





Nonlinearity of Optical Fibers

(The Good, the Bad, and the Ugly)

Govind P. Agrawal

The Institute of Optics University of Rochester Rochester, New York, USA

©2018 G. P. Agrawal





Introduction

Fiber Nonlinearities

- First studied during the 1970s soon after low-loss fibers were made.
- Ignored during the 1980s for single-channel lightwave systems.
- They were feared during the 1990s as multichannel (WDM) systems became widespread.
- Modern coherent systems are also affected by fiber nonlinearities.

Modern Perspective

- Telecom systems will always be affected by fiber nonlinearities. We should develop techniques to deal with them.
- Use the nonlinear effects to advantage whenever possible: Raman and parametric amplifiers, supercontinuum generation, wavelength tuning through soliton self-frequency shift.



Back Close

UNIVERSITY





Major Nonlinear Effects in Optical Fibers

- Self-Phase Modulation (SPM)
- Cross-Phase Modulation (XPM)
- Four-Wave Mixing (FWM)
- Stimulated Raman Scattering (SRS)
- Stimulated Brillouin Scattering (SBS)

Origin of Nonlinear Effects

- Third-order susceptibility $\chi^{(3)}$ of silica glass.
- Its real part leads to SPM, XPM, and FWM.
- Imaginary part leads to two-photon absorption (TPA), SRS and SBS (among other things).









Kerr Nonlinearity in Optical Fibers

- The tensorial nature of $\chi^{(3)}$ makes theory quite complicated.
- It can be simplified considerably when a single optical beam excites the fundamental mode of an optical fiber.
- Only the component $\chi^{(3)}_{1111}(-\omega;\omega,-\omega,\omega)$ is relevant in this case.
- Its real part provides the Kerr coefficient n_2 as

$$n_2(\boldsymbol{\omega}) = \frac{3}{4\varepsilon_0 c n_0^2} \operatorname{Re}[\boldsymbol{\chi}_{1111}^{(3)}].$$

• Refractive index depends on intensity as (Kerr effect):

$$n(\boldsymbol{\omega},I)=\bar{n}(\boldsymbol{\omega})+n_2I(t).$$

• For silica fibers, $n_2 = 3 \times 10^{-20} \text{ m}^2/\text{W}$ is relatively small; $\delta n = n_2 I = 3 \times 10^{-10}$ even for $I = 1 \text{ MW/cm}^2$.









Back

Close

Self-Phase Modulation

• Optical phase of a fiber mode changes with distance as

 $\phi(z,t) = \bar{n}k_0 z = [n_0 + n_2 I(t)]k_0 z.$

• Kerr nonlinearity leads to a nonlinear phase shift

$$\phi_{\mathrm{NL}}(t) = n_2 [P(t)/A_{\mathrm{eff}}] (2\pi/\lambda) z = \gamma P(t) z.$$

- Optical field modifies its own phase (SPM) inside an optical fiber.
- Since the nonlinear phase shift varies with time, each optical pulse becomes chirped.
- Chirping manifests as spectral broadening in the frequency domain.
- Such spectral changes are undesirable for telecom systems (the ugly aspect of SPM).





SPM-Induced Spectral Broadening

- First observed in 1978 by Stolen and Lin.
- 90-ps pulses transmitted through a 100-m-long fiber.
- Spectra are labelled using $\phi_{\max} = \gamma P_0 L.$
- Number *M* of spectral peaks: $\phi_{\max} = (M \frac{1}{2})\pi$.



- Output spectrum depends on shape and chirp of input pulses.
- Even spectral compression can occur for suitably chirped pulses.









SPM-Induced Spectral Broadening



- SPM-induced spectral broadening along the length of a silica fiber.
- Gaussian input pulse: $P_0(t) = P_0 \exp[-(t/T_0)^2]$ with $(\gamma P_0)^{-1} = 1$ m.





SPM: Good or Bad?

- SPM-induced spectral broadening can degrade performance of a lightwave system.
- SPM can lead to modulation instability that enhances system noise.
 On the positive side ...
- Modulation instability can be used to produce ultrashort pulses at high repetition rates.
- SPM is often used for fast optical switching.
- Formation of optical solitons is beneficial for some applications.
- SPM is useful for all-optical regeneration of WDM channels.
- Other applications include pulse compression, chirped-pulse amplification, passive mode-locking, etc.



