Multiphoton Interactions in Nonlinear Optical Waveguides

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Outline

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- Planar and Cylindrical Waveguides
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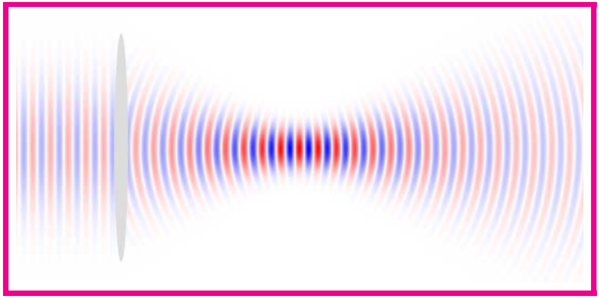
Introduction

- Nonlinear optical effects have been studied since 1962 and have found applications in many branches of optics.



- Nonlinear interaction length is limited in bulk materials because of tight focusing and diffraction of optical beams:

$$L_{\rm diff} = k w_0^2, \qquad (k = 2\pi/\lambda).$$

- Much longer interaction lengths become feasible in optical waveguides, which confine light through total internal reflection.
- Optical fibers allow interaction lengths > 1 km.

Advantage of Waveguides

- Efficiency of a nonlinear process in bulk media is governed by

$$(I_0 L_{\text{int}})_{\text{bulk}} = \left(\frac{P_0}{\pi w_0^2}\right)\frac{\pi w_0^2}{\lambda} = \frac{P_0}{\lambda}.$$

- In a waveguide, spot size w_0 remains constant across its length L.
- In this situation L_{int} is limited by the waveguide loss α.
- Using $I(z) = I_0 e^{-\alpha z}$, we obtain

$$(I_0 L_{\text{int}})_{\text{wg}} = \int_0^L I_0 e^{-\alpha z}\, dz \approx \frac{P_0}{\pi w_0^2 \alpha}.$$

- Nonlinear efficiency in a waveguide can be improved by

$$\frac{(I_0 L_{\text{int}})_{\text{wg}}}{(I_0 L_{\text{int}})_{\text{bulk}}} = \frac{\lambda}{\pi w_0^2 \alpha} \sim 10^6.$$

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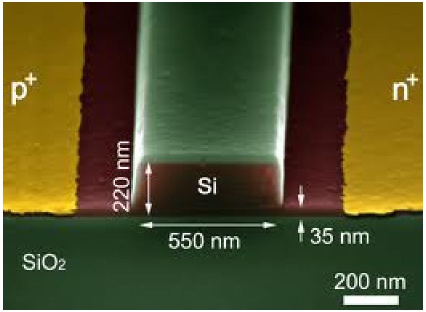
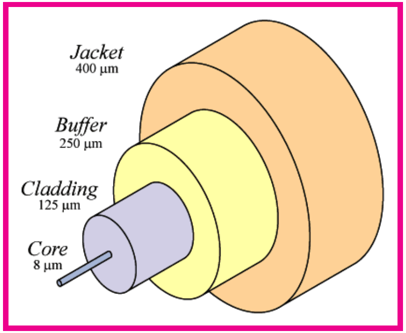
Planar and Cylindrical Waveguides





- Optical waveguides employ total internal reflection to confine light.
- The refractive index is larger inside a central region.
- Two main classes: Planar and cylindrical waveguides.
- In the planar case, waveguides use materials such as silicon, silicon nitride, and chalcogenide glass.
- In the cylindrical case, optical fibers are made of silica glass and used extensively for telecommunications.

Major Nonlinear Effects

- Self-Phase Modulation (SPM)
- Cross-Phase Modulation (XPM)
- Four-Wave Mixing (FWM)
- Stimulated Brillouin Scattering (SBS)
- Stimulated Raman Scattering (SRS)

Origin of Nonlinear Effects

- Third-order nonlinear susceptibility $\chi^{(3)}$ dominates when $\chi^{(2)} = 0$.
- Its real part leads to four-photon processes (SPM, XPM, and FWM).
- Its imaginary is responsible for two-photon absorption (TPA).

Third-order Nonlinear Susceptibility

- The tensorial nature of $\chi^{(3)}$ makes theory quite complicated.
- It can be simplified considerably when a single optical beam excites the fundamental mode of an optical waveguide.
- Only the component $\chi^{(3)}_{1111}(-\omega;\omega,-\omega,\omega)$ is relevant in this case.
- Its real and imaginary parts provide the Kerr coefficient n_2 and the TPA coefficient β_T as

$$n_2(\omega) + \frac{ic}{2\omega}\beta_T(\omega) = \frac{3}{4\varepsilon_0 c n_0^2}\chi^{(3)}_{1111}(-\omega;\omega,-\omega,\omega).$$

- A review paper on silicon waveguides provides more details:
  Q. Lin, O. Painter, G. P. Agrawal, Opt. Express **15**, 16604 (2007).

