Nonlinear Performance of SDM Systems Designed with Multimode or Multicore Fibers

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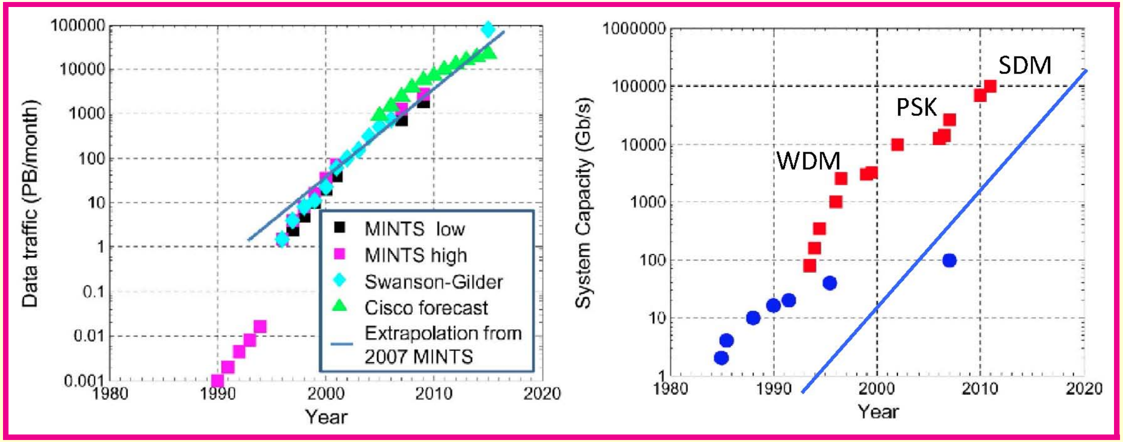
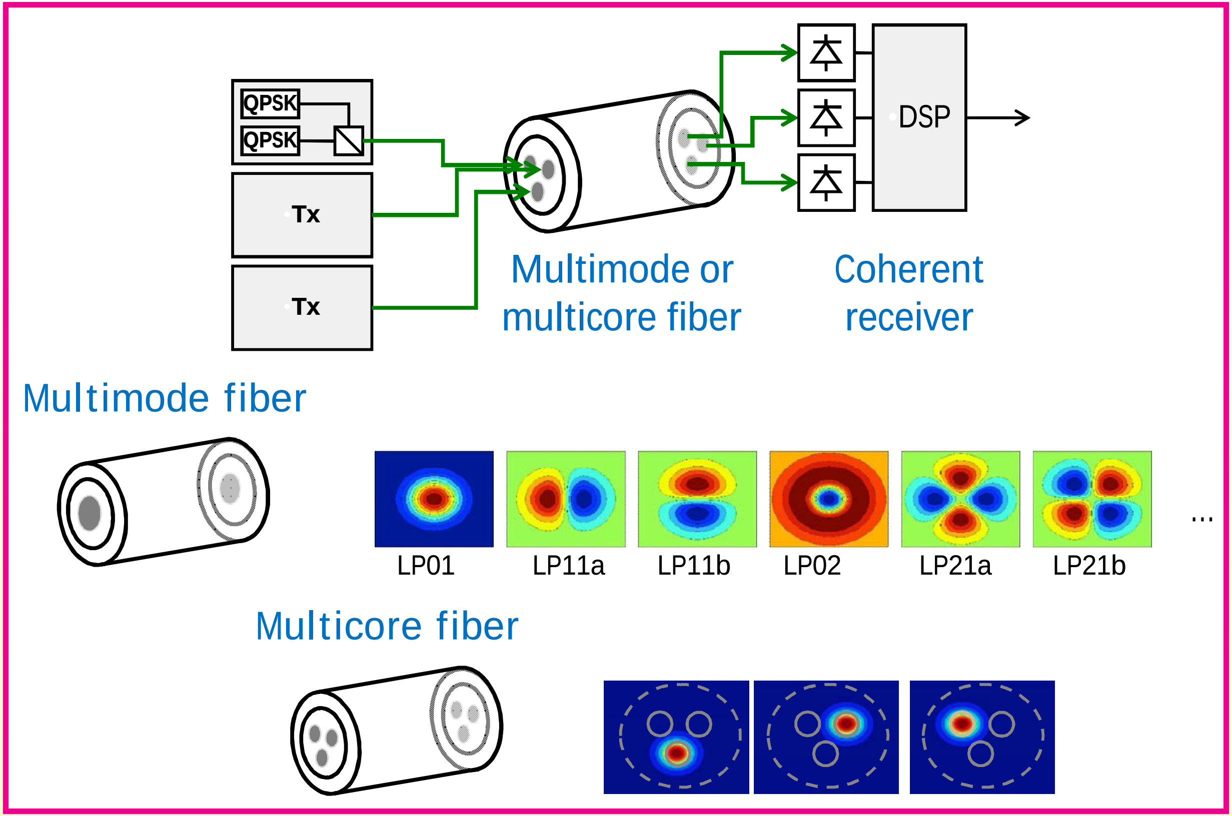
Introduction

- Internet traffic has been growing exponentially every year.
- System capacity must follow this growth to avoid a capacity crunch.
- The use of WDM, PSK, and PDM helped us during the last decade.
- Space-division multiplexing (SDM) may help us during this decade.



