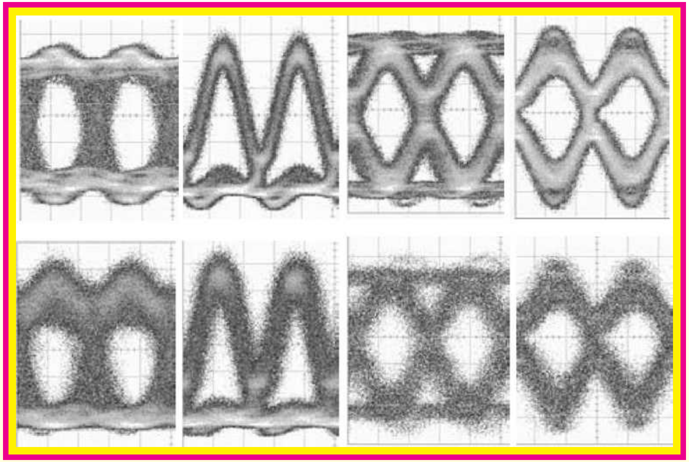
$$\delta_j = 10 \log_{10}\left(\frac{1 - b/2}{(1-b/2)^2 - b^2 Q^2/2}\right).$$

Timing Jitter (continued)



- Pulse curvature c_p at center of bit slot plays important role.
- Power penalty becomes infinitely large at a certain value of $B\sigma_t$.
- Tolerable value $B\sigma_t$ depends on c_p and decreases as c_p increases.
- Typically $c_p < 1$, and power penalty <0.5 dB) if $B\sigma_t < 0.08$.

Eye-Closure Penalty



- Eye diagrams at 40 Gb/s in the case of NRZ, CSRZ, NRZ-DPSK, and RZ-DPSK formats.
- $L = 0$ (top row) and $L = 263$ km (bottom row).
- Alternative measure of system performance is provided by the eye opening.

Forward Error Correction

- It is entirely possible that a specified BER cannot be achieved.
- Only viable alternative—Use an error-correction scheme.
- In one approach, errors are detected but not corrected.
- Suitable when packet switching is used (Internet protocol).
- In FEC, errors are detected and corrected at the receiver without any retransmission of bits.
- This scheme is best suited for lightwave systems operating with SONET or SDH protocol.
- Historically, lightwave systems did not employ FEC until the use of in-line optical amplifiers became common.

Error-Correcting Codes

- Basic idea: Add extra bits at transmitter using a suitable code.
- At the receiver end, a decoder uses these control bits to detect and correct errors.
- How many errors can be corrected depends on the coding scheme employed.
- In general, more errors can be corrected by adding more control bits to the signal.
- There is a limit to this process since bit rate of the system increases after the FEC coder.
- If B_e is effective bit rate after coding, FEC overhead is $B_e/B - 1$.
- Redundancy of a code is defined as $\rho = 1 - B/B_e$.

Error-Correcting Codes

- Classified under names such as linear, cyclic, Hamming, Reed–Solomon, convolutional, product, and turbo codes.
- Among these, Reed–Solomon (RS) codes have attracted most attention for lightwave systems.
- Denoted as RS(n,k), where k is the size of packet that is converted into a larger packet with n bits $(n = 2^m - 1)$.
- ITU recommendation: RS(255, 239) with $m = 8$. FEC overhead for this code is 6.7%.
- RS(255, 207) with an overhead of 23.2% is also used.
- Improvement in BER is quantified through the coding gain.

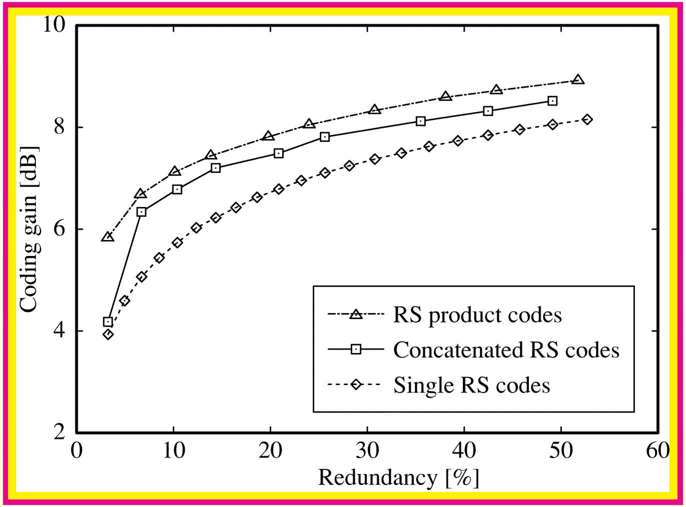
Coding Gain

- Coding gain: A measure of improvement in BER through FEC.
- It is expressed in terms of the equivalent value of Q as $G_c = 20\log_{10}(Q_c/Q)$.
- Q_c and Q are related to the BERs as

$$\text{BER}_c = \tfrac{1}{2}\text{erfc}(Q_c/\sqrt{2}), \qquad \text{BER} = \tfrac{1}{2}\text{erfc}(Q/\sqrt{2}).$$

- Factor of 20 is used in place of 10 because performance is often quantified through Q^2.
- If FEC decoder improves BER from 10^{-3} to 10^{-9}, Q increases from 3 to 6, resulting in a coding gain of 6 dB.
- Magnitude of coding gain increases with the FEC overhead.

Coding Gain



- For single RS codes, coding gain is 5.5 dB for 10% overhead and increases sublinearly, reaching 8 dB for 50% overhead.
- It can be improved by concatenating two or more RS codes or by employing the RS product codes.

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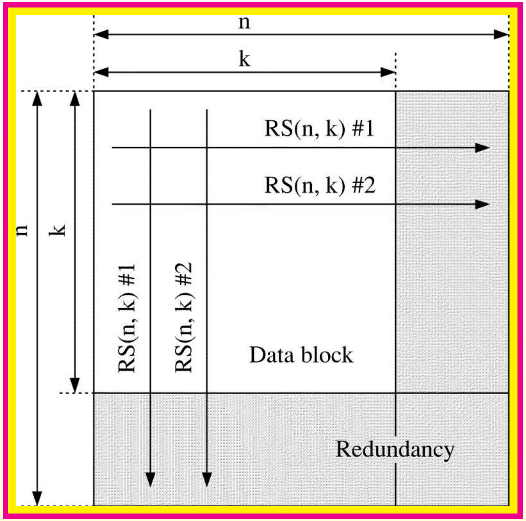
Product Codes

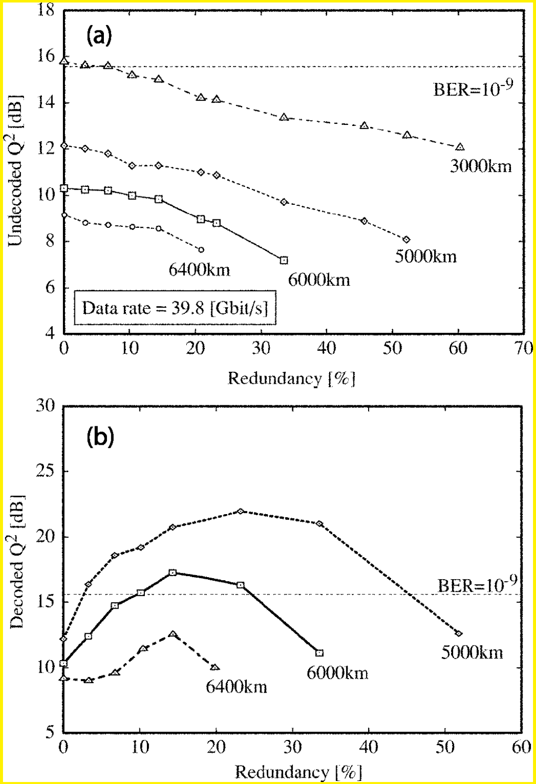


- Same code is applied along the rows and columns of a block.
- Overhead of $n^2/k^2 - 1$ for a RS product code is larger, but it also allows more error control.
- 6 dB of coding gain possible with only 5% overhead.

Coding Gain

- While implementing FEC, one faces a dilemma.
- As the overhead is increased to realize more coding gain, bit rate of the signal increases.
- Since Q factor realized at the receiver depends on the bit rate, its value is reduced, and BER actually worsens.
- Decoder improves it but it first has to overcome the degradation caused by the increased bit rate.
- If an aggressive FEC scheme is employed, BER may degrade so much that the system is not operable even with the FEC coder.
- An optimum range of coding overhead exists for every system designed to operate at a specific bit rate over a certain distance.