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Nonlinear Parameters

- Refractive index depends on intensity as (Kerr effect):

$$n(\omega, I) = \bar{n}(\omega) + n_2(1 + ir)I(t).$$

- Material parameter $n_2 = 3 \times 10^{-18}$ m^2/W is larger for silicon by a factor of 100 compared with silica fibers.
- Dimensionless parameter $r = \beta_T/(2k_0 n_2)$ is related to two-photon absorption (TPA).
- For silicon $\beta_T = 5 \times 10^{-12}$ m/W at wavelengths near 1550 nm.
- Dimensionless parameter $r \approx 0.1$ for silicon near 1550 nm.
- Negligible TPA occurs in silica glasses ($r \approx 0$).
- TPA is not negligible for chalcogenide glasses ($r \approx 0.2$).

Pulse Propagation in Waveguides

- It is governed by the Nonlinear Schrödinger Equation

$$\frac{\partial A}{\partial z} + \frac{i\beta_2}{2}\frac{\partial^2 A}{\partial t^2} = i\gamma|A|^2 A - \frac{\alpha}{2}A - \frac{\beta_T}{2}|A|^2 A$$

- Dispersive effects within the waveguide included through β_2.
- Nonlinear effects are included through $\gamma = 2\pi n_2/(\lambda a_{\text{eff}})$.
- If we ignore the dispersive effects, solution is $A = \sqrt{P}e^{i\phi}$ with

$$P(z,t) = \frac{P_0(t)e^{-\alpha z}}{1 + b_T P_0(t)}, \qquad \phi(z,t) = \ln[1 + b_T P_0(t)]\frac{\gamma_0}{b_T} z_{\text{eff}}.$$

- Here $z_{\text{eff}} = (1 - e^{-\alpha z})/\alpha$ is a reduced length because of single-photon losses.
- Two-photon effects are governed by $b_T = \beta_T z_{\text{eff}}/A_{\text{eff}}$.

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Self-Phase Modulation

- Optical pulse modifies its own phase as it travels inside a waveguide.
- Self-phase modulation (SPM) depends on the shape of input pulses.
- Since $\phi(z,t)$ varies with time, it leads to chirping of input pulses (frequency varying in time).
- Chirping manifests as spectral broadening in the frequency domain.
- Spectral broadening depends on the shape and the peak power of input pulses.
- The extent of broadening is reduced by two-photon absorption in waveguides made of materials with large β_T values.
- Although not immediately obvious, SPM is a four-photon nonlinear process since spectral broadening requires creation of new photons of different frequencies.

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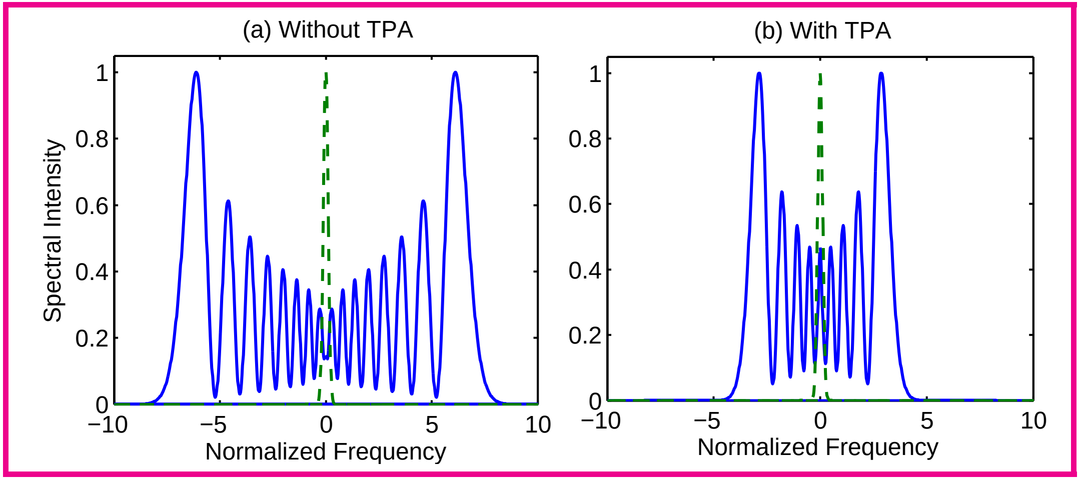
SPM-Induced Spectral Broadening



- Gaussian input pulses: $P_0(t) = P_0 \exp[-(t/T_0)^2]$ with $\gamma P_0 L = 50$.
- Dashed green curves show the spectrum of input pulses.
- Left: A silica fiber with negligible TPA ($\beta_T = 0$).
- Right: A silicon waveguide with appreciable TPA ($b_T P_0 = 2$).