(Essiambre and Tkach, IEEE Proc., May 2012)

Space-division multiplexing



System Design Issues

- Linear coupling among fiber modes would invariably lead to channel crosstalk.
- Nonlinear coupling among fiber modes will also occur through cross-phase modulation (XPM) an four-wave mixing (FWM).
- How does the linear coupling affect nonlinear penalties in a specific channel of a SDM system?
- If polarization-division-multiplexing (PDM) is employed in addition to WDM and SDM, what is the impact of birefringence fluctuations?
- To answer such questions, it is necessary to develop a comprehensive numerical model for SDM systems.
- In this talk, I present our theoretical model and recent numerical results. Mumtaz et al., PTL **24**, 1574-1576 (2012); JLT **31**, 398-406 (2013).

Theoretical Framework

- We start with the wave equation in the frequency domain

$$\nabla^2 \widetilde{\mathbf{E}}(\omega) + n^2(x,y)\frac{\omega^2}{c^2}\widetilde{\mathbf{E}}(\omega) = -\omega^2 \mu_0 \widetilde{\mathbf{P}}^{\mathrm{NL}}(\omega).$$

- Nonlinear effects included in the time domain through

$$\mathbf{P}^{\mathrm{NL}}(t) = \frac{\varepsilon_0}{4}\chi^{(3)}\left[(\mathbf{E}\cdot\mathbf{E})\mathbf{E}^* + 2(\mathbf{E}^*\cdot\mathbf{E})\mathbf{E}\right].$$

- Expand the optical field in terms of fiber modes

$$\widetilde{\mathbf{E}}(\boldsymbol{r},\omega) = \sum_{m=1}^{M} \mathrm{F}_m(x,y)\widetilde{\mathbf{A}}_m(z,\omega)e^{i\boldsymbol{\beta}_m(\omega)z},$$

- $\widetilde{\mathbf{A}}_m = [\widetilde{\mathrm{A}}_{mx} \;\; \widetilde{\mathrm{A}}_{my}]^{\mathrm{T}}$ includes both polarization components of the mth fiber mode (needed for PDM).

Nonlinear Propagation Equations

- The time-domain equation for the pth mode satisfies

$$\frac{\partial \mathbf{A}_p}{\partial z} - i(\boldsymbol{\beta}_{0p} - \beta_r)\mathbf{A}_p + \left(\boldsymbol{\beta}_{1p} - \frac{1}{v_{gr}}\right)\frac{\partial \mathbf{A}_p}{\partial t} + \frac{i\beta_{2p}}{2}\frac{\partial^2 \mathbf{A}_p}{\partial t^2}$$

$$= i\sum_m q_{mp}\mathbf{A}_m + \frac{i\gamma}{3}\sum_{lmn} f_{lmnp}\left[(\mathbf{A}_n^{\mathrm{T}}\mathbf{A}_m)\mathbf{A}_l^* + 2(\mathbf{A}_l^{\mathrm{H}}\mathbf{A}_m)\mathbf{A}_n\right].$$

- Linear and nonlinear couplings among spatial modes governed by

$$q_{mp} \propto k_0 \iint \Delta n(x,y,z) \mathrm{F}_p^* \mathrm{F}_m \, dx\, dy, \quad f_{lmnp} = \iint \mathrm{F}_l^* \mathrm{F}_m \mathrm{F}_n \mathrm{F}_p^* \, dx\, dy.$$