Coding Gain



- Numerically simulated Q factors (a) before and (b) after the FEC decoder as a function of code redundancy for a WDM system with 25 channels at 40 Gb/s.
- With FEC, Q factor becomes worse as overhead increases.

Chapter 6:
Optical Amplifier Noise

- Origin of Amplifier Noise
- Optical Signal-to-Noise Ratio
- Electrical Signal-to-Noise Ratio
- Receiver Sensitivity and Q Factor
- Role of Dispersive and Nonlinear Effects
- Periodically Amplified Lightwave Systems

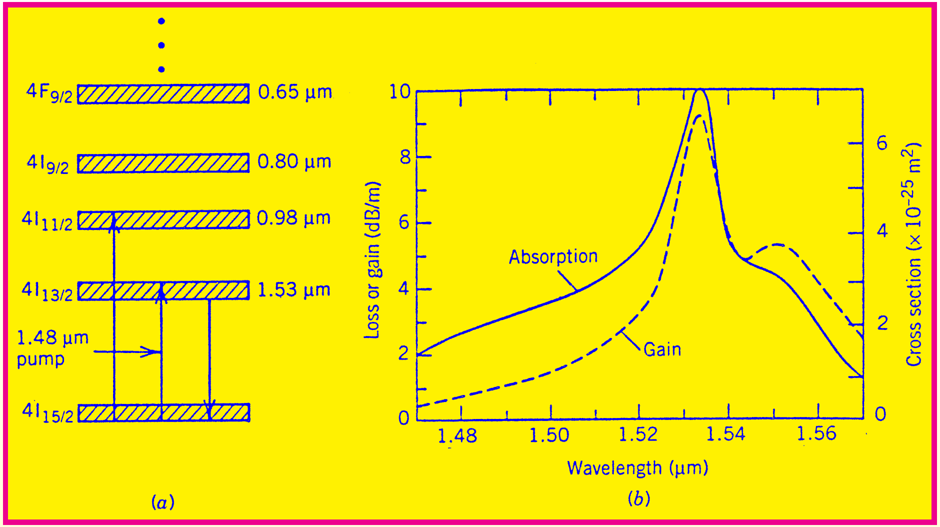
Optical Amplifiers

- Used routinely for loss compensation since 1995.
- Amplify the input signal but also add some noise.
- Several kinds of amplifiers have been developed:
  - ⋆ Semiconductor optical amplifiers
  - ⋆ Raman-based fiber amplifiers
  - ⋆ Erbium-doped fiber amplifiers
- EDFAs are used most commonly for lightwave systems.
- Raman amplifiers work better for long-haul systems.

Erbium-Doped Fiber Amplifiers

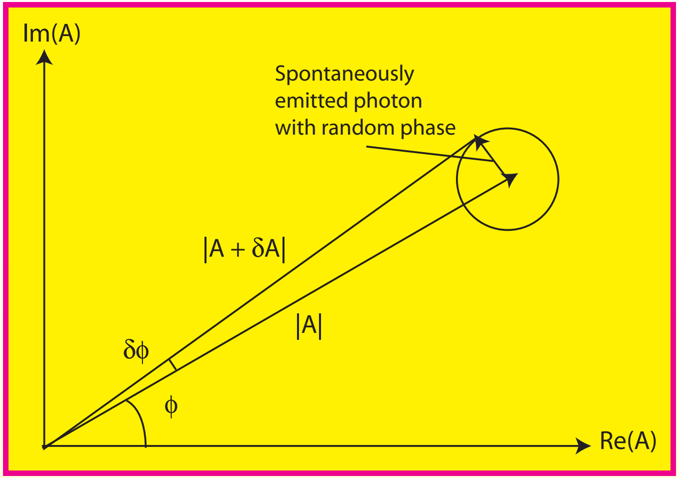
- Developed after 1987 and commercialized during the 1990s.
- Fiber core doped with erbium (length 20-200 m).
- Pumped using diode lasers operating at 980 or 1480 nm.
- Provide 20–30 dB gain at pump powers $<$50 mW.
- Gain bandwidth up to 40 nm possible.
- Relatively low noise; Noise figure 4 to 5 dB.
- Provide polarization-independent gain.
- Gain pattern independent (Response time $\sim$10 ms).
- Can be designed to work in both the C and L bands.

Pumping and Gain



- Semiconductor lasers at 980 or 1480 nm are used for pumping.
- Pumping efficiency up to 11 dB/mW possible at 980 nm.
- Amplification occurs when ions in the excited state emit coherent light through stimulated emission.

Origin of Amplifier Noise



- Source of noise: Spontaneous emission
- Spontaneous emitted photons have random phase and polarization.
- They perturb both A and phase ϕ in a random fashion.
- Such random perturbations are the source of amplifier noise.

Modeling of Amplifier Noise

- NLS equation including the gain and noise of optical amplifiers:

$$\frac{\partial A}{\partial z}+\frac{i\beta_2}{2}\frac{\partial^2 A}{\partial t^2}=i\gamma|A|^2A+\frac{1}{2}(g_0-\alpha)A+f_n(z,t).$$

- Gain coefficient $g_0=\sigma_e N_2-\sigma_a N_1$; σ_e and σ_a are emission and absorption cross sections.
- Noise term vanishes on average, i.e, $\langle f_n(z,t)\rangle=0$.
- Noise modeled as a Markovian process with Gaussian statistics

$$\langle f_n^*(z,t)f_n(z',t')\rangle=n_{\rm sp}h\nu_0 g_0\delta(z-z')\delta(t-t').$$

- Spontaneous-emission factor $n_{\rm sp}=\sigma_e N_2/(\sigma_e N_2-\sigma_a N_1)$.
- Two delta functions ensure that all spontaneous-emission events are independent of each other in time and space.

Noise of Lumped Amplifiers

- Amplifier Length l_a is much shorter than amplifier spacing L_A.
- Neither loss, nor dispersion, nor nonlinearities are important within the amplifier.
- Neglecting them and integrating, we obtain: $A_{\text{out}}(t) = \sqrt{G}A_{\text{in}}(t) + a_n(t)$ with $G = \exp(g_0 l_a)$.
- Amplified spontaneous emission (ASE) at the amplifier output:

$$a_n(t) = \int_0^{l_a} f_n(z,t) \exp\left[\tfrac{1}{2} g_0 (l_a - z)\right] dz.$$

- Since $\langle f_n(z,t) \rangle = 0$, $a_n(t)$ also vanishes on average.
- Second moment of $a_n(t)$ is found to be

$$\langle a_n(t) a_n(t') \rangle = S_{\text{ASE}} \delta(t - t'), \qquad S_{\text{ASE}} = n_{\text{sp}} h \nu_0 (G - 1).$$

Total Noise Power

- It is important to note that $a_n(t)$ represents only the portion of ASE that is coupled to the mode occupied by the signal.
- One must add up noise over the entire bandwidth of amplifier.
- If an optical filter is used, ASE power becomes

$$P_{\mathrm{ASE}} = 2\int_{-\infty}^{\infty} S_{\mathrm{ASE}} H_f(\nu - \nu_0)\, d\nu \approx 2S_{\mathrm{ASE}}\Delta\nu_o.$$

- $\Delta\nu_o$ is the effective bandwidth of optical filter.
- Factor of 2 takes into account two orthogonally polarized modes of fiber.
- Only half the noise power is copolarized with the optical signal.

