Nonlinear Fiber Optics and its Applications in Optical Signal Processing

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Outline

- Introduction
- Self-Phase Modulation
- Cross-Phase Modulation
- Four-Wave Mixing
- Stimulated Raman Scattering
- Stimulated Brillouin Scattering
- Concluding Remarks

Introduction

Fiber nonlinearities

- Studied during the 1970s.
- Ignored during the 1980s.
- Feared during the 1990s.
- Are being used in this decade.

Objective:

- Review of *Nonlinear Effects in Optical Fibers*.

Major Nonlinear Effects

- Self-Phase Modulation (SPM)
- Cross-Phase Modulation (XPM)
- Four-Wave Mixing (FWM)
- Stimulated Raman Scattering (SRS)
- Stimulated Brillouin Scattering (SBS)

Origin of Nonlinear Effects in Optical Fibers

- Ultrafast third-order susceptibility $\chi^{(3)}$.
- Real part leads to SPM, XPM, and FWM.
- Imaginary part leads to SBS and SRS.

Self-Phase Modulation

- Refractive index depends on optical intensity as (Kerr effect)

$$n(\omega, I) = n_0(\omega) + n_2 I(t).$$

- Frequency dependence leads to dispersion and pulse broadening.
- Intensity dependence leads to nonlinear phase shift

$$\phi_{\mathrm{NL}}(t) = (2\pi/\lambda) n_2 I(t) L.$$

- An optical field modifies its own phase (thus, SPM).
- Phase shift varies with time for pulses (chirping).
- As a pulse propagates along the fiber, its spectrum changes because of SPM.

Nonlinear Phase Shift

- Pulse propagation governed by Nonlinear Schrödinger Equation

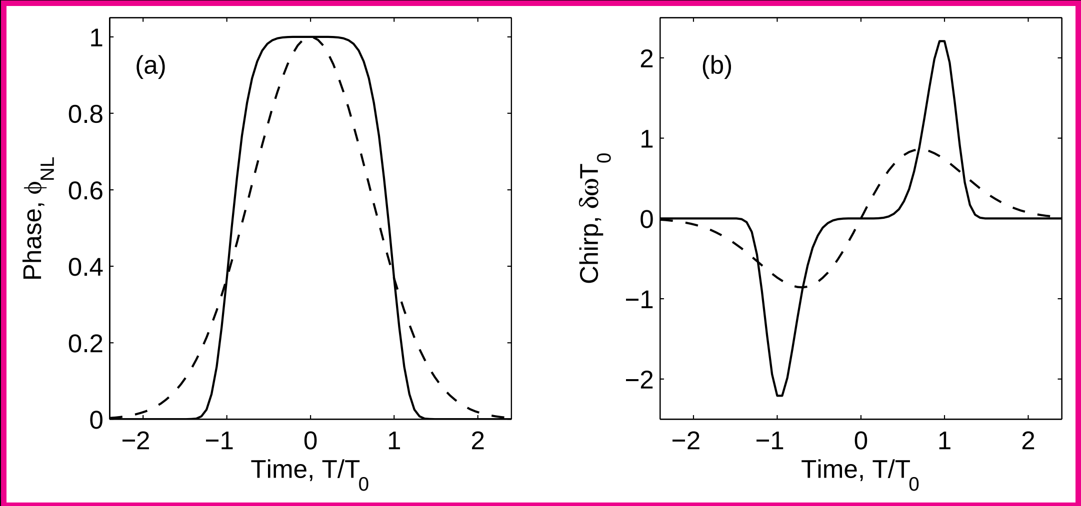
$$i\frac{\partial A}{\partial z} - \frac{\beta_2}{2}\frac{\partial^2 A}{\partial t^2} + \gamma|A|^2 A = 0.$$

- Dispersive effects within the fiber included through β_2.
- Nonlinear effects included through $\gamma = 2\pi n_2/(\lambda A_{\text{eff}})$.
- If we ignore dispersive effects, solution can be written as

$$A(L,t) = A(0,t)\exp(i\phi_{\text{NL}}), \quad \text{where} \quad \phi_{\text{NL}}(t) = \gamma L|A(0,t)|^2.$$

- Nonlinear phase shift depends on input pulse shape.
- Maximum Phase shift: $\phi_{\max} = \gamma P_0 L = L/L_{\text{NL}}$.
- Nonlinear length: $L_{\text{NL}} = (\gamma P_0)^{-1} \sim 1$ km for $P_0 \sim 1$ W.

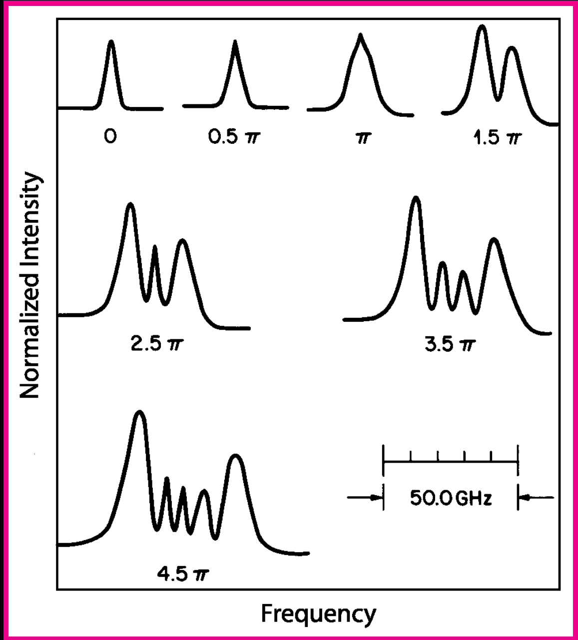
SPM-Induced Chirp



- Super-Gaussian pulses: $P(t) = P_0 \exp[-(t/T)^{2m}]$.
- Gaussian pulses correspond to the choice $m = 1$.
- Chirp is related to the phase derivative $d\phi/dt$.
- SPM creates new frequencies and leads to spectral broadening.