UNIVERSITY







Back Close

Group-Velocity Dispersion

• Frequency dependence of the mode index included using

$$\beta(\boldsymbol{\omega}) = \bar{n}(\boldsymbol{\omega})\boldsymbol{\omega}/c = \beta_0 + \beta_1(\boldsymbol{\omega} - \boldsymbol{\omega}_0) + \beta_2(\boldsymbol{\omega} - \boldsymbol{\omega}_0)^2 + \cdots,$$

where ω_0 is the carrier frequency of optical pulse.

- Group velocity is related to $\beta_1 = (d\beta/d\omega)_{\omega=\omega_0}$ as $v_g = 1/\beta_1$.
- Different frequency components of a pulse travel at different speeds and result in pulse broadening governed by $\beta_2 = (d^2\beta/d\omega^2)_{\omega=\omega_0}$.







10/51

Back Close

Pulse Evolution in Optical Fibers

• Pulse propagation is governed by Nonlinear Schrödinger Equation

$$i\frac{\partial A}{\partial z} - \frac{\beta_2}{2}\frac{\partial^2 A}{\partial t^2} + \gamma |A|^2 A = 0.$$

- Dispersive effects inside the fiber are included through β_2 .
- Nonlinear effects included through $\gamma = 2\pi n_2/(\lambda A_{\rm eff})$.
- Useful to normalize as $Z=z/L, \ au=t/T_0, \ U=A/\sqrt{P_0}$ to obtain

$$i\frac{\partial U}{\partial Z} - \operatorname{sgn}(\beta_2)\frac{L}{2L_D}\frac{\partial^2 A}{\partial t^2} + \frac{L}{L_{\mathrm{NL}}}|U|^2U = 0.$$

- Dispersion length $L_D = T_0^2/|\beta_2|$; Nonlinear length $L_{\rm NL} = (\gamma P_0)^{-1}$ for pulses of width T_0 and peak power P_0 .
- Dispersion negligible if $L_D \gg L$; nonlinearity negligible if $L_D \gg L_{\rm NL}$.
- Both of them become comparable if $L_D \sim L_{\rm NL} < L$.



ROCHESTER

Optical Solitons

- Combination of SPM and anomalous GVD produces solitons.
- Solitons preserve their shape in spite of the dispersive and nonlinear effects occurring inside fibers.
- This is useful for optical communications systems.



- Dispersive and nonlinear effects balanced when N = 1 or $L_{NL} = L_D$.
- Two lengths become equal if peak power and width of a pulse satisfy $P_0T_0^2 = |eta_2|/\gamma$.









Fundamental and Higher-Order Solitons

- NLS equation: $i\frac{\partial A}{\partial z} \frac{\beta_2}{2}\frac{\partial^2 A}{\partial t^2} + \gamma |A|^2 A = 0.$
- Solution depends on a single parameter: $N^2 = \frac{\gamma P_0 T_0^2}{|B_2|}$.
- Fundamental (N = 1) solitons preserve shape:

 $A(z,t) = \sqrt{P_0} \operatorname{sech}(t/T_0) \exp(iz/2L_D).$

• Higher-order solitons evolve in a periodic fashion.









13/51

Back

Close

Stability of Optical Solitons

- Solitons are remarkably stable.
- Fundamental solitons can be excited with any pulse shape.



Gaussian pulse with N = 1. Pulse eventually acquires a 'sech' shape.

- Can be interpreted as temporal modes of a SPM-induced waveguide.
- $\Delta n = n_2 I(t)$ larger near the pulse center.
- Some pulse energy is lost through dispersive waves.





Higher-Order Solitons











Cross-Phase Modulation

- Consider two distinct optical fields propagating simultaneously.
- Nonlinear refractive index seen by one wave depends on intensity of the other wave as $\Delta n_{\rm NL} = n_2(|A_1|^2 + b|A_2|^2)$.
- b = 2/3 for orthogonal polarizations; b = 2 for different wavelengths.
- Total nonlinear phase shift:

 $\phi_{\rm NL} = (2\pi L/\lambda)n_2[I_1(t) + bI_2(t)].$

- An optical pulse modifies not only its own phase but also of other co-propagating pulses (XPM) of different wavelengths.
- XPM induces nonlinear coupling among optical pulses belonging to different channels (nonlinear crosstalk).