Distributed Amplification

- In the case of distributed amplification, NLS equation should be solved along the entire fiber link.
- Gain $g_0(z)$ is not constant along the fiber length.
- It is not easy to solve the NLS equation. If we set $\beta_2 = 0$ and $\gamma = 0$, the solution is $A(L,t) = \sqrt{G(L)}A(0,t) + a_n(t)$ with

$$a_n(t) = \sqrt{G(L)} \int_0^L \frac{f_n(z,t)}{\sqrt{G(z)}} dz, \quad G(z) = \exp\left(\int_0^z [g_0(z') - \alpha]\, dz'\right).$$

- $a_n(t)$ vanishes on average and its second moment is given by

$$\langle a_n(t)a_n(t')\rangle = G(L)\int_0^L dz \int_0^L dz' \frac{\langle f_n(z,t)f_n(z',t')\rangle}{\sqrt{G(z)G(z')}} = S_{\mathrm{ASE}}\delta(t-t'),$$

- Spectral density: $S_{\mathrm{ASE}} = n_{\mathrm{sp}}h\nu_0 G(L)\int_0^L \frac{g_0(z)}{G(z)} dz.$

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Distributed Raman Amplification

- The origin of noise is related to spontaneous Raman scattering.
- Spontaneous-emission factor $n_{\rm sp}$ has a different meaning than that in the case of EDFAs.
- No electronic transitions involved during Raman amplification.
- Spontaneous Raman scattering is affected by phonon population that depends on temperature of the fiber.
- More precisely, $n_{\rm sp}$ is given by

$$n_{\rm sp}(\Omega) = 1 + \frac{1}{\exp(\hbar\Omega/k_BT) - 1} \equiv \frac{1}{1 - \exp(-\hbar\Omega/k_BT)}.$$

- At room temperature $n_{\rm sp} = 1.14$ near the Raman-gain peak.

Total ASE Power

- Total ASE power is obtained by adding contributions over the Raman-gain bandwidth or the bandwidth of optical filter.
- Assuming a filter is used, the total ASE power is given by

$$P_{\text{ASE}} = 2\int_{-\infty}^{\infty} S_{\text{ASE}} H_f(\nu - \nu_0)\, d\nu = 2S_{\text{ASE}}\Delta\nu_o.$$

- Factor of 2 includes both polarization components.
- Substituting the expression for S_{ASE}, ASE power becomes

$$P_{\text{ASE}} = 2n_{\text{sp}} h\nu_0 \Delta\nu_o G(L) \int_0^L \frac{g_0(z)}{G(z)}\, dz.$$

- ASE power depends on the pumping scheme through $g_0(z)$.

Optical SNR

- Optical SNR = Ratio of optical power to ASE power.
- Assume that all amplifiers are spaced apart by L_A and have the same gain $G = \exp(\alpha L_A)$.
- Total ASE power for a chain of N_A amplifiers:

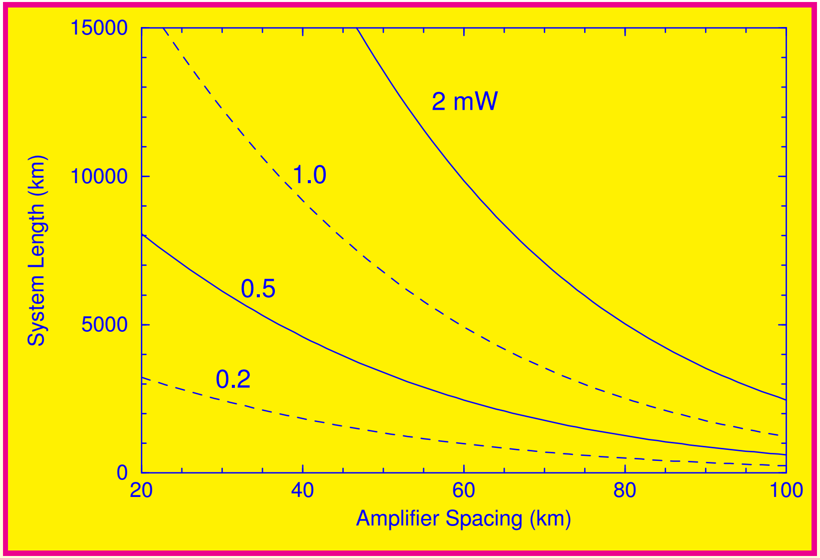
$$P_{\rm ASE}^{\rm tot} = 2N_A S_{\rm ASE} \Delta\nu_o = 2n_{\rm sp} h\nu_0 N_A (G-1)\Delta\nu_o.$$

- Factor of 2 takes into account unpolarized nature of ASE.
- Optical SNR is thus given by

$$\mathrm{SNR}_o = \frac{P_{\rm in}}{P_{\rm ASE}^{\rm tot}} = \frac{P_{\rm in} \ln G}{2n_{\rm sp} h\nu_0 \Delta\nu_o \alpha L_T (G-1)}.$$

- We used $N_A = L_T/L_A = \alpha L_T/\ln G$ for a link of total length L_T.

Optical SNR



- SNR can be enhanced by reducing the gain of each amplifier.
- ASE-limited system length as a function of L_A for several values of input power using $\alpha = 0.2$ dB/km, $n_{sp} = 1.6$, $\Delta\nu_o = 100$ GHz.
- It is assumed that an SNR of 20 is required by the system.

Optimum Amplifier Spacing

- Optimum L_A becomes smaller as system length increases.
- Amplifier spacing can be improved by increasing input power P_{in}.
- In practice, maximum launched power is limited by the onset of various nonlinear effects.
- Typically, P_{in} is limited to close to 1 mW.
- At such power levels, L_A should be in the range of 40 to 50 km for submarine lightwave systems with lengths of 6,000 km or more.
- Amplifier spacing can be increased to 80 km for terrestrial systems with link lengths under 3,000 km.

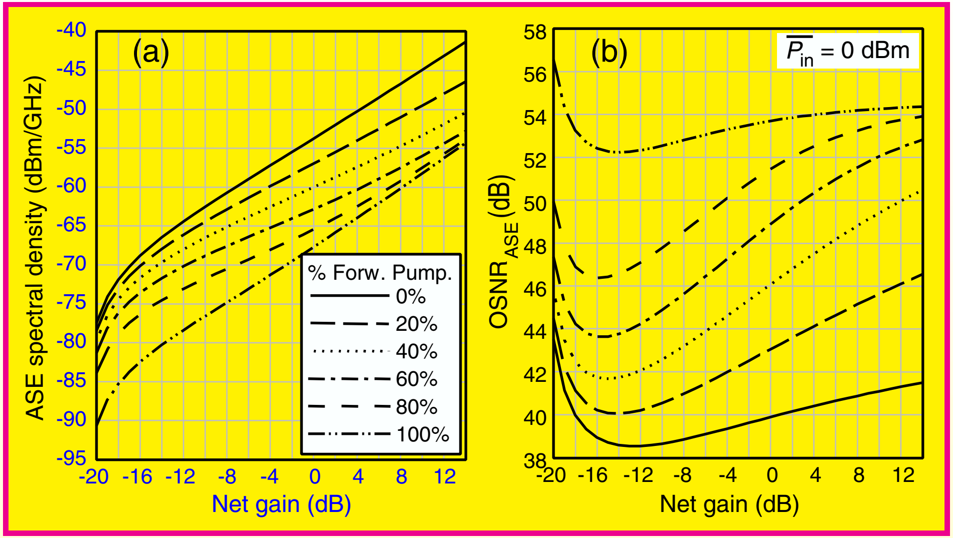
Case of Distributed Amplification

- Optical SNR in this case takes the form

$$\mathrm{SNR}_o = \frac{P_{in}}{2N_A S_{\mathrm{ASE}} \Delta\nu_o}, \qquad S_{\mathrm{ASE}} = n_{\mathrm{sp}} h\nu_0 G(L) \int_0^L \frac{g_0(z)}{G(z)}\, dz.$$

- Pump power can be injected in the forward, backward, or both directions.
- $g(z)$ depends on the pumping scheme, and S_{ASE} depends on $g(z)$.
- We can control optical SNR by adopting a suitable pumping scheme.
- Consider a 100-km-long fiber section pumped bidirectionally to provide distributed Raman amplification.
- ASE spectral density and optical SNR are shown as a function of net gain when $P_{\mathrm{in}} = 1$ mW.

SNR for Raman Amplification



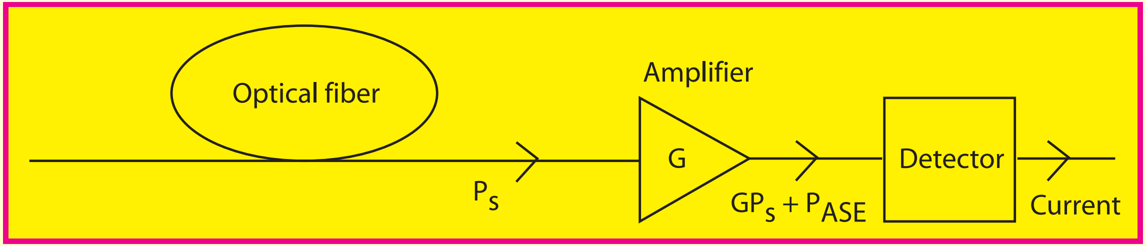
- Fraction of forward pumping varies from 0 to 100%.
- Losses are 0.26 and 0.21 dB/km at pump and signal wavelengths.
- Other parameters are $n_{sp} = 1.13$, $h\nu_0 = 0.8$ eV, and $g_R = 0.68$ W^{-1}/km.