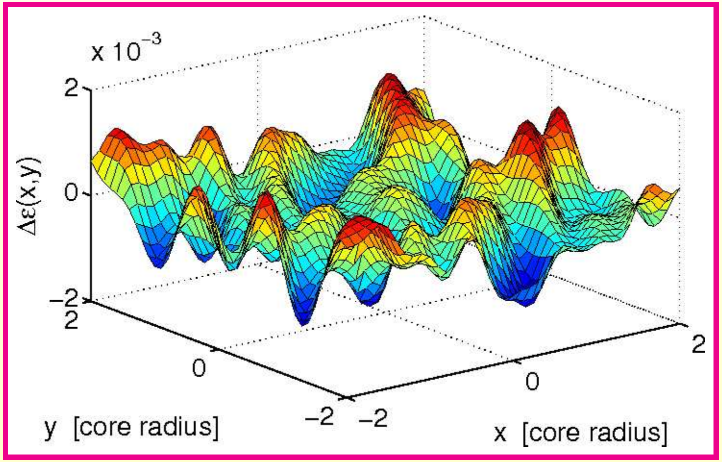
- Fiber modes are normalized such that

$$\iint \mathrm{F}_p^*(x,y)\mathrm{F}_m(x,y)\, dx\, dy = \delta_{mp}.$$

Linear Coupling in Multimode Fibers

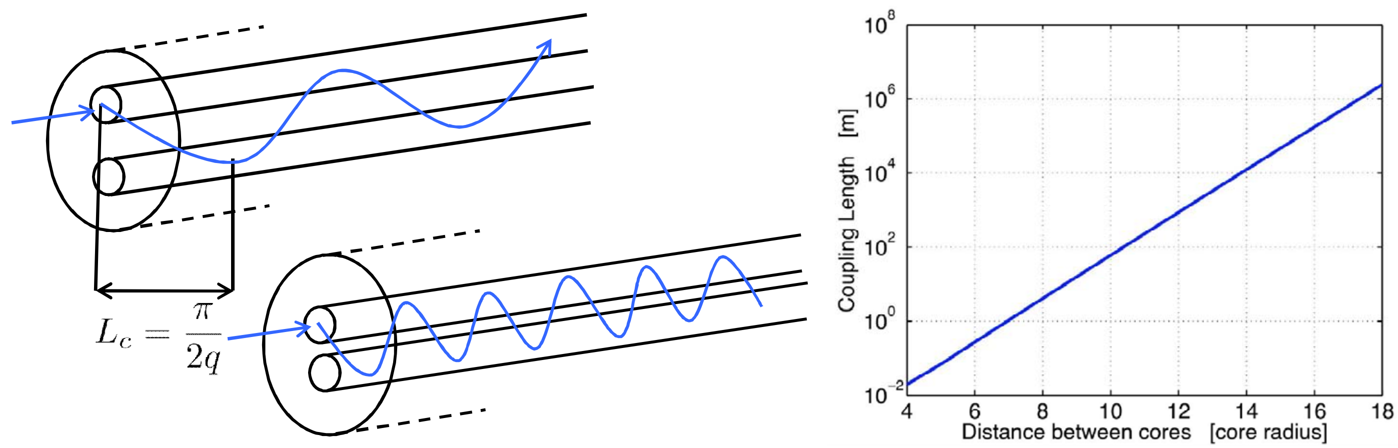
- Coupling between modes does not occur in a perfect multimode fiber.
- In practice, spatial modes experience random coupling owing to fiber imperfections.
- Coupling is governed by small fluctuations in the refractive index $[\Delta n(x,y,z) \sim 10^{-4}]$.

This case will be discussed later in this talk.



Linear Coupling in Multicore Fibers

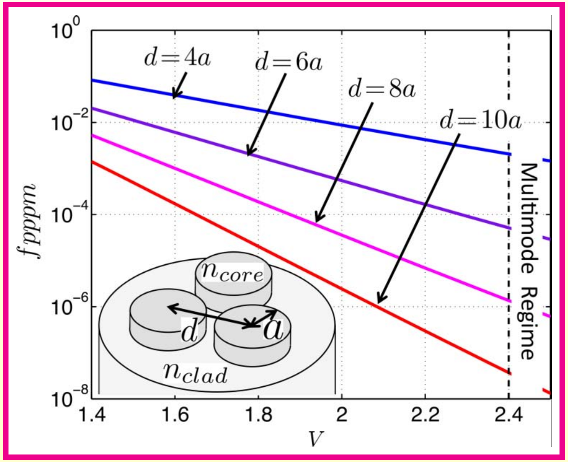
- Coupling between two cores depends on the spacing between them.
- Coupling coefficient q_{mp} is a measure of this coupling.
- Coupling length $L_c = \pi/(2q)$ can vary from <1 cm to >1000 km depending on how close the two cores are.



Nonlinear Coupling in Multicore Fibers

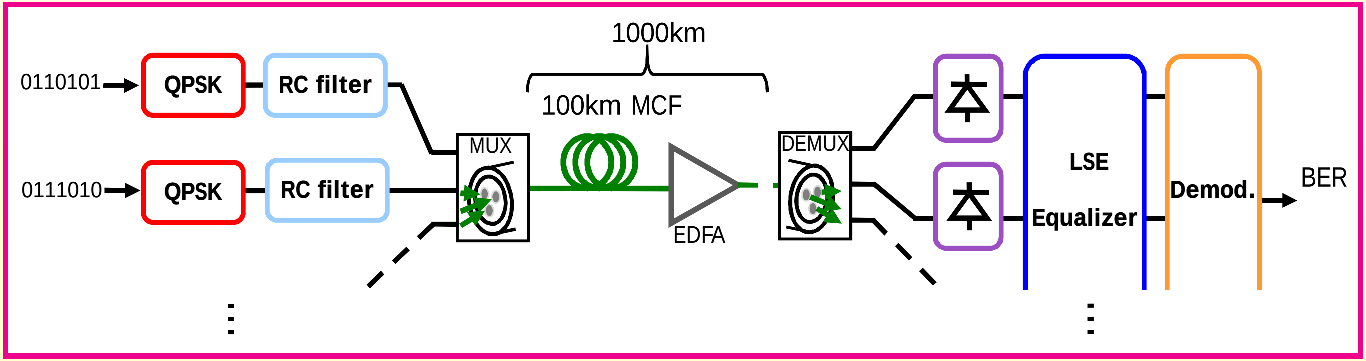
- Governed by the mode-overlap factor $f_{lmnp} \propto \iint \mathrm{F}_l^* \mathrm{F}_m \mathrm{F}_n \mathrm{F}_p^* \, dx\, dy$.
- Self-phase modulation (SPM) occurs through $f_{pppp} = 1$.
- Cross-phase modulation (XPM) governed by terms like f_{ppmm}.
- Other Combinations represent FWM-like effects.