XPM: Good or Bad?

- XPM leads to interchannel crosstalk in WDM systems.
- It degrades optical SNR through amplitude noise and timing jitter.

On the other hand ...

XPM can be used beneficially for

- Nonlinear Pulse Compression
- Passive mode locking
- Ultrafast optical switching
- Demultiplexing of OTDM channels
- Wavelength conversion of WDM channels









XPM-Induced Crosstalk



- A CW probe propagated with 10-Gb/s pump channel.
- Probe phase modulated through XPM.
- Dispersion converts phase modulation into amplitude modulation.
- Probe power after 130 (middle) and 320 km (top) exhibits large fluctuations (Hui et al., JLT, 1999).







XPM-Induced Pulse Compression



- An intense pump pulse is copropagated with the low-energy pulse requiring compression.
- Pump produces XPM-induced chirp on the weak pulse.
- Fiber dispersion compresses the pulse.







19/51

Back Close

Four-Wave Mixing (FWM)



- FWM is a nonlinear process that transfers energy from pumps to signal and idler waves.
- FWM requires conservation of (notation: $E = \operatorname{Re}[Ae^{i(\beta z \omega t)}])$
 - $\begin{array}{ll}\star \mbox{ Energy} & \omega_1 + \omega_2 = \omega_3 + \omega_4 \\ \star \mbox{ Momentum} & \beta_1 + \beta_2 = \beta_3 + \beta_4 \end{array}$
- Degenerate FWM: Single pump ($\omega_1 = \omega_2$).





FWM: Good or Bad?

- FWM leads to interchannel crosstalk in WDM systems.
- It generates additional noise and degrades system performance.

On the other hand ...

FWM can be used beneficially for

- Optical amplification and wavelength conversion
- Phase conjugation and dispersion compensation
- Ultrafast optical switching and signal processing
- Generation of correlated photon pairs









Fiber-optic Parametric Amplifiers





- Pump wavelength is close to fiber's ZDWL
- Wide but nonuniform gain spectrum with a dip

- Pumps at opposite ends
- Much more uniform gain
- Lower pump powers (\sim 0.5 W)







Wavelength Conversion



- FOPAs can transfer data to a different wavelength.
- A CW pump beam is launched into the fiber together with the signal channel.
- Pump wavelength is chosen half way from the desired shift.
- FWM transfers the data from signal to the idler wave at the new wavelength.





Multichannel Wavelength Conversion



- Islam et al., IEEE JSTQE 8, 527 (2002).
- 860-mW peak power pump at 1532 nm; 315-m-long fiber.
- 32 channels converted into S band with 4.7 dB conversion efficiency.



UNIVERSITY of







Stimulated Raman Scattering

- Scattering of light from vibrating silica molecules.
- Amorphous nature of silica turns vibrational state into a band.
- Raman gain spectrum extends over 40 THz or so.



- Raman gain is maximum near 13 THz.
- Scattered light red-shifted by 100 nm in the 1.5 μ m region.







SRS: Good or Bad?

- Raman gain introduces interchannel crosstalk in WDM systems.
- Crosstalk can be reduced by lowering channel powers but it limits the number of channels.

On the other hand \dots

- Raman amplifiers are a boon for WDM systems.
- They can be used in the entire 1300–1650 nm range.
- Distributed nature of Raman amplification lowers noise.
- Intrapulse Raman scattering can be used to tune the wavelength of short-pulse lasers toward longer wavelengths.



UNIVERSITY



26/51

Back Close

Intrapulse Raman scattering

- Spectrum of ultrashort pulses propagating as solitons shifts toward the red side; no distinct Stokes pulse generated.
- This effect was first observed in 1986. It is known as the soliton self-frequency shift (SSFS) or Raman-induced frequency shift (RIFS).
- RIFS scales with pulse width as T_0^{-4} and becomes quite large for short pulses.
- It can also occur in the normal-dispersion regime but its magnitude is relatively small: Santhanam and Agrawal, Opt. Commun. 222, 413 (2003).
- Very large RIFS can occur in highly nonlinear fibers if pulse energy is large enough to excite a high-order soliton.
- RIFS can be used for tuning wavelength of a mode-locked laser.