SPM-Induced Spectral Broadening

- First observed in 1978 by Stolen and Lin.
- 90-ps pulses transmitted through a 100-m-long fiber.
- Spectra are labelled using $\phi_{\max} = \gamma P_0 L$.
- Number M of spectral peaks: $\phi_{\max} = (M - \frac{1}{2})\pi$.



- Output spectrum depends on shape and chirp of input pulses.
- Even spectral compression can occur for suitably chirped pulses.

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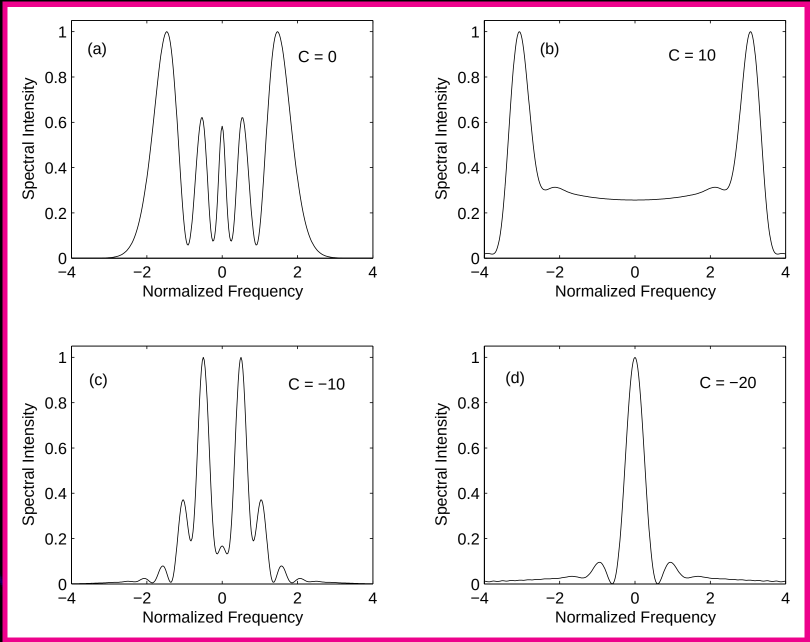
SPM-Induced Spectral Narrowing



- Chirped Gaussian pulses with $A(0,t) = A_0 \exp[-\frac{1}{2}(1+iC)(t/T_0)^2]$.
- If $C < 0$ initially, SPM produces spectral narrowing.

SPM: Good or Bad?

- SPM-induced spectral broadening can degrade performance of a lightwave system.
- Modulation instability often enhances system noise.

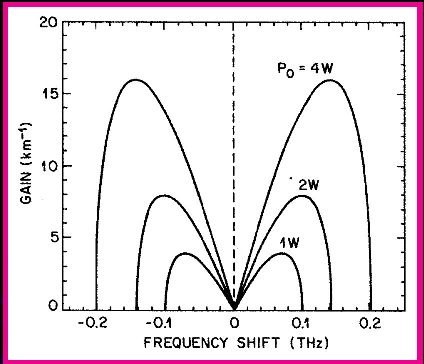
On the positive side . . .

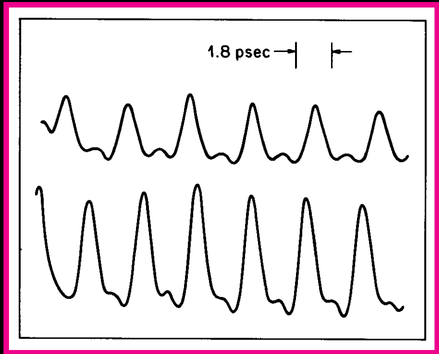
- Modulation instability can be used to produce ultrashort pulses at high repetition rates.
- SPM often used for fast optical switching (NOLM or MZI).
- Formation of standard and dispersion-managed optical solitons.
- Useful for all-optical regeneration of WDM channels.
- Other applications (pulse compression, chirped-pulse amplification, passive mode-locking, etc.)

Modulation Instability

Nonlinear Schrödinger Equation

$$i\frac{\partial A}{\partial z} - \frac{\beta_2}{2}\frac{\partial^2 A}{\partial t^2} + \gamma |A|^2 A = 0.$$

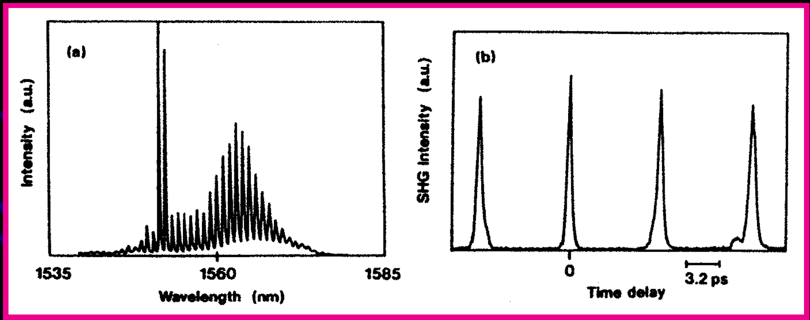




- CW solution unstable for anomalous dispersion $(\beta_2 < 0)$.
- Useful for producing ultrashort pulse trains at tunable repetition rates [Tai et al., PRL 56, 135 (1986); APL 49, 236 (1986)].

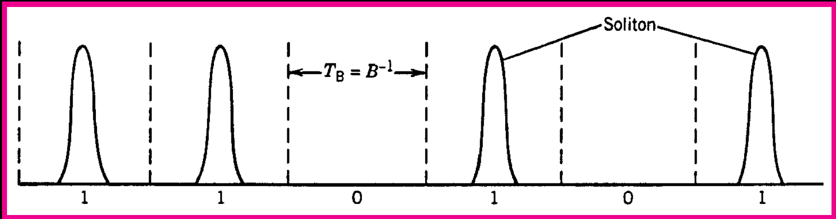
Modulation Instability

- A CW beam can be converted into a pulse train.
- Two CW beams at slightly different wavelengths can initiate modulation instability and allow tuning of pulse repetition rate.
- Repetition rate is governed by their wavelength difference.
- Repetition rates ∼100 GHz realized by 1993 using DFB lasers (Chernikov et al., APL 63, 293, 1993).