XPM and FWM terms are negligible in multicore fibers whose cores of radius a are separated by $d > 4a$.

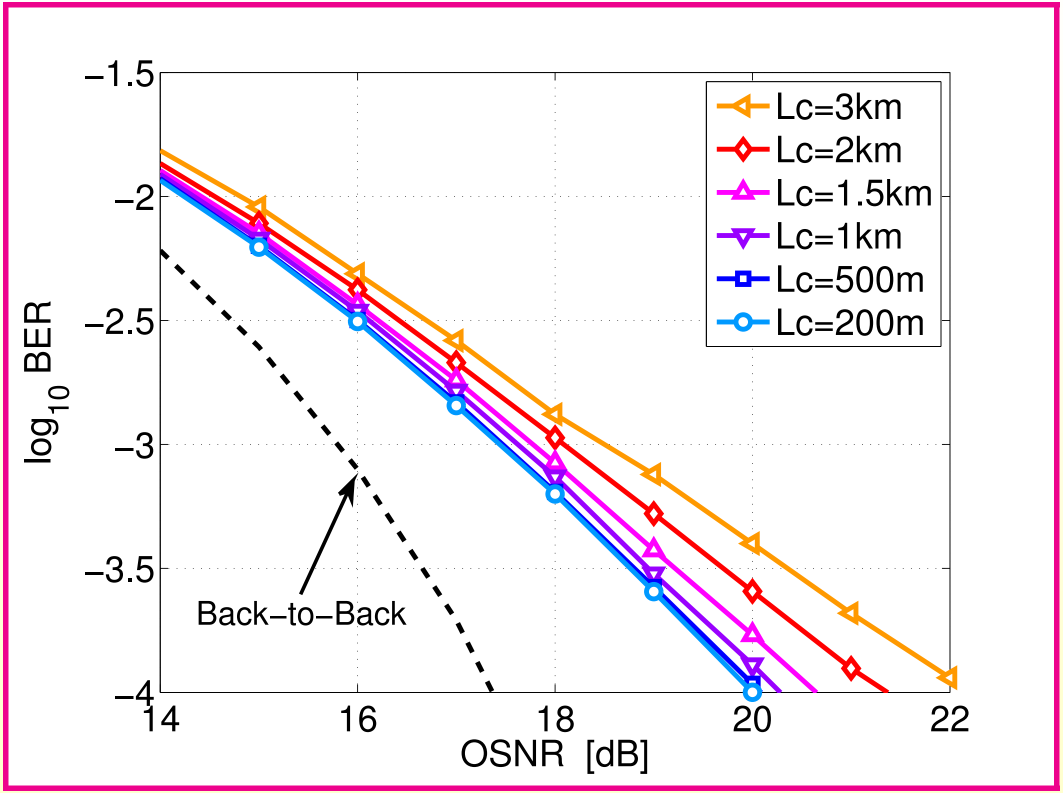


Numerical Results for Multicore Fibers

- We considered multicore fibers with up to 19 cores.
- Two 28.5-Gbaud PDM-QPSK symbol streams launched into each core.
- Propagation over 1000 km by the split-step Fourier method.
- Least square equalizer (LSE) is based on a training sequence.
- Actual bit error rate (BER) calculated using 2^{20} bits during simulations.