Distributed Raman Amplification

- Optical SNR is highest in the case of purely forward pumping.
- It degrades by as much as 15 dB as the fraction of backward pumping is increased from 0 to 100%.
- ASE generated near the input end experiences losses over the full length of the fiber in the case of forward pumping.
- It experiences only a fraction of losses for backward pumping.
- If N_A such sections are employed to form a long-haul fiber link, SNR is reduced by a factor of N_A.
- Even when $L_T = 10,000$ km ($N_A = 100$), SNR$_o$ remains >20 dB.
- Such high values of optical SNR are difficult to maintain when EDFAs are used.

Electrical SNR

- Optical SNR is not what governs the BER at the receiver.
- Electrical SNR of the current generated is more relevant for signal recovery at the receiver.
- Assume that a single optical amplifier is used before receiver to amplify a low-power signal before it is detected.
- This configuration is sometimes used to improve receiver sensitivity through optical preamplifcation.



ASE-Induced Current Fluctuations

- Photocurrent $I = R_d(|\sqrt{G}E_s + E_{\text{cp}}|^2 + |E_{\text{op}}|^2) + i_s + i_T$.
- It is necessary to separate the ASE into two parts because only its copolarized part can beat with the signal.
- ASE-induced current noise has its origin in beating of E_s with E_{cp} and beating of ASE with itself.
- Useful to divide bandwidth $\Delta\nu_o$ into M bins, each of bandwidth $\Delta\nu_s$, and write

$$E_{\text{cp}} = \sum_{m=1}^{M} (S_{\text{ASE}}\Delta\nu_s)^{1/2} \exp(i\phi_m - i\omega_m t).$$

- ϕ_m is the phase of noise component at $\omega_m = \omega_l + m(2\pi\Delta\nu_s)$.
- An identical form applies for E_{op}.

ASE-Induced Current Fluctuations

- Using $E_s = \sqrt{P_s}\exp(i\phi_s - i\omega_s t)$ and including all beating terms,

$$I = R_d G P_s + i_{\text{sig-sp}} + i_{\text{sp-sp}} + i_s + i_T.$$

- $i_{\text{sig-sp}}$ and $i_{\text{sp-sp}}$ represent current fluctuations resulting from signal–ASE and ASE–ASE beating:

$$i_{\text{sig-sp}} = 2R_d(GP_s S_{\text{ASE}}\Delta\nu_s)^{1/2}\sum_{m=1}^{M}\cos[(\omega_s-\omega_m)t+\phi_m-\phi_s],$$

$$i_{\text{sp-sp}} = 2R_d S_{\text{ASE}}\Delta\nu_s \sum_{m=1}^{M}\sum_{n=1}^{M}\cos[(\omega_n-\omega_m)t+\phi_m-\phi_n].$$

- $\langle i_{\text{sp-sp}}\rangle = 2R_d S_{\text{ASE}}\Delta\nu_s M \equiv 2R_d S_{\text{ASE}}\Delta\nu_o \equiv R_d P_{\text{ASE}}.$
- Variances of two noise currents are found to be

$$\sigma^2_{\text{sig-sp}} = 4R_d^2 G P_s S_{\text{ASE}}\Delta f, \qquad \sigma^2_{\text{sp-sp}} = 4R_d^2 S_{\text{ASE}}^2 \Delta f(\Delta\nu_o - \Delta f/2).$$

Impact of ASE on SNR

- Total variance σ^2 of current fluctuations is given by

$$\sigma^2 = \sigma^2_{\text{sig}-\text{sp}} + \sigma^2_{\text{sp}-\text{sp}} + \sigma^2_s + \sigma^2_T.$$

- Electrical SNR at the receiver becomes

$$\text{SNR}_e = \frac{\langle I \rangle^2}{\sigma^2} = \frac{R_d^2 (GP_s + P_{\text{ASE}})^2}{\sigma^2_{\text{sig}-\text{sp}} + \sigma^2_{\text{sp}-\text{sp}} + \sigma^2_s + \sigma^2_T}.$$

- SNR realized in the absence of optical amplifier:

$$\text{SNR}'_e = \frac{R_d^2 P_s^2}{\sigma^2_s + \sigma^2_T}.$$

- For an ideal receiver with no thermal noise and $R_d = q/h\nu_0$, $\text{SNR}'_e = P_s/(2h\nu_0 \Delta f)$.

Noise Figure of Amplifier

- In practice, current variance is dominated by $\sigma^2_{\mathrm{sig-sp}}$.
- Neglecting $\sigma^2_{\mathrm{sp-sp}}$, the SNR is found to be

$$\mathrm{SNR}_e = \frac{GP_s}{(4S_{\mathrm{ASE}} + 2h\nu_0)\Delta f}.$$

- Using $S_{\mathrm{ASE}} = n_{\mathrm{sp}} h\nu_0 (G-1)$, optical amplifier is found to degrade the electrical SNR by a factor of

$$F_o = \frac{\mathrm{SNR}_e'}{\mathrm{SNR}_e} = 2n_{\mathrm{sp}}\left(1 - \frac{1}{G}\right) + \frac{1}{G}.$$

- F_o is known as the noise figure of an optical amplifier.
- In the limit $G \gg 1$, SNR is degraded by $F_o = 2n_{\mathrm{sp}}$.
- Even when $n_{\mathrm{sp}} = 1$, SNR is reduced by 3 dB.

Impact of Thermal Noise

- Preceding conclusion holds for an ideal receiver.
- In practice, thermal noise exceeds shot noise by a large amount.
- It should be included before concluding that an optical amplifier always degrades the electrical SNR.
- Retaining only the dominant term $\sigma^2_{\text{sig-sp}}$:

$$\frac{\text{SNR}_e}{\text{SNR}'_e} = \frac{G\sigma_T^2}{4R_d^2 P_s S_{\text{ASE}} \Delta f}.$$

- This ratio can be made quite large by lowering P_s.
- Electrical SNR can be improved by 20 dB or more compared with its value possible without amplification.

Electrical SNR

- Thermal noise is the most important factor that limits the electrical SNR.
- Optical preamplification helps to mask thermal noise, resulting in an improved SNR.
- If we retain only dominant noise term, the electrical SNR becomes

$$\mathrm{SNR}_e = \frac{GP_s}{4S_{\mathrm{ASE}}\Delta f} = \frac{GP_s\Delta\nu_o}{2P_{\mathrm{ASE}}\Delta f}.$$

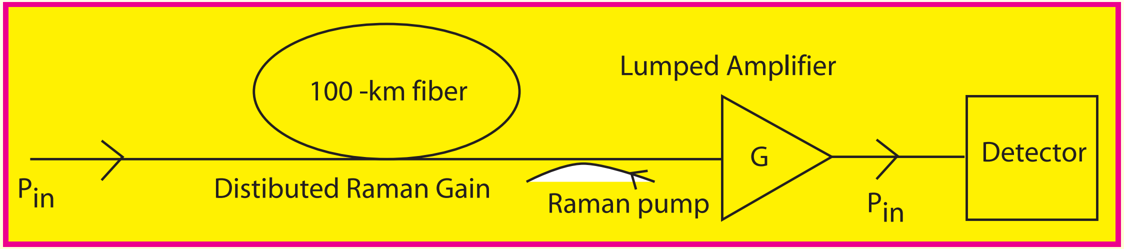
- This should be compared with the optical SNR of GP_s/P_{ASE}.
- Electrical SNR is higher by a factor of $\Delta\nu_o/(2\Delta f)$ under identical conditions.
- The reason is that ASE noise contributes only over the receiver bandwidth Δf that is much narrower than filter bandwidth $\Delta\nu_o$.