Optical Solitons

- Combination of SPM and anomalous GVD produces solitons.
- Solitons preserve their shape in spite of the dispersive and nonlinear effects occurring inside fibers.
- Useful for optical communications systems.



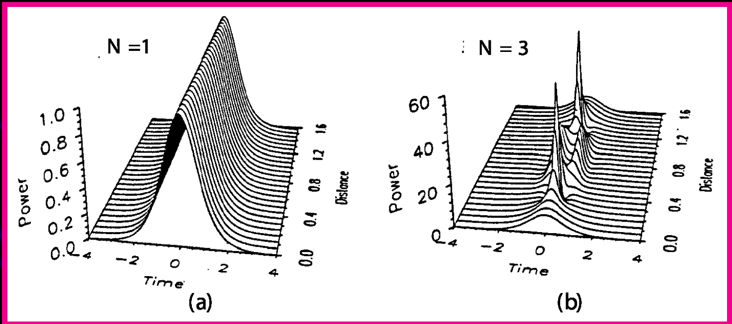
- Dispersive and nonlinear effects balanced when $L_{NL} = L_D$.
- Nonlinear length $L_{NL} = 1/(\gamma P_0)$; Dispersion length $L_D = T_0^2/|\beta_2|$.
- Two lengths become equal if peak power and width of a pulse satisfy $P_0 T_0^2 = |\beta_2|/\gamma$.

Fundamental and Higher-Order Solitons

- NLS equation: $i\frac{\partial A}{\partial z} - \frac{\beta_2}{2}\frac{\partial^2 A}{\partial t^2} + \gamma|A|^2 A = 0.$
- Solution depends on a single parameter: $N^2 = \frac{\gamma P_0 T_0^2}{|\beta_2|}.$
- Fundamental ($N = 1$) solitons preserve shape:

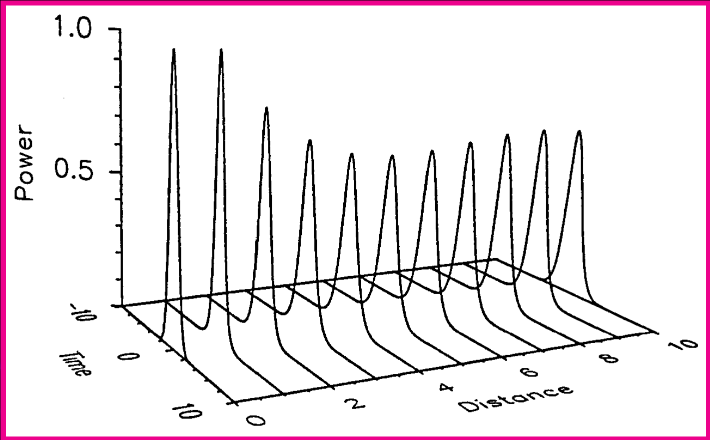
$$A(z,t) = \sqrt{P_0}\,\text{sech}(t/T_0)\exp(iz/2L_D).$$

- Higher-order solitons evolve in a periodic fashion.



Stability of Optical Solitons

- Solitons are remarkably stable.
- Fundamental solitons can be excited with any pulse shape.



Gaussian pulse with $N = 1$.
Pulse eventually acquires a 'sech' shape.

- Can be interpreted as temporal modes of a SPM-induced waveguide.
- $\Delta n = n_2 I(t)$ larger near the pulse center.
- Some pulse energy is lost through dispersive waves.

Cross-Phase Modulation

- Consider two optical fields propagating simultaneously.
- Nonlinear refractive index seen by one wave depends on the intensity of the other wave as

$$\Delta n_{\rm NL} = n_2(|A_1|^2 + b|A_2|^2).$$

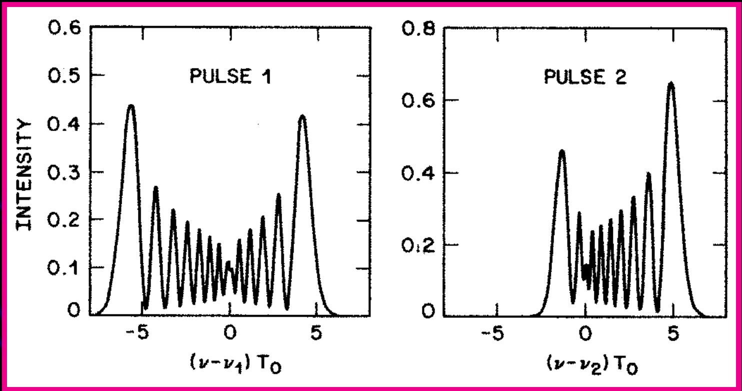
- Total nonlinear phase shift:

$$\phi_{\rm NL} = (2\pi L/\lambda)n_2[I_1(t) + bI_2(t)].$$

- An optical beam modifies not only its own phase but also of other copropagating beams (XPM).
- XPM induces nonlinear coupling among overlapping optical pulses.

XPM-Induced Chirp

- Fiber dispersion affects the XPM considerably.
- Pulses belonging to different WDM channels travel at different speeds.
- XPM occurs only when pulses overlap.
- Asymmetric XPM-induced chirp and spectral broadening.



XPM: Good or Bad?