Wavelength tuning through SSFS

(Liu et al, Opt. Lett. 26, 358, 2001)

Left: Experiment: Output after 15 cm for 1300-nm, 200-fs pulses **Right:** Numerics: Rapid SSFS through soliton fission.

SSFS tuning in Mid-Infrared

Left: Experiment: Spectra of 100-fs pulses after 2m fluoride fiber **Right:** Numerics: Wavelength tunable from 2 to 4 μ m.

Stimulated Brillouin Scattering

- Origin of SBS lies in scattering of light from acoustic waves.
- Most of the power launched into a single-mode fiber is reflected backward if it exceeds the SBS threshold.
- Threshold of SBS relatively low for long fibers (\sim 5 mW).

- Pump at frequency ω_p creates a Stokes wave at ω_s when scattered by the acoustic wave of frequency Ω_A inside an optical fiber.
- Conservation of energy and momentum during this process requires:

$$\Omega_A = \omega_p - \omega_s, \qquad \vec{k}_A = \vec{k}_p - \vec{k}_s.$$

• Acoustic waves traveling at speed v_A satisfy the dispersion relation:

$$\Omega_A = v_A |\vec{k}_A| \approx 2 v_A |\vec{k}_p| \sin(\theta/2).$$

• In a single-mode fiber $heta=180^\circ$, resulting in the Brillouin shift

$$v_B = \Omega_A/2\pi = 2n_p v_A/\lambda_p \approx 11 \text{ GHz},$$

if we use $v_A = 5.96$ km/s, $n_p = 1.45$, and $\lambda_p = 1.55$ μ m.

Back Close

Brillouin Gain Spectrum

- Measured spectra for (a) silica-core (b) depressed-cladding, and (c) dispersion-shifted fibers.
- Brillouin gain spectrum is quite narrow (bandwidth ${\sim}50$ MHz).
- Brillouin shift depends on GeO₂ doping within the core.
- Multiple peaks are due to the excitation of different acoustic modes.
- Each acoustic mode propagates at a different velocity v_A and thus leads to a different Brillouin shift $(v_B = 2n_p v_A / \lambda_p)$.

SBS: Good or Bad?

- SBS is not of concern for WDM systems because of low channel powers (\sim 1 mW) used in practice.
- SBS becomes of major concern in high-power fiber lasers and amplifiers made with Yb-doped fibers.
- SBS must be controlled in parametric and Raman amplifiers requiring high pump powers (> 100 mW).

On the other hand \dots

- SBS can be used for making Brillouin lasers and amplifiers.
- Its narrow gain bandwidth can be used for channel selection in some applications.

Back Close

33/51

Back

Close

Techniques for Controlling SBS

- Modulation of the pump phase at several frequencies >0.1 GHz or randomly using a pseudorandom pattern (spectral broadening).
- Cross-phase modulation by launching a pseudorandom pulse train at a different wavelength.
- Temperature gradient along the fiber: Changes in $v_B = 2n_p v_A / \lambda_p$ through temperature dependence of n_p .
- Built-in strain along the fiber: Changes in v_B through n_p .
- Nonuniform core radius and dopant density: mode index n_p also depends on fiber design parameters (a and Δ).
- Control of overlap between the optical and acoustic modes.
- Use of Large-core fibers: A wider core reduces SBS threshold by enhancing $A_{\rm eff}$.

Supercontinuum Generation

(Ranka et al., Opt. Lett. 25, 25, 2000)

- Spectrum at the end of a 75-cm-long microstructured fiber when 100-fs pules with 0.8 pJ energy were launched.
- Spectrum extends over a wide spectral range (400 to 1600 nm).
- It is also relatively flat over the whole range (on a log scale).
- Useful in biomedical imaging as a broadband source.