Noise Figure of Distributed Amplifiers

- Because of gain variations, noise figure is given by

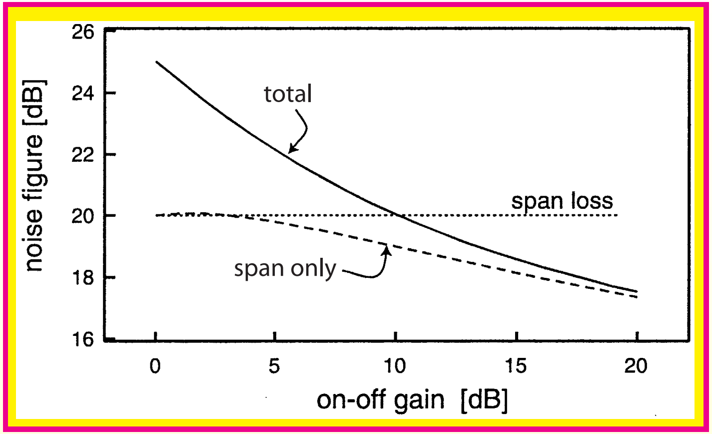
$$F_o = 2n_{sp}\int_0^L \frac{g_0(z)}{G(z)}\,dz + \frac{1}{G(L)}.$$

- Consider the following hybrid scheme:



- The predicted F_o can exceed 15 dB depending on the span length.
- This does not mean distributed amplifiers are more noisy than lumped amplifiers.

Noise Figure of Distributed Amplifiers



- When $G_R = 0$ (no pumping), 100-km-long passive fiber has a noise figure of 20 dB.
- If signal is amplified using a lumped amplifier, additional 5-dB degradation results in a total noise figure of 25 dB.
- This value decreases as G_R increases, reaching a level of 17.5 dB for $G_R = 20$ dB (no lumped amplification).

Noise Figure of Distributed Amplifiers

- It is common to introduce the concept of an *effective* noise figure using $F_{\text{eff}} = F_o \exp(-\alpha L)$.
- $F_{\text{eff}} < 1$ is negative on the decibel scale by definition.
- It is this feature of distributed amplification that makes it so attractive for long-haul WDM lightwave systems.
- In the preceding example, $F_{\text{eff}} \approx -2.5$ dB when pure distributed amplification is employed.
- Effective noise figure of a Raman amplifier depends on the pumping scheme used.
- Forward pumping provides the highest SNR, and the smallest noise figure.

Receiver Sensitivity and Q Factor

- BER can be calculated following the method used in Chapter 5.
- $\text{BER} = p(1)P(0/1) + p(0)P(1/0) = \frac{1}{2}[P(0/1) + P(1/0)]$.
- Conditional probabilities require PDF for the current I.
- Strictly speaking, PDF does not remain Gaussian when optical amplifiers are used.
- If we assume it to remain Gaussian, $\text{BER} = \frac{1}{2}\text{erfc}\left(\frac{Q}{\sqrt{2}}\right)$.
- Q factor: defined as $Q = \frac{I_1 - I_0}{\sigma_1 + \sigma_0}$, where

$$\sigma_1^2 = \sigma_{\text{sig-sp}}^2 + \sigma_{\text{sp-sp}}^2 + \sigma_s^2 + \sigma_T^2,$$

$$\sigma_0^2 = \sigma_{\text{sp-sp}}^2 + \sigma_T^2.$$

Approximate Q Factor

- In the case of 0 bits, σ_s^2 and $\sigma_{\text{sig-sp}}^2$ can be neglected as they are signal-dependent.
- Even for 1 bits σ_s^2 can be neglected in comparison with $\sigma_{\text{sig-sp}}^2$.
- Thermal noise σ_T^2 can also be neglected when optical power at the receiver is relatively large (>0.1 mW).
- Noise currents σ_1 and σ_0 are then approximated by

$$\sigma_1 = (\sigma_{\text{sig-sp}}^2 + \sigma_{\text{sp-sp}}^2)^{1/2}, \qquad \sigma_0 = \sigma_{\text{sp-sp}}.$$

- We calculate the Q factor using

$$Q = \frac{I_1 - I_0}{\sigma_1 + \sigma_0} = \frac{I_1 - I_0}{\sqrt{\sigma_{\text{sig-sp}}^2 + \sigma_{\text{sp-sp}}^2} + \sigma_{\text{sp-sp}}}.$$

Receiver Sensitivity

- Assume that no energy is contained in 0 bits so that $I_0 = 0$ and $I_1 = 2R_d\bar{P}_{\rm rec}$.
- Using Q and expressions for σ_1 and σ_0,

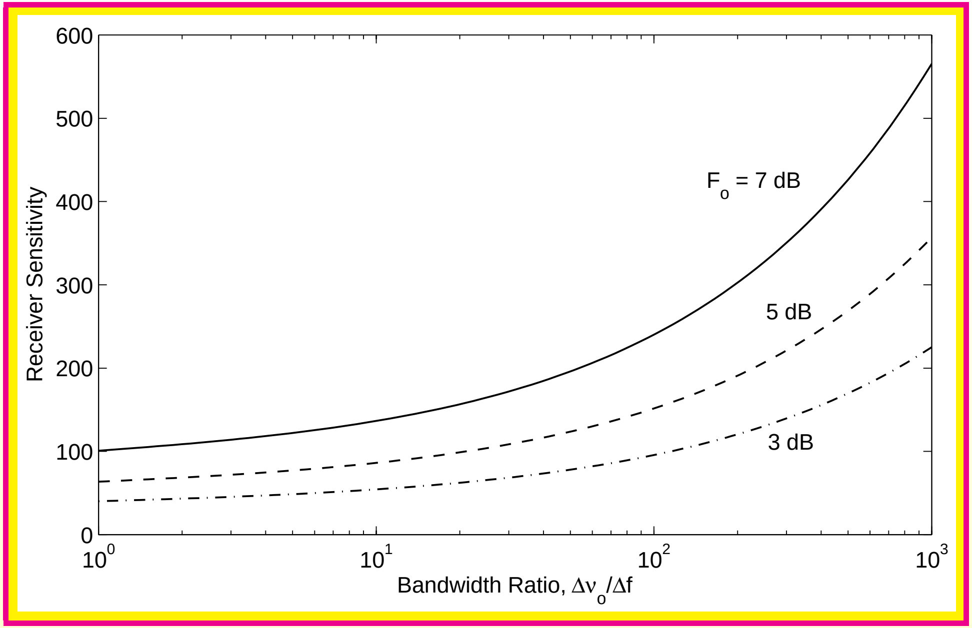
$$\bar{P}_{\rm rec} = h\nu_0 F_o \Delta f [Q^2 + Q(\Delta\nu_o/\Delta f - \tfrac{1}{2})^{1/2}].$$

- Using $\bar{P}_{\rm rec} = \bar{N}_p h\nu_0 B$ and $\Delta f = B/2$, $\bar{N}_p$ is given by

$$\bar{N}_p = \tfrac{1}{2}F_o[Q^2 + Q(r_f - \tfrac{1}{2})^{1/2}].$$

- $r_f = \Delta\nu_o/\Delta f$ is the factor by which the optical filter bandwidth exceeds the receiver bandwidth.
- A remarkably simple expression for the receiver sensitivity.
- It shows why amplifiers with a small noise figure must be used.
- It also shows how narrowband optical filters can help.

Receiver Sensitivity



- Using $Q = 6$ with $F_o = 2$ and $r_f = 2$, the minimum value $\bar{N}_p = 43.3$ photons/bit.
- Without optical amplifiers, $\bar{N}_p$ exceeds 1000.

Non-Gaussian Receiver Noise

- Even though the ASE itself has a Gaussian PDF, detector current does not follow Gaussian statistics.
- Detector current $I = R_d(|E_s + E_{\rm cp}|^2 + |E_{\rm op}|^2)$.
- Orthogonal part of noise can be suppressed by placing a polarizer in front of the receiver.
- Using $E_{\rm cp} = \sum_{m=1}^{M} (S_{\rm ASE}\Delta\nu_s)^{1/2} \exp(i\phi_m - i\omega_m t)$:

$$I = I_s + 2\sqrt{I_N I_s}\sum_{m=1}^{M} c_m + I_N \sum_{m=1}^{pM} (c_m^2 + s_m^2).$$

- Signal $I_s = R_d|E_s|^2$ and noise current $I_N = R_d S_{\rm ASE}\Delta\nu_s$.
- Random variables c_m and s_m defined as $c_m + i s_m = \exp(i\phi_m)$.
- Integer $p = 1$ or 2 depending on whether a polarizer is used or not.