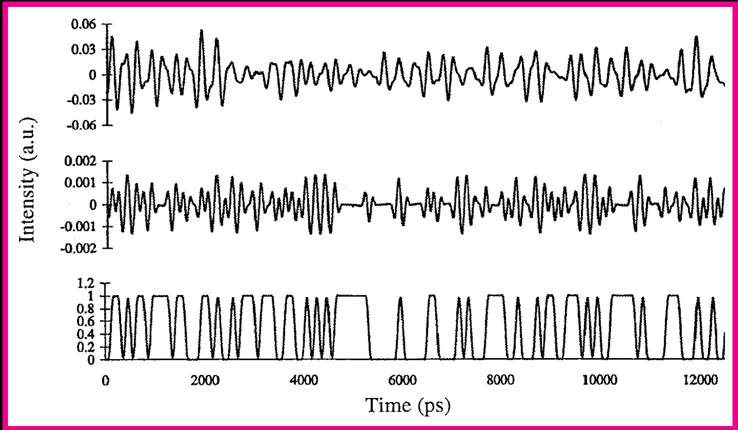
- XPM leads to interchannel crosstalk in WDM systems.
- It can produce amplitude and timing jitter.

On the other hand ...

XPM can be used beneficially for

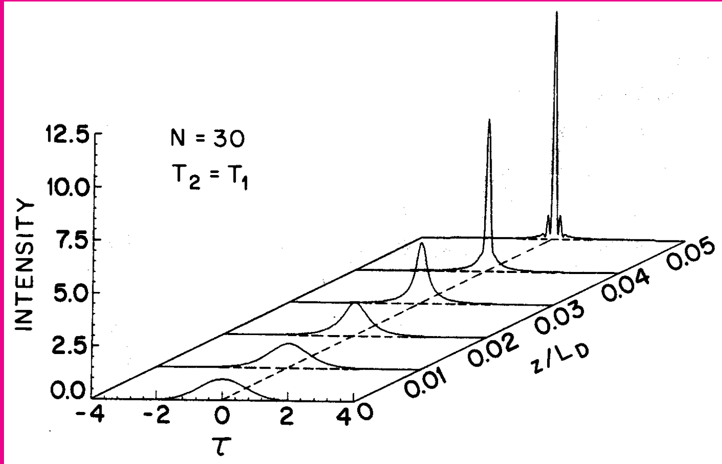
- Nonlinear Pulse Compression
- Passive mode locking
- Ultrafast optical switching
- Demultiplexing of OTDM channels
- Wavelength conversion of WDM channels

XPM-Induced Crosstalk



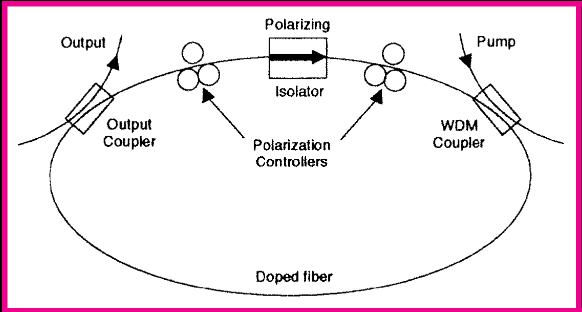
- A CW probe propagated with 10-Gb/s pump channel.
- Probe phase modulated through XPM.
- Dispersion converts phase modulation into amplitude modulation.
- Probe power after 130 (middle) and 320 km (top) exhibits large fluctuations (Hui et al., JLT, 1999).

XPM-Induced Pulse Compression



- An intense pump pulse is copropagated with the low-energy pulse requiring compression.
- Pump produces XPM-induced chirp on the weak pulse.
- Fiber dispersion compresses the pulse.

XPM-Induced Mode Locking

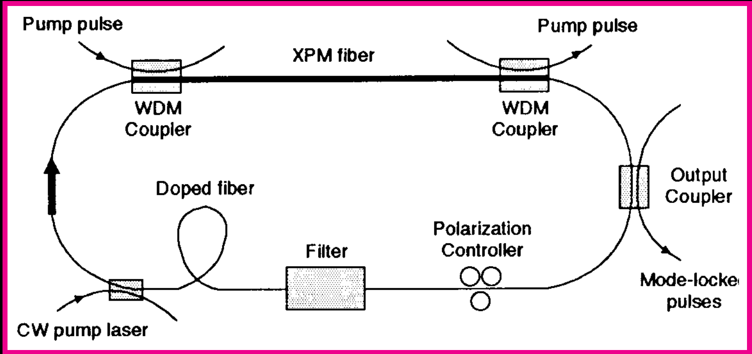


- Different nonlinear phase shifts for the two polarization components: nonlinear polarization rotation.

$$\phi_x - \phi_y = (2\pi L/\lambda) n_2 [(I_x + bI_y) - (I_y + bI_x)].$$

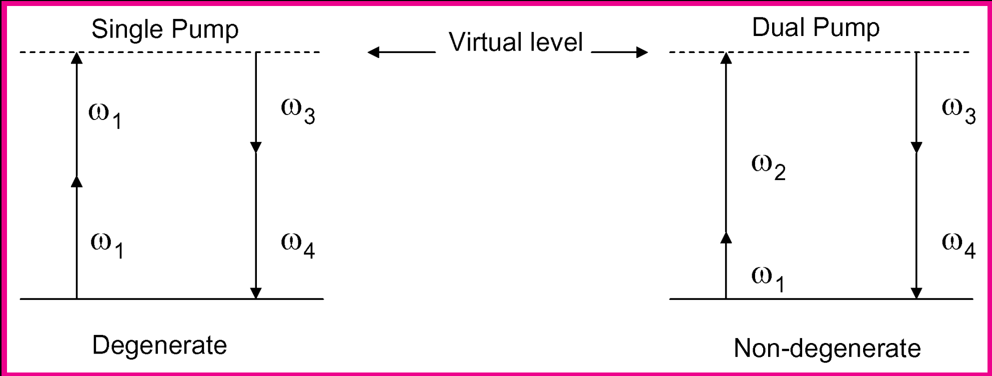
- Pulse center and wings develop different polarizations.
- Polarizing isolator clips the wings and shortens the pulse.
- Can produce ~100 fs pulses.

Synchronous Mode Locking



- Laser cavity contains the XPM fiber (few km long).
- Pump pulses produce XPM-induced chirp periodically.
- Pulse repetition rate set to a multiple of cavity mode spacing.
- Situation equivalent to the FM mode-locking technique.
- 2-ps pulses generated for 100-ps pump pulses (Noske et al., Electron. Lett, 1993).