Physics Behind SC Generation

- 100-fs input pulses propagate as high-order solitons (N > 10).
- Third-order dispersion (TOD) leads to their fission into multiple narrower fundamental solitons: $T_k = T_0/(2N+1-2k)$.
- Each of these solitons is affected by intrapulse Raman scattering that transfers energy from the blue side to the red side.
- Spectrum of each soliton shifts toward longer and longer wavelengths with propagation inside the fiber.
- At the same time, each soliton emits dispersive waves at different wavelengths on the blue side of the input wavelength.
- Cross-phase modulation (XPM) and four-wave mixing generate additional bandwidth to produce the observed supercontinuum.

Back

Close

36/51

Back Close

Numerical Modeling of Supercontinuum

• Supercontinuum is modeled by solving the generalized NLS equation

$$\frac{\partial A}{\partial z} + \frac{\alpha}{2}A + i\sum_{m=2}^{M} \frac{i^{m}\beta_{m}}{m!} \frac{\partial^{m}A}{\partial t^{m}} \\ = i\gamma \left(1 + \frac{i}{\omega_{0}} \frac{\partial}{\partial t}\right) \left(A(z,t) \int_{0}^{\infty} R(t') |A(z,t-t')|^{2} dt'\right).$$

- It is important to include the dispersive effects (β_m) and intrapulse Raman scattering (through R(t)) as accurately as possible.
- Terms up to M = 8 are often included in numerical simulations.
- Raman response included through the measured gain spectrum.
- Most features observed experimentally can be understood, at least qualitatively, by such a theory.

Supercontinuum Generation

• Fission of a N = 8 soliton inside a silica fiber.

- Multiple solitons and dispersive waves produce new frequencies.
- Supercontinuum formed after one dispersion length.

↓
↓
Back
Close

Close

Multimode or Multicore Fibers

39/51

Back Close

Fiber Modes (cont.)

- Number of modes governed by $V = k_0 a \sqrt{n_1^2 n_2^2}$.
- Mode diagrams provide \bar{n} for all modes of a fiber. Right figure applies when $\Delta = (n_1 n_2)/n_1 \ll 1$ (weakly guiding approximation).
- LP modes represent a linear combination of one or more degenerate modes and are approximately linearly polarized.

LP Modes of a Fiber

Multicore fibers for SDM

- Multicore fibers provide a simple way to enhance the capacity of WDM systems. Each core carries a different WDM bit stream.
- If cores are kept relatively far apart, linear and nonlinear cross-talk levels can be made acceptably small.
- The system capacity is boosted by the number of cores employed.
- Starting in 2010 many laboratory demonstrations explored this scheme.
- A 12-core fiber was used in 2012 to demonstrate 1 Pbit/s transmission over 52 km (Takara et al., ECOC PD paper Th.3.C.1).
- Experiment employed 222 WDM channels with 456 Gb/s/channel.
- A 32-core fiber was used in a 2017 experiment for 1 Pbit/s transmission over 205 km (Kobayashi et al., OFC PD paper Th5B.1).

44

Back Close

Back Close

Multimode fibers for SDM

- Few-mode fibers have a single core with V > 2.405 so that the fiber supports several modes.
- Linear and nonlinear cross talk cannot be avoided for such fibers since all modes share the same physical path.
- The impact of linear crosstalk can be eliminated at the receiver end though digital signal processing (MIMO).
- Starting in 2011 several laboratory demonstrations explored the use of few-mode fibers for SDM applications.
- A 6-mode fiber (3 spatial modes) was used in 2011 to demonstrate transmission over 96 km (Ryf et al., JLT **30**, 521, 2012).
- A 2013 experiment employed a 12-mode fiber with a spectral efficiency of 32 bit/s/Hz (Ryf et al., OFC, paper PDP5A.1, 2013).