Non-Gaussian Receiver Noise

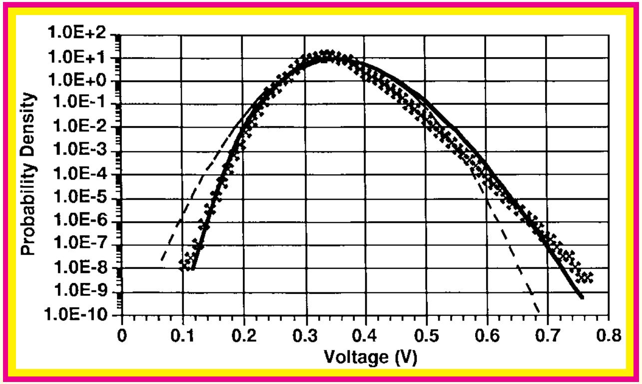
- I is a function of a large number of random variables, each of which follows Gaussian statistics.
- Without ASE–ASE beating, I follows a Gaussian PDF.
- However, this beating term cannot be ignored, and the statistics of I are generally non-Gaussian.
- PDF can be obtained in an analytic form. In the case of 0 bits

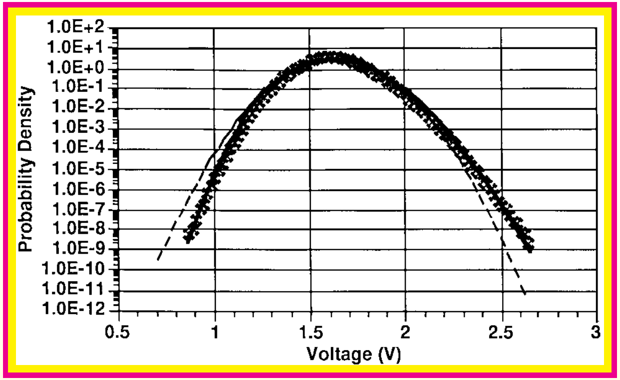
$$p_0(I) = \frac{I^{pM-1}}{(pM-1)!I_N^{pM}} \exp\left(-\frac{I}{I_N}\right).$$

- In the case of 1 bits (using $I_s = I_1$)

$$p_1(I) = \frac{1}{I_N}\left(\frac{I}{I_1}\right)^{\frac{1}{2}(pM-1)} \exp\left(-\frac{I+I_1}{I_N}\right) \mathscr{I}_{pM-1}\left(-\frac{2\sqrt{II_1}}{I_N}\right).$$

Non-Gaussian Receiver Noise





- Measured and predicted PDFs for 0 (top) and 1 bits (bottom). A dashed line shows the Gaussian approximation.
- PDF is far from Gaussian for 0 bits.
- Deviations relatively small in the case of 1 bits.
- Gaussian approximation holds better as the bandwidth of optical filter increases.

Q Factor and Optical SNR

- Assume $I_0 \approx 0$ and $I_1 = R_d P_1$.
- $\sigma^2_{\mathrm{sig-sp}} = 2R_d\sqrt{P_1 P_{\mathrm{ASE}}}/M, \qquad \sigma^2_{\mathrm{sp-sp}} = P^2_{\mathrm{ASE}}/M.$
- We assumed $M = \Delta\nu_o/\Delta f \gg 1$.
- Using σ_1 and σ_0 in the expression for Q,

$$Q = \frac{\mathrm{SNR}_o\sqrt{M}}{\sqrt{2\mathrm{SNR}_o + 1} + 1}.$$

- $\mathrm{SNR}_o \equiv P_1/P_{\mathrm{ASE}}$ is the optical SNR.
- This relation can be inverted to find

$$\mathrm{SNR}_o = \frac{2Q^2}{M} + \frac{2Q}{\sqrt{M}}.$$

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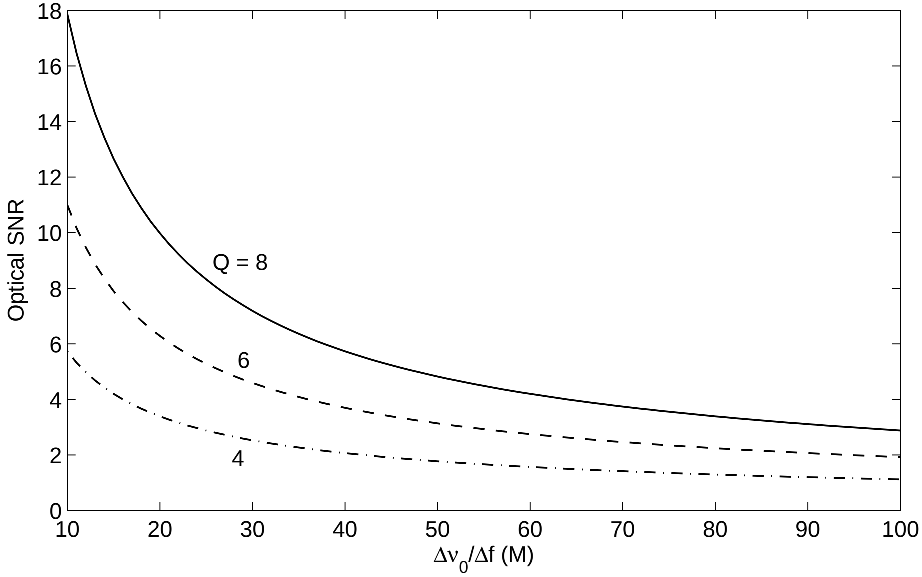
Q Factor and Optical SNR



- Optical SNR as a function of M for several values of Q factor.
- We only need $\text{SNR}_o = 7.5$ when $M = 16$ to maintain $Q = 6$.

Noise Growth through Modulation Instability

- Each amplifier adds ASE noise that propagates with the signal.
- In a purely linear system, noise power would not change.
- Modulation instability amplifies ASE noise.
- Using $A(z,t) = \sqrt{p(z)}B(z,t)$, NLS equation becomes

$$\frac{\partial B}{\partial z} + \frac{i\beta_2}{2}\frac{\partial^2 B}{\partial t^2} = i\gamma p(z)|B|^2 B + f_n(z,t)/\sqrt{p(z)}.$$

- $p(z)$ is defined such that $p(z) = 1$ at the location of amplifiers.
- A numerical approach is necessary in general.
- Assuming a CW signal, the solution is of the form $B(z,t) = [\sqrt{P_0} + a(z,t)]\exp(i\phi_{\rm NL})$.
- $\phi_{\rm NL} = \gamma P_0 \int_0^z p(z)\,dz$ is the SPM-induced nonlinear phase shift.

Noise Growth

- Assuming noise is much weaker than signal ($|a|^2 \ll P_0$),

$$\frac{\partial a}{\partial z} + \frac{i\beta_2}{2}\frac{\partial^2 a}{\partial t^2} = i\gamma P_0 e^{-\alpha z}(a + a^*).$$

- This linear equation is easier to solve in the Fourier domain and leads to two coupled equations:

$$\frac{db_1}{dz} = \frac{i}{2}\beta_2\Omega^2 b_1 + i\gamma P_0 e^{-\alpha z}(b_1 + b_2^*),$$
$$\frac{db_2}{dz} = \frac{i}{2}\beta_2\Omega^2 b_2 + i\gamma P_0 e^{-\alpha z}(b_2 + b_1^*),$$

- $b_1(z) = \tilde{a}(z,\Omega)$, $b_2(z) = \tilde{a}(z,-\Omega)$, and $\Omega = \omega_n - \omega_0$.
- When Ω falls within the gain bandwidth of modulation instability, the two noise components are amplified.

Noise Growth

- Coupled linear equations can be solved easily when $\alpha = 0$.
- They can also be solved when $\alpha \neq 0$. but the solution involves Hankel functions of complex order and argument.
- In a simple approach, fiber is divided into multiple segments.
- Propagation through each segment of length h is governed by

$$\begin{pmatrix} b_1(z_n+h) \\ b_2(z_n+h) \end{pmatrix} = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \begin{pmatrix} b_1(z_n) \\ b_2(z_n) \end{pmatrix}.$$

- Matrix elements M_{mn} are constants in each fiber segment but change from segment to segment.
- Solution at the end of fiber is obtained by multiplying individual matrices.