Four-Wave Mixing (FWM)



- FWM is a nonlinear process that transfers energy from pumps to signal and idler waves.
- FWM requires conservation of (notation: $E = \mathrm{Re}[Ae^{i(\beta z - \omega t)}]$)
  - ⋆ Energy $\omega_1 + \omega_2 = \omega_3 + \omega_4$
  - ⋆ Momentum $\beta_1 + \beta_2 = \beta_3 + \beta_4$
- Degenerate FWM: Single pump ($\omega_1 = \omega_2$).

Theory of Four-Wave Mixing

- Third-order polarization: $\mathbf{P}_{\rm NL} = \varepsilon_0\chi^{(3)}\vdots\mathbf{EEE}$ (Kerr nonlinearity).

$$\mathbf{E} = \frac{1}{2}\hat{x}\sum_{j=1}^{4} F_j(x,y)A_j(z,t)\exp[i(\beta_j z - \omega_j t)] + \text{c.c.}$$

- The four slowly varying amplitudes satisfy

$$\frac{dA_1}{dz} = \frac{in_2\omega_1}{c}\left[\left(f_{11}|A_1|^2 + 2\sum_{k\neq 1} f_{1k}|A_k|^2\right)A_1 + 2f_{1234}A_2^*A_3A_4e^{i\Delta kz}\right]$$

$$\frac{dA_2}{dz} = \frac{in_2\omega_2}{c}\left[\left(f_{22}|A_2|^2 + 2\sum_{k\neq 2} f_{2k}|A_k|^2\right)A_2 + 2f_{2134}A_1^*A_3A_4e^{i\Delta kz}\right]$$

$$\frac{dA_3}{dz} = \frac{in_2\omega_3}{c}\left[\left(f_{33}|A_3|^2 + 2\sum_{k\neq 3} f_{3k}|A_k|^2\right)A_3 + 2f_{3412}A_1A_2A_4^*e^{-i\Delta kz}\right]$$

$$\frac{dA_4}{dz} = \frac{in_2\omega_4}{c}\left[\left(f_{44}|A_4|^2 + 2\sum_{k\neq 4} f_{4k}|A_k|^2\right)A_4 + 2f_{4312}A_1A_2A_3^*e^{-i\Delta kz}\right]$$

Simplified Scalar Theory

- Linear phase mismatch: $\Delta k = \beta_3 + \beta_4 - \beta_1 - \beta_2$.
- Overlap integrals $f_{ijkl} \approx f_{ij} \approx 1/A_{\text{eff}}$ in single-mode fibers.
- Full problem quite complicated (4 coupled nonlinear equations)
- Undepleted-pump approximation $\Longrightarrow$ two linear coupled equations:
- Using $A_j = B_j \exp[2i\gamma(P_1 + P_2)z]$, the signal and idler satisfy:

$$\frac{dB_3}{dz} = 2i\gamma\sqrt{P_1 P_2} B_4^* e^{-i\kappa z}, \qquad \frac{dB_4}{dz} = 2i\gamma\sqrt{P_1 P_2} B_3^* e^{-i\kappa z}.$$

- Total phase mismatch: $\kappa = \beta_3 + \beta_4 - \beta_1 - \beta_2 + \gamma(P_1 + P_2)$.
- Nonlinear parameter: $\gamma = n_2\omega_0/(cA_{\text{eff}}) \sim 10\ \text{W}^{-1}/\text{km}$.
- Signal power P_3 and Idler power P_4 are much smaller than pump powers P_1 and P_2 ($P_n = |A_n|^2 = |B_n|^2$).

General Solution

- Signal and idler fields satisfy:

$$\frac{dB_3}{dz} = 2i\gamma\sqrt{P_1 P_2} B_4^* e^{-i\kappa z}, \qquad \frac{dB_4^*}{dz} = -2i\gamma\sqrt{P_1 P_2} B_3 e^{i\kappa z}.$$

- General solution when both the signal and idler are present at $z = 0$:

$$\begin{aligned} B_3(z) = \{B_3(0)[\cosh(gz) &+ (i\kappa/2g)\sinh(gz)] \\ &+ (i\gamma/g)\sqrt{P_1 P_2} B_4^*(0)\sinh(gz)\}e^{-i\kappa z/2} \\ B_4^*(z) = \{B_4^*(0)[\cosh(gz) &- (i\kappa/2g)\sinh(gz)] \\ &- (i\gamma/g)\sqrt{P_1 P_2} B_3(0)\sinh(gz)\}e^{i\kappa z/2} \end{aligned}$$

- If an idler is not launched at $z = 0$ (phase-insensitive amplification):

$$B_3(z) = B_3(0)[\cosh(gz) + (i\kappa/2g)\sinh(gz)]e^{-i\kappa z/2}$$
$$B_4^*(z) = B_3(0)(-i\gamma/g)\sqrt{P_1 P_2}\sinh(gz)e^{i\kappa z/2}$$

Gain Spectrum

- Signal amplification factor for a FOPA:

$$G(\omega) = \frac{P_3(L,\omega)}{P_3(0,\omega)} = \left[1 + \left(1 + \frac{\kappa^2(\omega)}{4g^2(\omega)}\right)\sinh^2[g(\omega)L]\right].$$

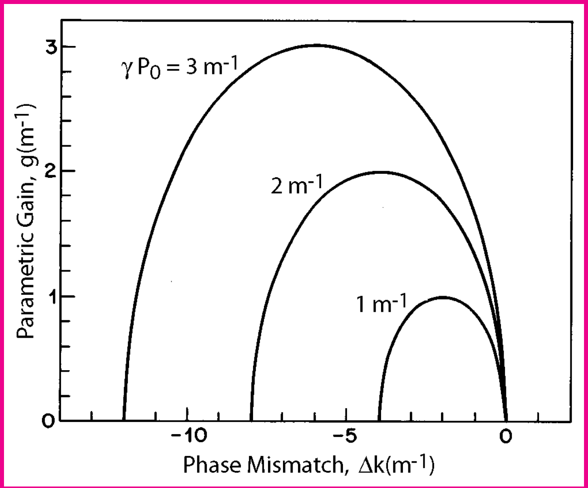
- Parametric gain: $g(\omega) = \sqrt{4\gamma^2 P_1 P_2 - \kappa^2(\omega)/4}$.
- Wavelength conversion efficiency:

$$\eta_c(\omega) = \frac{P_4(L,\omega)}{P_3(0,\omega)} = \left(1 + \frac{\kappa^2(\omega)}{4g^2(\omega)}\right)\sinh^2[g(\omega)L].$$