Nonlinear Propagation Equations

• Numerical modeling requires solution of coupled NLS equations:

$$\frac{\partial \mathbf{A}_p}{\partial z} - i(\boldsymbol{\beta}_{0p} - \boldsymbol{\beta}_r)\mathbf{A}_p + \left(\boldsymbol{\beta}_{1p} - \frac{1}{v_{g_r}}\right)\frac{\partial \mathbf{A}_p}{\partial t} + \frac{i\boldsymbol{\beta}_{2p}}{2}\frac{\partial^2 \mathbf{A}_p}{\partial t^2}$$
$$= i\sum_m q_{mp}\mathbf{A}_m + \frac{i\boldsymbol{\gamma}}{3}\sum_{lmn} f_{lmnp}\left[(\mathbf{A}_n^{\mathrm{T}}\mathbf{A}_m)\mathbf{A}_l^* + 2(\mathbf{A}_l^{\mathrm{H}}\mathbf{A}_m)\mathbf{A}_n\right].$$

• Linear and nonlinear couplings among spatial modes governed by

$$q_{mp} \propto k_0 \iint \Delta n(x, y, z) \mathbf{F}_p^* \mathbf{F}_m dx dy, \quad f_{lmnp} \propto \iint \mathbf{F}_l^* \mathbf{F}_m \mathbf{F}_n \mathbf{F}_p^* dx dy.$$

- Fiber modes normalized such that $\iint F_p^*(x,y)F_m(x,y)dxdy = \delta_{mp}$.
- For a fiber with *M* spatial modes, one needs to solve 2*M* coupled NLS equations (2 accounts for two orthogonal polarizations).

Back Close

Numerical Results for Multicore Fibers

PDM-QPSK simulation over 1000 km with strongly coupled cores

UNIVERSITY of

FR

45/51

Back Close

Soliton Propagation in Multimode Fibers

- Soliton dynamics is much more complex in multimode fibers since modes become nonlinearly coupled through cross-phase modulation.
- Can a soliton launched into a specific mode survive inside a fiber if only noise is present in other modes?
- What happens if multiple solitons are launched simultaneously into different fiber modes?
- Numerical results show nonlinear energy transfer from higher-order modes to the fundamental mode [Buch and Agrawal, Opt. Lett. 40, 225 (2015)].
- Differential group delay among modes plays an important role.
- Its effects can be minimized using graded-index multimode fibers.

Supercontinuum Generation

Wright et al., Nature Photon. 9, 306 (2015)

- 500-fs pulse excites 5 modes of a 1-m-long GRIN multimode fiber.
- Pulse spectra at the fiber output for individual modes shown on top.
- Evolution of total spectra along fiber length is shown at bottom.

Supercontinuum Generation (cont.)

Wright et al., Nature Photon. 9, 306 (2015)

- Infrared and visible (middle) spectra for several pulse energies.
- Output beam profiles are also shown in each case.
- Right part shows different colors in the spectrally resolved output.

Back Close

UNIVERSITY of

Self-Focusing and Filamentation

Wright et al., Nature Photon. 9, 306 (2015)

- Near filed images at different pulse energies showing self-focusing, speckling, refocusing, and filamentation.
- Pulse spectra at the fiber output are shown on the right side.
- Pulse evolution exhibits a complicated spatio-temporal dynamics.

Back Close

Intermodal FWM

400-ps pump pulses at 1060 nm launched with peak powers:

23.6 kW (blue) 43.7 kW (green) 51.7 kW (red) 70.5 kW (black)

49/51

Bendahmane et al., JOSAB **35**, 295 (2018)

- Graded-index fiber had a 100- μ m core.
- Multiple spectral bands generated through intermodal FWM.

Intermodal SRS

Antikainen et al., CLEO (2018)

- Pump pulse excites LP_{0,19} mode of a 20-m-long fiber (90-μm core) with widths 60-100 fs.
- Output spectra for pulse widths ranging from 60 to 100 fs.
- Stokes pulse forms in a neighboring mode for which group-velocity matching occurs.

Concluding Remarks

- Optical fibers enhance nonlinear effects by confining light to a narrow core and maintaining high intensities over long distances.
- Five major nonlinear phenomena were covered in this tutorial.
- The use of short pulses enhances peak intensities and allows supercontinuum generation with a multitude of practical applications.
- SPM, XPM and FWM limit the performance of modern telecom systems; they are also useful for ultrafast signal processing and optical switching, among other things.
- SRS is of concern for high-power fiber lasers but is also useful for broadband amplifiers and for shifting the wavelength of lasers.
- Multimode fibers allow exploring of intermodal nonlinear effects. They may also enhance the capacity of lightwave systems.

UNIVERSITY of