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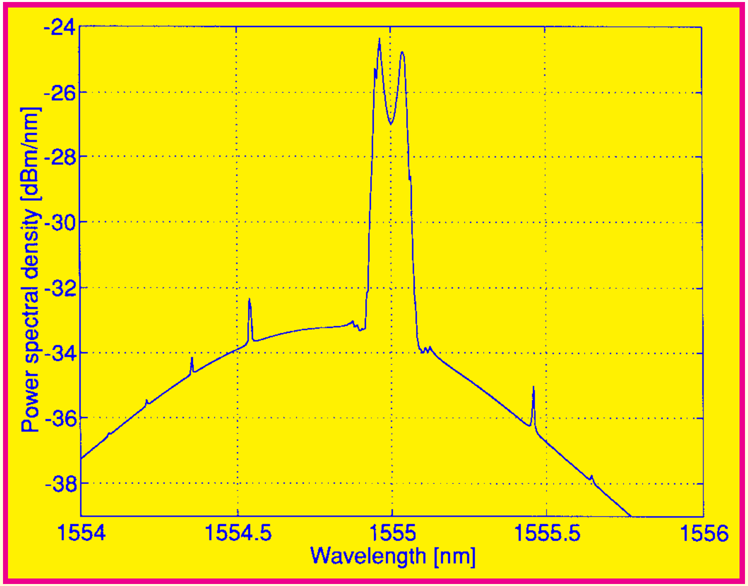
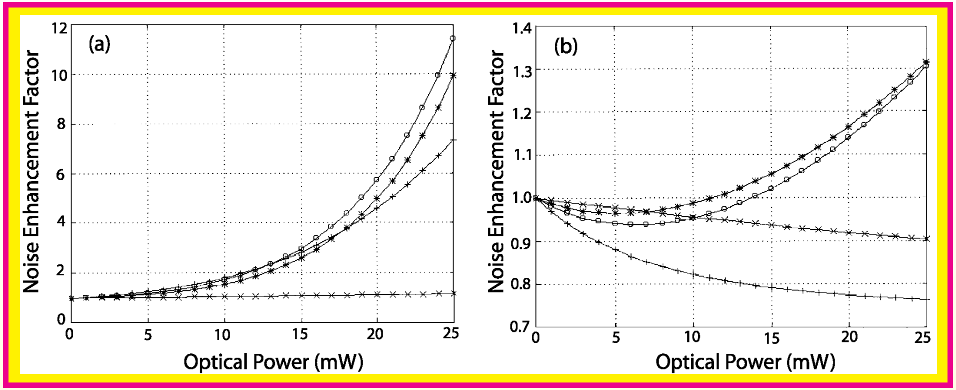
Noise Growth



- An example of numerically simulated spectrum at the end of a 2,500-km fiber link with 50 amplifiers placed 50 km apart.
- Broad pedestal represents the ASE spectrum expected even in the absence of nonlinear effects.

Noise Growth



- Possible to calculate factor F_v by which $\sigma^2_{\text{sig-sp}}$ changes.
- F_v as a function of launched power (four 100-km-long sections).
- (a) anomalous [$D = 2$ ps/(km-nm)];
  (b) normal dispersion [$D = -2$ ps/(km-nm)].
- $\Delta f = 2$ GHz (crosses), 8 GHz (pluses), 20 GHz (stars), and 30 GHz (circles).

Noise-Induced Signal Degradation

- Optical signal degrades as ASE noise is added by amplifiers.
- As expected, ASE induces power fluctuations (reduced SNR).
- Surprisingly, ASE also induces timing jitter.
- Physical origin of ASE-induced jitter: Amplifiers affect not only amplitude but also phase of amplified signal.
- Chirping of pulses shifts signal frequency from ω_0 by a small amount after each amplifier.
- Since group velocity depends on frequency (because of dispersion), speed at which a pulse propagates is affected by each amplifier.
- Speed changes produce random shifts in pulse position at receiver.

Moment Method Revisited

- Moment method can be used by introducing two new moments.
- q and Ω represent pulse position and shift in the carrier frequency:

$$q(z) = \frac{1}{E}\int_{-\infty}^{\infty} t|B(z,t)|^2\,dt, \quad \Omega(z) = \frac{i}{2E}\int_{-\infty}^{\infty}\left(B^*\frac{\partial B}{\partial t} - B\frac{\partial B^*}{\partial t}\right)dt.$$

- $E(z) \equiv \int_{-\infty}^{\infty}|B(z,t)|^2\,dt$ is related to pulse energy.
- Differentiating E, q, and Ω with respect to z,

$$\frac{dE}{dz} = 0, \qquad \frac{dq}{dz} = \beta_2\Omega, \qquad \frac{d\Omega}{dz} = 0.$$

- Energy E and frequency Ω do not change during propagation.
- Pulse position shifts for a finite value of Ω as $q(z) = \beta_2\Omega z$.

Moment Method Revisited

- Because of ASE added by the amplifier, E, Ω, and q change by random amounts δE_k, $\delta \Omega_k$, and δq_k after each amplifier:

$$\frac{dE}{dz} = \sum_k \delta E_k \delta(z - z_k),$$

$$\frac{dq}{dz} = \beta_2 \Omega + \sum_k \delta q_k \delta(z - z_k),$$

$$\frac{d\Omega}{dz} = \sum_k \delta \Omega_k \delta(z - z_k).$$

- The sum is over the total number of amplifiers encountered by the pulse before it arrives at z.
- ASE-induced timing jitter can be reduced by operating a lightwave system near the zero-dispersion wavelength of fiber.

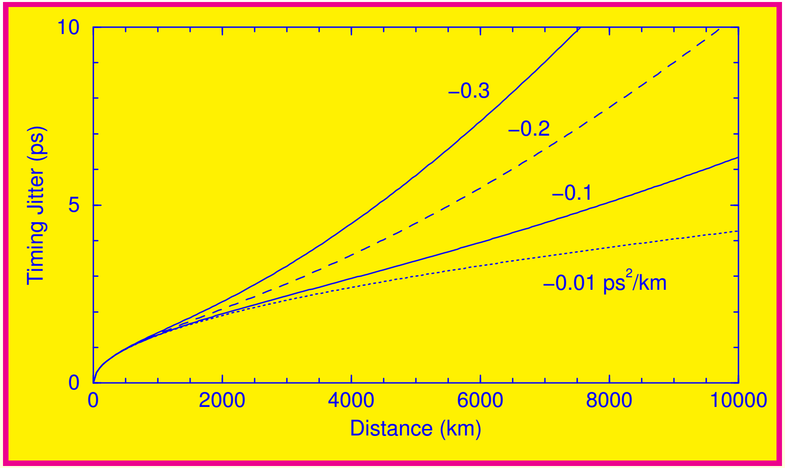
Noise-Induced Timing Jitter

- Total jitter at the end of the fiber link: $\sigma_t^2 = \langle q_f^2 \rangle - \langle q_f \rangle^2$.
- Angle brackets denote averaging over amplifier noise.
- Final result turns out to be relatively simple:

$$\sigma_t^2 = (S_{\mathrm{ASE}}/E_0) T_0^2 N_A [(1 + (C_0 + N_A d_a/T_0^2)^2].$$

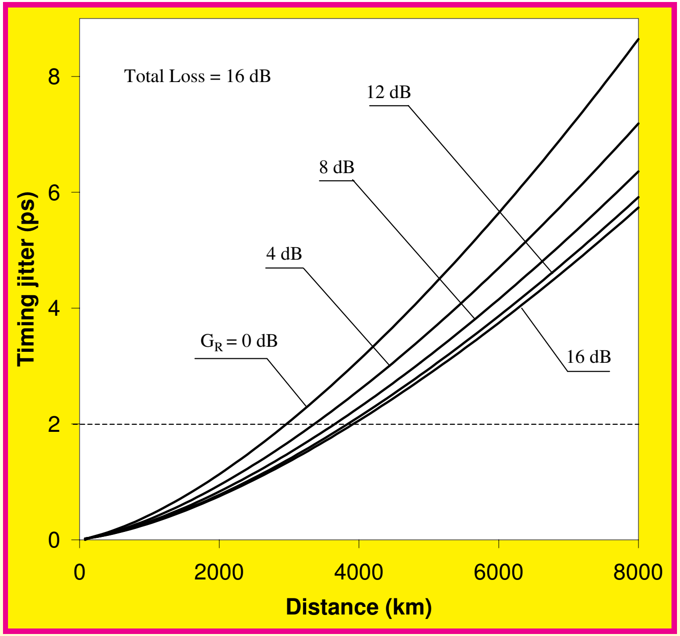
- $d_a = \int_0^{L_A} \beta_2(z)\, dz$ is the dispersion accumulated over the entire link.
- In the case of perfect dispersion compensation ($d_a = 0$), σ_t^2 increases linearly with the number N_A of amplifiers.
- When $d_a \neq 0$, it increases with N_A in a cubic fashion.