- Best performance for perfect phase matching ($\kappa = 0$):

$$G(\omega) = \cosh^2[g(\omega)L], \qquad \eta_c(\omega) = \sinh^2[g(\omega)L].$$

Parametric Gain and Phase Matching



In the case of a single pump:
$g(\omega) = \sqrt{(\gamma P_0)^2 - \kappa^2(\omega)/4}$.

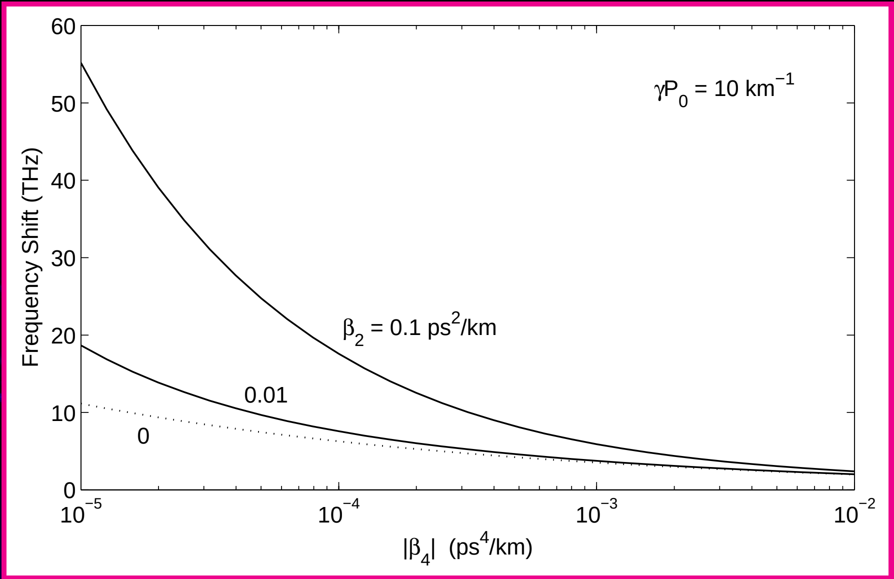
Phase mismatch $\kappa = \Delta k + 2\gamma P_0$

Parametric gain maximum when $\Delta k = -2\gamma P_0$.

- Linear mismatch: $\Delta k = \beta_2\Omega^2 + \beta_4\Omega^4/12 + \cdots$, where $\Omega = \omega_s - \omega_p$.
- Phase matching realized by detuning pump wavelength from fiber's ZDWL slightly such that $\beta_2 < 0$.
- In this case $\Omega = \omega_s - \omega_p = (2\gamma P_0/|\beta_2|)^{1/2}$.

Highly Nondegenerate FWM

- Some fibers can be designed such that $\beta_4 < 0$.
- If $\beta_4 < 0$, phase matching is possible for $\beta_2 > 0$.
- Ω can be very large in this case.



FWM: Good or Bad?

- FWM leads to interchannel crosstalk in WDM systems.
- It generates additional noise and degrades system performance.

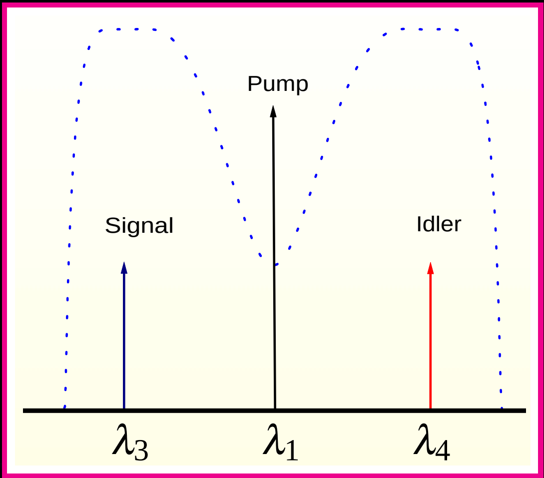
On the other hand ...
FWM can be used beneficially for

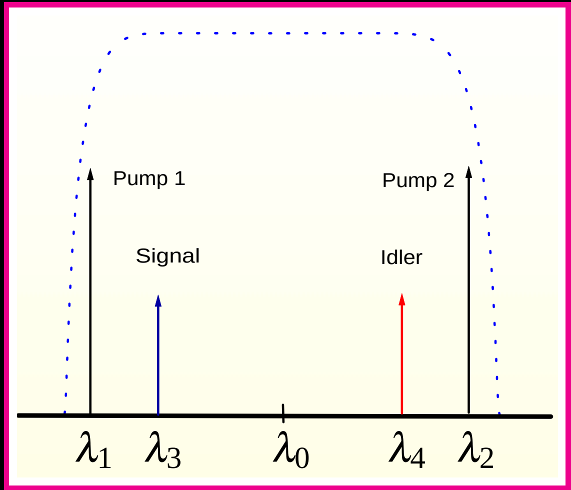
- Optical amplification and wavelength conversion
- Phase conjugation and dispersion compensation
- Ultrafast optical switching and signal processing
- Generation of correlated photon pairs

Parametric Amplification

- FWM can be used to amplify a weak signal.
- Pump power is transferred to signal through FWM.
- Peak gain $G_p = \frac{1}{4}\exp(2\gamma P_0 L)$ can exceed 20 dB for $P_0 \sim 0.5$ W and $L \sim 1$ km.
- Parametric amplifiers can provide gain at any wavelength using suitable pumps.
- Two pumps can be used to obtain 30–40 dB gain over a large bandwidth (>40 nm).
- Such amplifiers are also useful for ultrafast signal processing.
- They can be used for all-optical regeneration of bit streams.