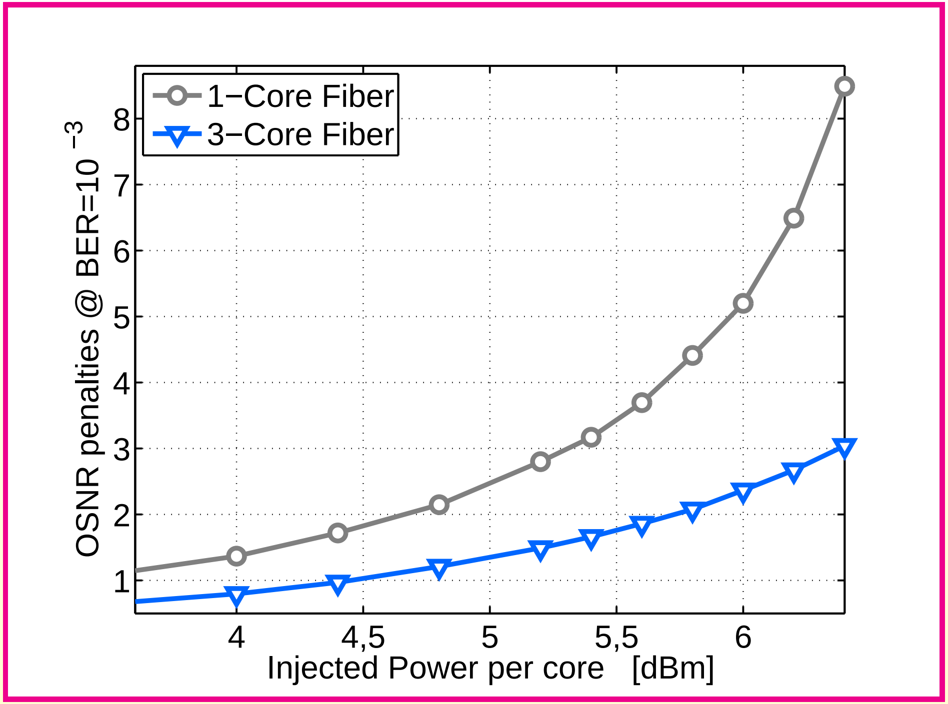


Three-core Fibers



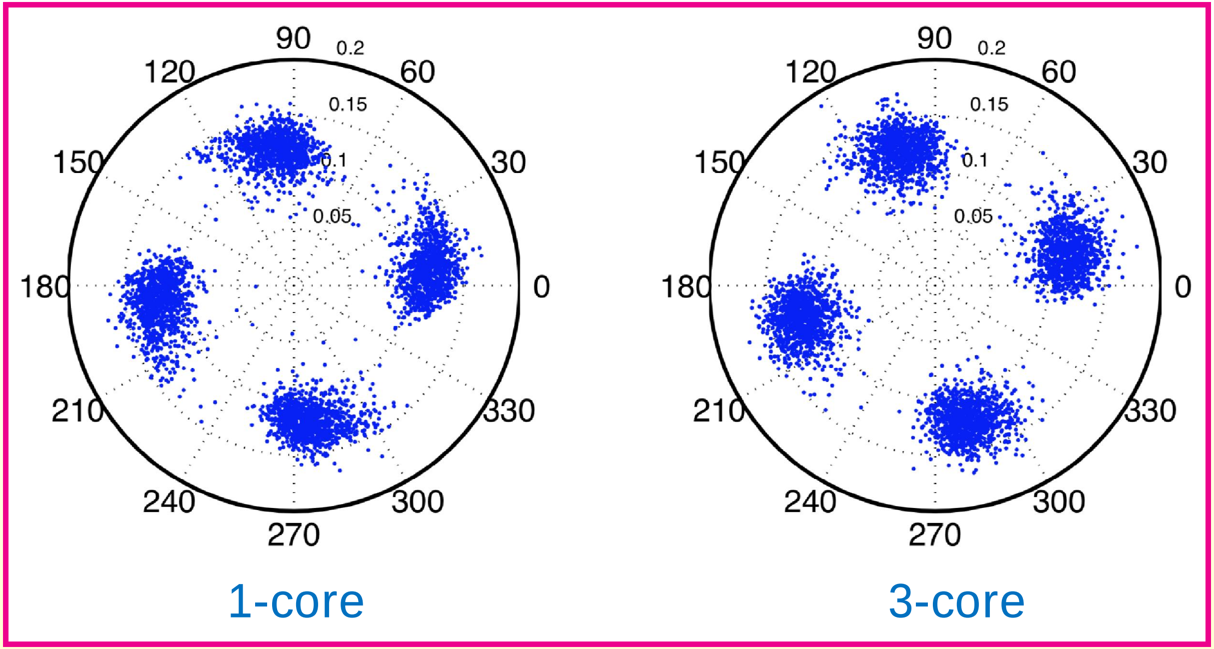
Stronger linear coupling (shorter L_c) improves the BER at a given OSNR.

OSNR penalties



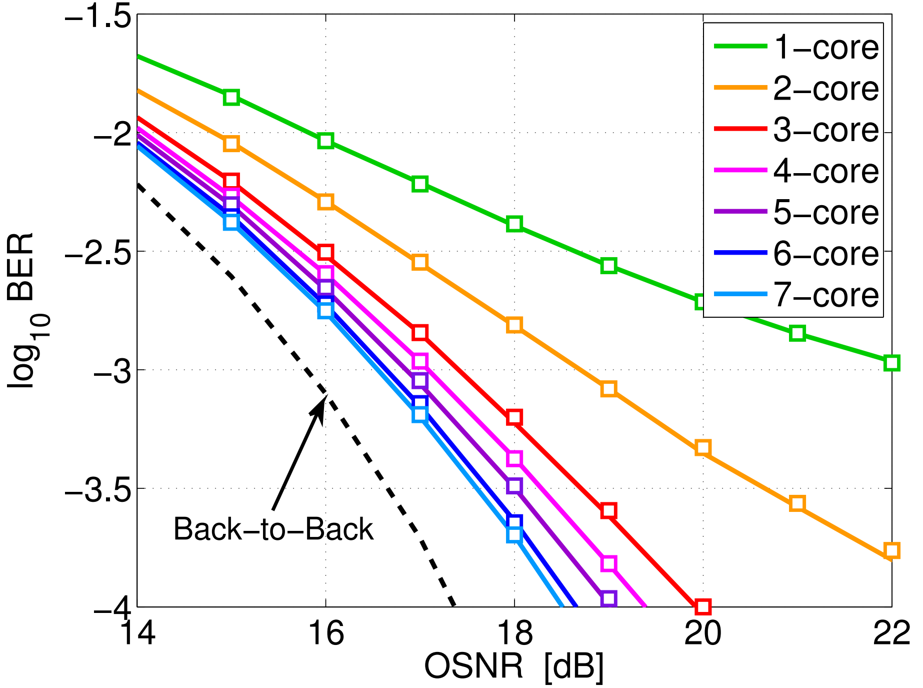
OSNR penalties calculated after 1000 km using $L_c = 100$ m.