Noise Growth



- ASE-induced timing jitter as a function of system length for several values of average dispersion $\bar{\beta}_2$.
- Results are for a 10-Gb/s system with $T_0 = 30$ ps, $L_A = 50$ km, $C_0 = 0.2$, and $S_{\rm ASE}/E_0 = 10^{-4}$.
- ASE-induced jitter becomes a significant fraction of pulse width because of the cubic dependence of σ_t^2 on system length L_T.

Distributed Amplification

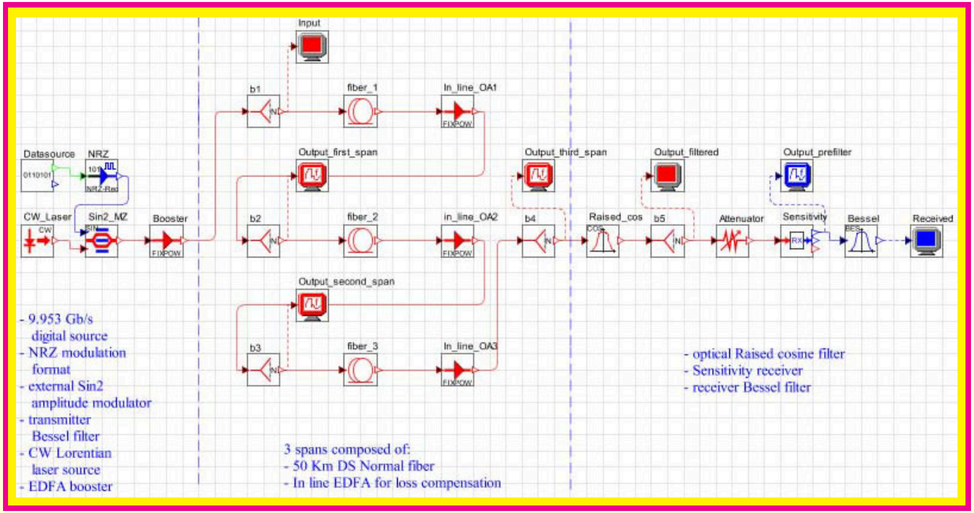


- Raman gain is varied from 0 to 16 dB (total loss over 80 km).
- Dashed line shows the tolerable value of timing jitter.

Numerical Approach

- Nonlinear and dispersive effects act on a noisy optical signal simultaneously.
- Their mutual interplay cannot be studied analytically.
- Most practical approach for designing modern lightwave system consists of solving the NLS equation numerically.
- Numerical simulations indeed show that nonlinear effects often limit the system performance.
- System design requires optimization of various parameters such as amplifier spacing and input power launched.
- Several software packages are available commercially.
- One such package called OptSim 4.0 is provided on the CD.

OptSim Simulation Package



- Layout of a typical lightwave system for modeling based on the software package OptSim.
- Main advantage: Optimum values of various system parameters can be found such that design objectives are met at a minimum cost.

Numerical Approach

- Input to optical transmitter is a pseudo-random sequence of electrical pulses, representing 1 and 0 bits.
- The length N of this bit pattern determines the computing time and should be chosen judiciously.
- Typically, $N = 2^M$, where M is in the range of 6 to 10.
- Optical bit stream obtained by solving the rate equations that govern the modulation response of the laser or modulator.
- Deformation of optical bit stream during its transmission calculated by solving the NLS equation.
- Method most commonly used for solving this equation is known as the spit-step Fourier method.

Numerical Approach

- Two equivalent techniques used for adding ASE noise to the signal during numerical simulations.
- In one case, noise is added in the time domain, while ensuring that it follows Gaussian statistics with $\langle a_n(t)a_n(t')\rangle = S_{\text{ASE}}\delta(t-t')$.
- Because of a finite temporal resolution Δt, delta function is replaced with a "rect function" of width Δt.
- Its height is chosen to be $1/\Delta t$ so that $\int_{-\infty}^{-\infty} \delta(t)\,dt = 1$ is satisfied.
- Alternatively, noise can be added in the frequency domain:

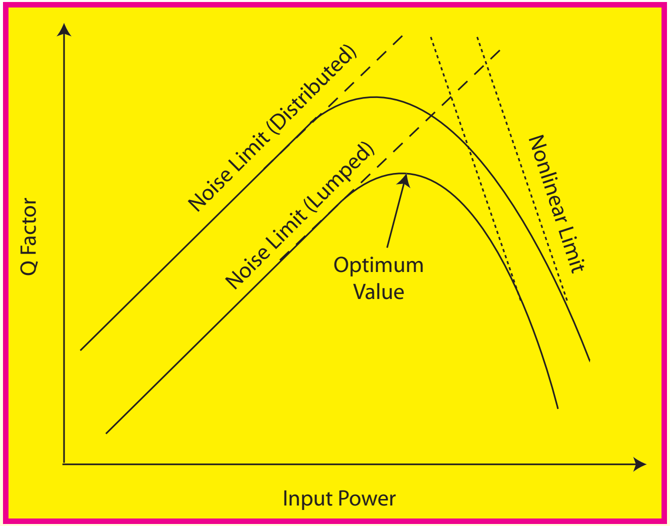
$$\tilde{A}_{\text{out}}(\nu) = \sqrt{G}\tilde{A}_{\text{in}}(\nu) + \tilde{a}_n(\nu).$$

- Real and imaginary parts of $\tilde{a}_n(\nu)$ follow Gaussian statistics.
- Noise is assumed to be white (same variance at each frequency).

Numerical Approach

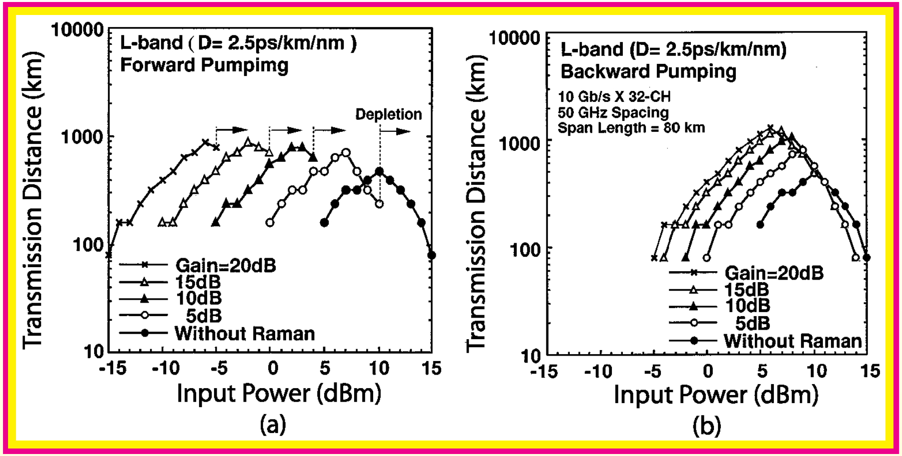
- A receiver model converts optical signal into electric domain.
- An electric filter used with its bandwidth Δf smaller than bit rate B (typically $\Delta f/B = 0.6$–0.8).
- Electric bit stream is used to find the instantaneous values of currents, I_0 and I_1 by sampling it at the center of each bit slot.
- Eye diagram is constructed using the filtered bit stream.
- System performance is quantified through the Q factor, related directly to the BER.
- Calculation of Q factor requires that the NLS equation be solved a large number of times with different seeds for amplifier noise.
- Such an approach becomes quite time-consuming for WDM systems.

Optimum Launched Power



- Q-factor variations with launched power in long-haul systems.
- Q factor increases initially with launched power, reaches a peak value, and then decreases with a further increase in power because of the onset of the nonlinear effects.
- Use of distributed amplification improves system performance.

Optimum Launched Power



- Numerical results for a 32-channel WDM system.
- Maximum distance plotted as a function of input power.
- Fiber link contains 80-km sections whose 20-dB loss compensated using (a) forward or (b) backward pumping configuration.
- Pump depletion becomes significant at arrow location.