Single- and Dual-Pump FOPAs



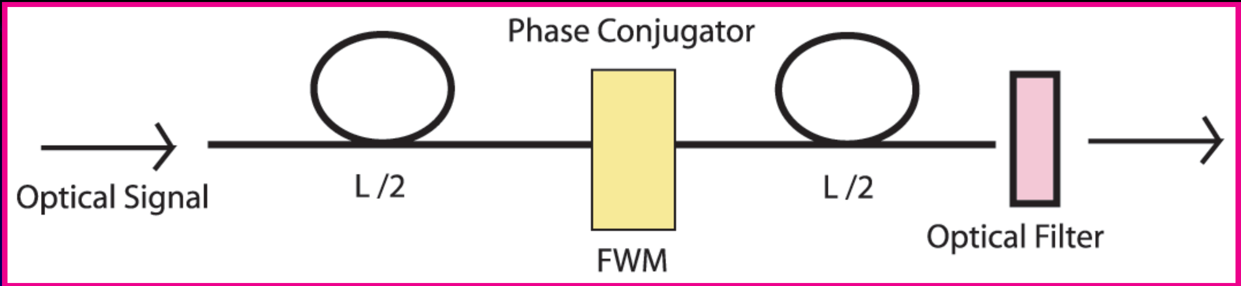


- Pump close to fiber's ZDWL
- Wide but nonuniform gain spectrum with a dip

- Pumps at opposite ends
- Much more uniform gain
- Lower pump powers ($\sim$0.5 W)

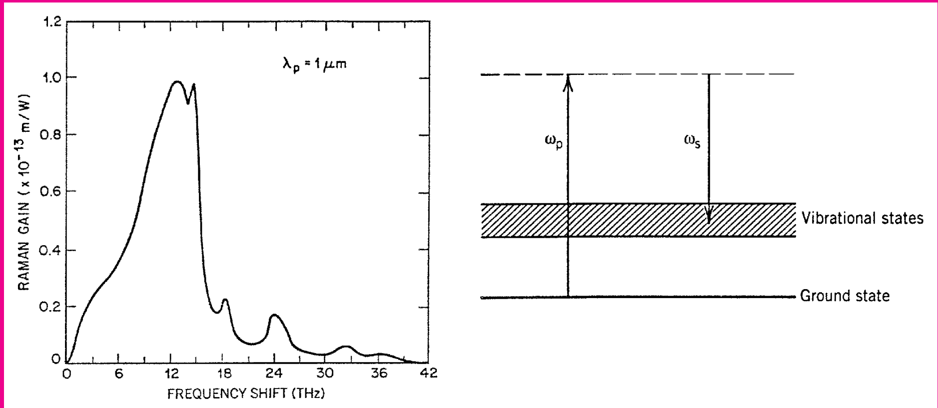
Optical Phase Conjugation

- FWM generates an idler wave during parametric amplification.
- Its phase is complex conjugate of the signal field ($A_4 \propto A_3^*$) because of spectral inversion.
- Phase conjugation can be used for dispersion compensation by placing a parametric amplifier midway.
- It can also reduce timing jitter in lightwave systems.



Stimulated Raman Scattering

- Scattering of light from vibrating silica molecules.
- Amorphous nature of silica turns vibrational state into a band.
- Raman gain spectrum extends over 40 THz or so.



- Raman gain is maximum near 13 THz.
- Scattered light red-shifted by 100 nm in the 1.5 μm region.

Raman Threshold

- Raman threshold is defined as the input pump power at which Stokes power becomes equal to the pump power at the fiber output:

$$P_s(L) = P_p(L) \equiv P_0 \exp(-\alpha_p L).$$

- $P_0 = I_0 A_{\text{eff}}$ is the input pump power.
- For $\alpha_s \approx \alpha_p$, threshold condition becomes

$$P_{s0}^{\text{eff}} \exp(g_R P_0 L_{\text{eff}}/A_{\text{eff}}) = P_0,$$

- Assuming a Lorentzian shape for the Raman-gain spectrum, Raman threshold is reached when (Smith, Appl. Opt. **11**, 2489, 1972)

$$\frac{g_R P_{th} L_{\text{eff}}}{A_{\text{eff}}} \approx 16 \quad \Longrightarrow \quad P_{th} \approx \frac{16 A_{\text{eff}}}{g_R L_{\text{eff}}}.$$

Estimates of Raman Threshold

Telecommunication Fibers

- For long fibers, $L_{\rm eff} = [1 - \exp(-\alpha L)]/\alpha \approx 1/\alpha \approx 20$ km for $\alpha = 0.2$ dB/km at 1.55 μm.
- For telecom fibers, $A_{\rm eff}$ = 50–75 μm^2.
- Threshold power P_{th} ~1 W is too large to be of concern.
- Interchannel crosstalk in WDM systems because of Raman gain.

Yb-doped Fiber Lasers and Amplifiers

- Because of gain, $L_{\rm eff} = [\exp(gL) - 1]/g > L$.
- For fibers with a large core, $A_{\rm eff} \sim 1000$ μm^2.
- P_{th} exceeds 10 kW for short fibers ($L < 10$ m).
- SRS may limit fiber lasers and amplifiers if $L \gg 10$ m.

SRS: Good or Bad?