SPM-Induced Noise



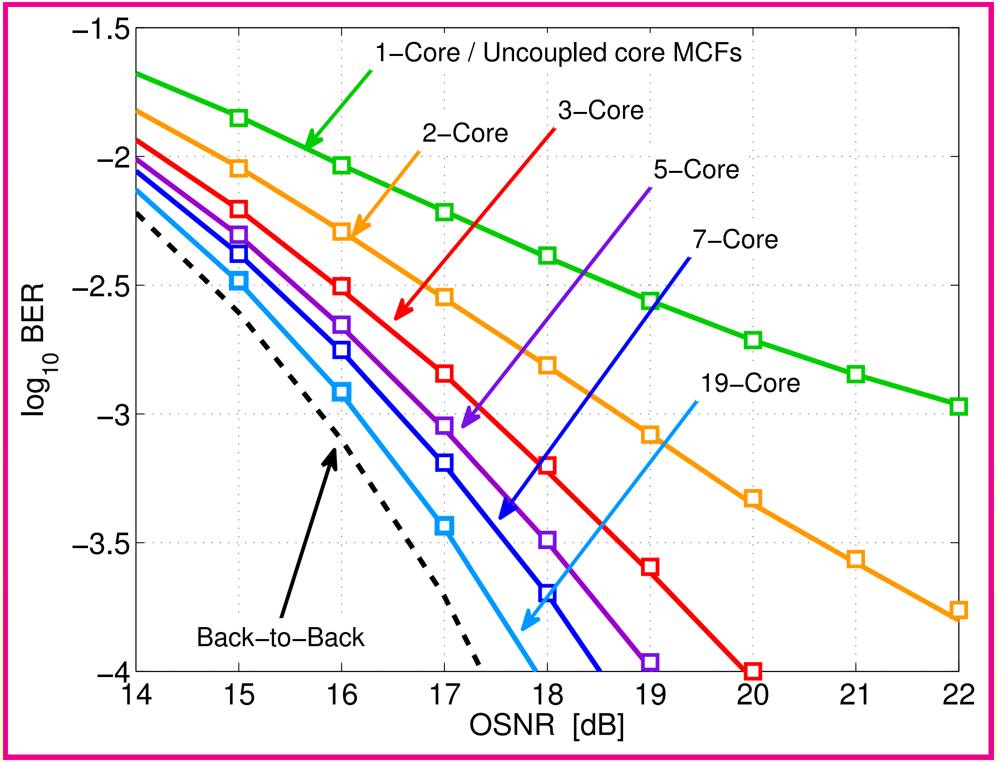
- Nonlinear noise after 100 km for an input power of 12 dBm.
- Degradations induced by SPM are reduced in a 3-core fiber.

Multicore Fibers



Simulated BER curves for fibers containing up to 7 cores.

Multicore Fibers (cont.)



Simulated BER curves for fibers containing up to 19 cores.

Birefringence Fluctuations

- Birefringence fluctuations must be included in realistic simulations.
- Their inclusion makes all mode-propagation equations stochastic.
- Numerical simulations become time-consuming as one must solve these equations many times before averaging.
- In the case of single-mode fibers, Manakov equations are found to be quite useful for predicting the average behavior.
- We have derived new Manakov equations for multimode fibers assuming strong polarization coupling but no coupling among spatial modes.
- Fiber divided into many sections with randomly oriented phase plates [Mumtaz et al., JLT **31**, 398 (2013)].