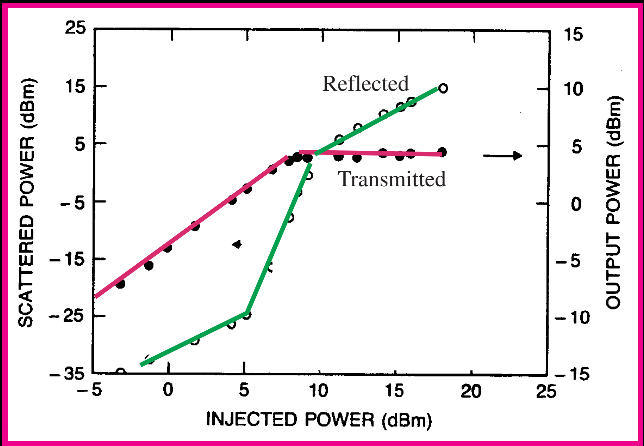
- Raman gain introduces interchannel crosstalk in WDM systems.
- Crosstalk can be reduced by lowering channel powers but it limits the number of channels.

On the other hand . . .

- Raman amplifiers are a boon for WDM systems.
- Can be used in the entire 1300–1650 nm range.
- EDFA bandwidth limited to ∼40 nm near 1550 nm.
- Distributed nature of Raman amplification lowers noise.
- Needed for opening new transmission bands in telecom systems.

Stimulated Brillouin Scattering

- Scattering of light from acoustic waves.
- Becomes a stimulated process when input power exceeds a threshold level.
- Low threshold power for long fibers (∼5 mW).



- Most of the power reflected backward after SBS threshold is reached.

Brillouin Shift

- Pump produces density variations through electrostriction, resulting in an index grating which generates Stokes wave through Bragg diffraction.
- Energy and momentum conservation require:

$$\Omega_B = \omega_p - \omega_s, \qquad \vec{k}_A = \vec{k}_p - \vec{k}_s.$$

- Acoustic waves satisfy the dispersion relation:

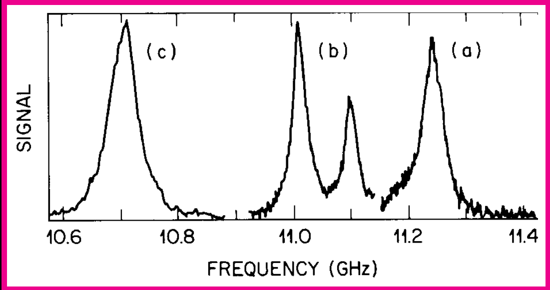
$$\Omega_B = v_A|\vec{k}_A| \approx 2v_A|\vec{k}_p|\sin(\theta/2).$$

- In a single-mode fiber $\theta = 180^\circ$, resulting in

$$\nu_B = \Omega_B/2\pi = 2n_p v_A/\lambda_p \approx 11 \text{ GHz},$$

if we use $v_A = 5.96$ km/s, $n_p = 1.45$, and $\lambda_p = 1.55\ \mu$m.

Brillouin Gain Spectrum



- Measured spectra for (a) silica-core (b) depressed-cladding, and (c) dispersion-shifted fibers.
- Brillouin gain spectrum is quite narrow (~50 MHz).
- Brillouin shift depends on GeO_2 doping within the core.
- Multiple peaks are due to the excitation of different acoustic modes.
- Each acoustic mode propagates at a different velocity v_A and thus leads to a different Brillouin shift ($\nu_B = 2n_p v_A/\lambda_p$).

Brillouin Threshold

- Pump and Stokes evolve along the fiber as

$$-\frac{dI_s}{dz} = g_B I_p I_s - \alpha I_s, \qquad \frac{dI_p}{dz} = -g_B I_p I_s - \alpha I_p.$$

- Ignoring pump depletion, $I_p(z) = I_0 \exp(-\alpha z)$.
- Solution of the Stokes equation:

$$I_s(L) = I_s(0) \exp(g_B I_0 L_{\text{eff}} - \alpha L).$$

- Brillouin threshold is obtained from

$$\frac{g_B P_{th} L_{\text{eff}}}{A_{\text{eff}}} \approx 21 \quad \Longrightarrow \quad P_{th} \approx \frac{21 A_{\text{eff}}}{g_B L_{\text{eff}}}.$$

- Brillouin gain $g_B \approx 5 \times 10^{-11}$ m/W is nearly independent of the pump wavelength.

Estimates of Brillouin Threshold

Telecommunication Fibers

- For long fibers, $L_{\rm eff} = [1 - \exp(-\alpha L)]/\alpha \approx 1/\alpha \approx 20$ km for $\alpha = 0.2$ dB/km at 1.55 μm.
- For telecom fibers, $A_{\rm eff}$ = 50–75 μm^2.
- Threshold power P_{th} ~1 mW is relatively small.

Yb-doped Fiber Lasers and Amplifiers

- Because of gain, $L_{\rm eff} = [\exp(gL) - 1]/g > L$.
- P_{th} exceeds 20 W for a 1-m-long standard fibers.
- Further increase occurs for large-core fibers; $P_{th} \sim 400$ W when $A_{\rm eff} \sim 1000$ μm^2.
- SBS is the dominant limiting factor at power levels $P_0 > 1$ kW.

Techniques for Controlling SBS

- Pump-Phase modulation: Sinusoidal modulation at several frequencies >0.1 GHz or with a pseudorandom bit pattern.
- Cross-phase modulation by launching a pseudorandom pulse train at a different wavelength.
- Temperature gradient along the fiber: Changes in $\nu_B = 2n_p v_A/\lambda_p$ through temperature dependence of n_p.
- Built-in strain along the fiber: Changes in ν_B through n_p.
- Nonuniform core radius and dopant density: mode index n_p also depends on fiber design parameters (a and Δ).
- Control of overlap between the optical and acoustic modes.
- Use of Large-core fibers: A wider core reduces SBS threshold by enhancing A_{eff}.

Concluding Remarks

- Optical fibers exhibit a variety of nonlinear effects.
- Fiber nonlinearities are feared by telecom system designers because they can affect system performance adversely.
- Fiber nonlinearities can be managed thorough proper system design.
- Nonlinear effects are useful for many device and system applications: optical switching, soliton formation, wavelength conversion, broadband amplification, channel demultiplexing, etc.
- New kinds of fibers have been developed for enhancing nonlinear effects (microstrctured fibers with air holes).
- Nonlinear effects in such fibers are finding new applications.