Manakov Equations for Multimode Fibers

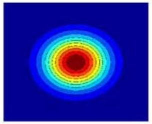
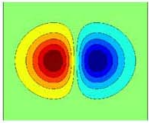
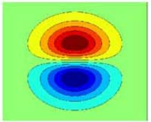
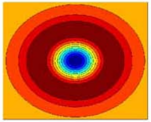
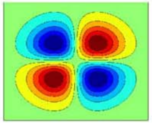
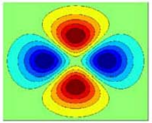
- After averaging over birefringence fluctuations, we obtain

$$\frac{\partial \bar{\mathbf{A}}_p}{\partial z} + d_p \frac{\partial \bar{\mathbf{A}}_p}{\partial t} + \frac{i\beta_{2p}}{2}\frac{\partial^2 \bar{\mathbf{A}}_p}{\partial t^2} = i\gamma\Big(f_{pppp}\frac{8}{9}|\bar{\mathbf{A}}_p|^2 + \sum_{m\neq p} f_{mmpp}\frac{4}{3}|\bar{\mathbf{A}}_m|^2\Big)\bar{\mathbf{A}}_p.$$

- Modal group-velocity mismatch included through d_p.
- The SPM term contains the usual factor of 8/9 as in single-mode fibers.
- The factor of 2 in the XPM terms is reduced to 4/3.
- This reduction indicates that XPM penalties are reduced in multimode fibers in the presence of birefringence fluctuations.
- The contribution of all FWM terms averages out to zero.
- Numerical simulation support these conclusions.

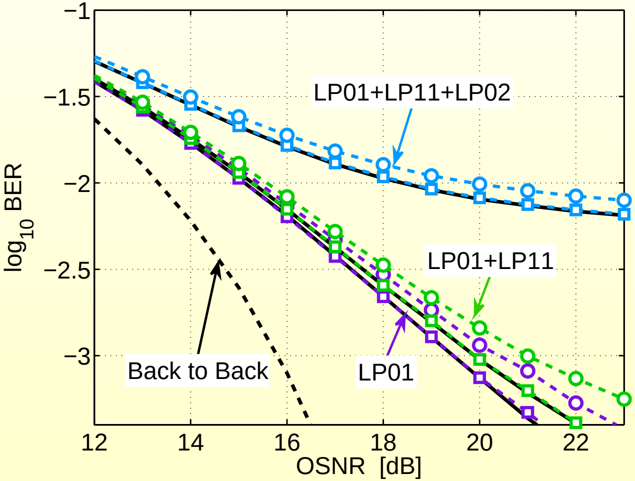
Numerical Results for Multimode Fibers

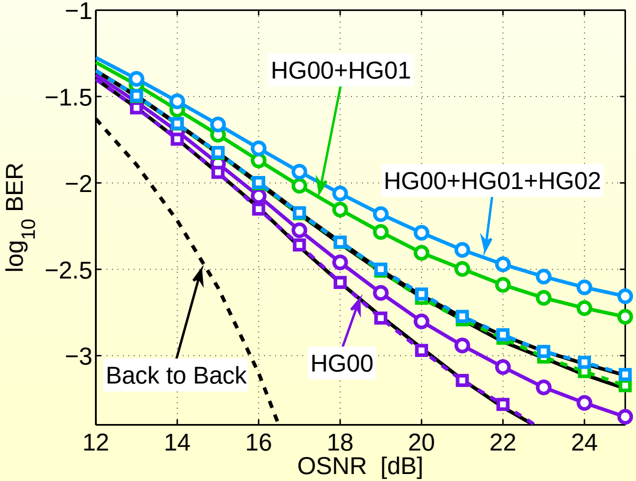
- We consider both step-index and graded-index multimode fibers.
- Core diameter $12\ \mu$m, $\Delta = 0.01$, $V = 5$ at 1550 nm.

Spatial distribution shapes	Step-index fiber	Graded-index fiber
	LP01	HG00
 	LP11a, LP11b	HG01, HG10
	LP02	HG02+HG20
 	LP21a, LP21b	HG11a, HG11b

Numerical Results for Multimode Fibers

- PDM-QPSK symbol streams launched into each mode (bit rate 114 G/s).
- Propagation over 1000 km by the split-step Fourier method.
- Solid curves, Manakov; Squares, full model; circles, no birefringence.





Manakov Equations in Strong-Coupling Regime

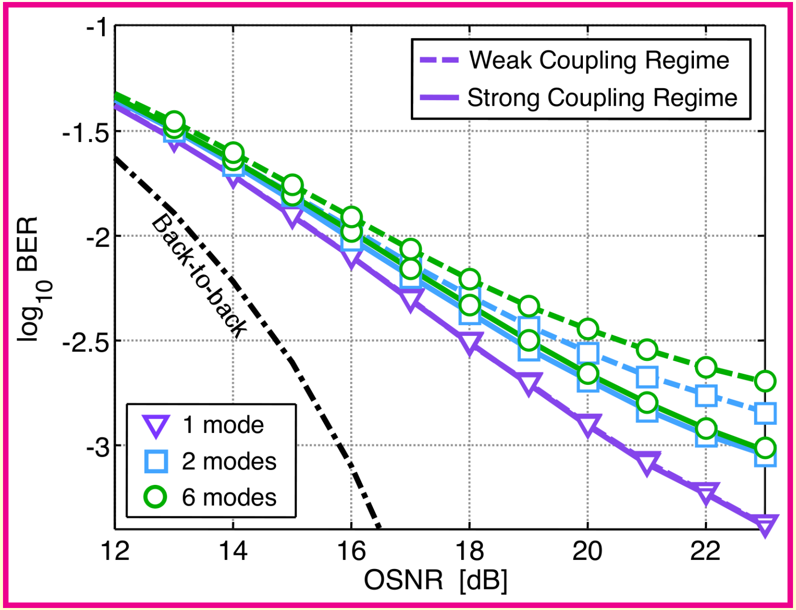
- Coupling between spatial modes can occur in practice.
- In general, coupling strength varies for various mode pairs and is strongest for nearly degenerate modes.
- Strong-coupling regime (all modes strongly coupled) is easy to deal with [(Mecozzi et al., Opt. Exp. **20**, 11673 (2012)].
- Manakov equation in this regime takes the form

$$\frac{\partial \bar{\mathscr{A}}}{\partial z} + \frac{i\bar{\beta}_2}{2}\frac{\partial^2 \bar{\mathscr{A}}}{\partial t^2} = i\gamma\kappa|\bar{\mathscr{A}}|^2\bar{\mathscr{A}}, \quad \kappa = \sum_{k=1}^{M}\sum_{l\le k}^{M}\frac{32}{2^{\delta_{kl}}}\frac{f_{kkll}}{6M(2M+1)}.$$

- Here $\bar{\mathscr{A}}$ is a column vector with 2M elements (M modes, 2 polarizations).

Numerical Results

- PDM-QPSK symbol streams launched into each mode (bit rate 114 Gb/s).
- Propagation over 1000 km by the split-step Fourier method.
- Coupling regimes compared in the case of a graded index fiber.



Mode Coupling in Multimode Fibers

- Neither of the two preceding coupling models is realistic.
- Spatial modes of an ideal fiber are not coupled. In this case, two orthogonally polarized components of each mode are coupled through birefringence fluctuations (no coupling among spatial modes).
- In practice, random fluctuations in the refractive index of a multimode fibre result in coupling of all spatial modes.
- However, not all modes are strongly coupled since coupling also depends on how close the propagation constants of the two modes are.
- A more general coupling model is needed that takes into account such mode-dependent coupling.

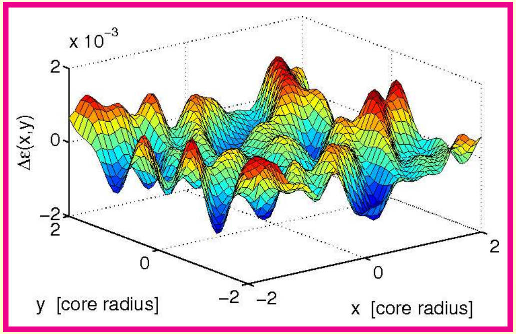
General Mode-Coupling Model

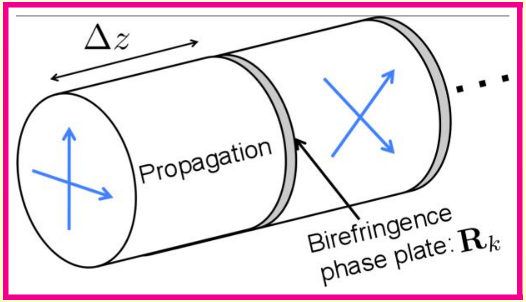
- We are developing a model that takes into account of the fact that the strength of mode coupling varies from one mode pair to another.

- To account for random refractive index fluctuations, we assume $\varepsilon(x,y,z) = n^2(x,y) + \Delta\varepsilon(x,y,z)$, where $\Delta\varepsilon(x,y,z)$ is a random variable.

- $\Delta\varepsilon(x,y,z)$ vanishes on average at every point in the transverse plane such that

$$\langle \Delta\varepsilon(x,y,z)\Delta\varepsilon(x,y,z') \rangle = \sigma_\varepsilon^2 \exp(-|z-z'|/l_d).$$





Coupling and Transfer Matrices

- Fluctuations $\Delta\varepsilon(x,y,z)$ couple all spatial modes of a multimode fiber through a random coupling matrix.
- Average value of all elements of this martix is zero but the variance depends on the standard deviation of index fluctuations.
- A transfer matrix can be used to propagate all modes simultaneously along the fiber.
- It turn out that the strength of coupling between two modes is governed by the ratio $q_{mn} = \kappa_{mn}/|\beta_m - \beta_n|$.
- Here κ_{mn} is the root-mean square value of the coupling coefficient and β_m is the propagation constant of the mth mode.
- As expected, modes with smaller values of $|\beta_m - \beta_n|$ exhibit stronger coupling.

Concluding Remarks

- The use of SDM through multimode or multicore fibers is promising for future telecom systems.
- We have studied the impact of mode coupling on nonlinear penalties in such systems.
- In a multicore fiber, nonlinear penalties are reduced when coupling is enhanced by bringing cores together.
- Random birefringence must be included in all cases.
- Two new Manakov equations derived in the weak and strong coupling regimes by averaging over random birefringence.
- A new coupling model is under development. It will allow us to study the transition from no coupling to strong coupling.