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SPM as a Four-Photon Process

- Convert the NLS equation to frequency domain by Fourier transforming it.
- The Fourier transform $B(z,\omega)$ of $A(z,t)$ satisfies

$$\frac{\partial B}{\partial z} + \frac{1}{2}(\alpha - i\beta_2\omega^2)B = i(\gamma + i\beta_T/2) \times \iint_{-\infty}^{\infty} B^*(\omega - \omega_1 - \omega_2)B(\omega_2)B(\omega_1)\,d\omega_1 d\omega_2.$$

- Dispersive effects are included in this equation through β_2.
- This equation shows how the interaction of three photons of different frequencies creates a fourth photon (intrapulse FWM).
- Two 'pump photons' use their energy create two photons of different frequencies such that both energy and momentum are conserved.

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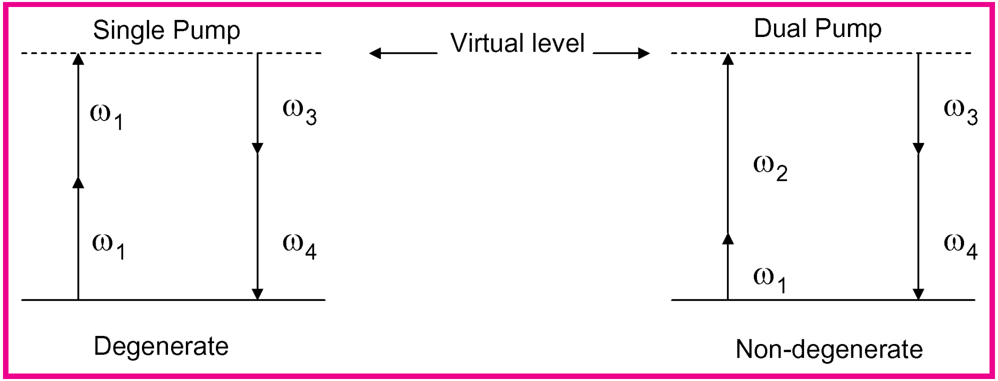
Four-Wave Mixing (FWM)

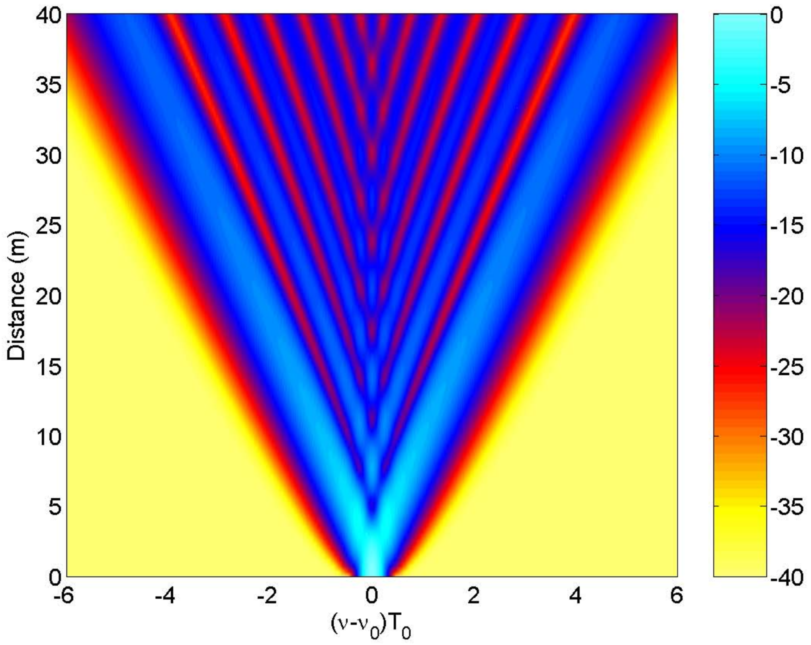


- FWM is a nonlinear process involving four photons.
- FWM requires conservation of energy and momentum:

$$\omega_1 + \omega_2 = \omega_3 + \omega_4, \qquad \beta_1 + \beta_2 = \beta_3 + \beta_4.$$

- Degenerate FWM: Single pump ($\omega_1 = \omega_2$).

SPM as Intrapulse FWM



- SPM-induced spectral broadening along the length of a silica fiber.
- Gaussian input pulse: $P_0(t) = P_0 \exp[-(t/T_0)^2]$ with $(\gamma P_0)^{-1} = 1$ m.

Optical Solitons

- Combination of SPM and anomalous dispersion produces solitons.
- Dispersive and nonlinear effects balanced when $N^2 = L_D/L_{\rm NL} = 1$.
- Nonlinear length $L_{\rm NL} = 1/(\gamma P_0)$;   Dispersion length $L_D = T_0^2/|\beta_2|$.
- Fundamental solitons ($N = 1$) preserve their "sech" shape as they propagate if losses (both linear and nonlinear) are negligible.
- Higher-order solitons ($N > 1$) evolve in a periodic fashion.
- Any perturbation of such solitons though higher-order dispersive and nonlinear effects breaks them into N fundamental solitons (called soliton fission).
- Soliton fission leads to extreme spectral broadening through a combination of several multi-photon processes.

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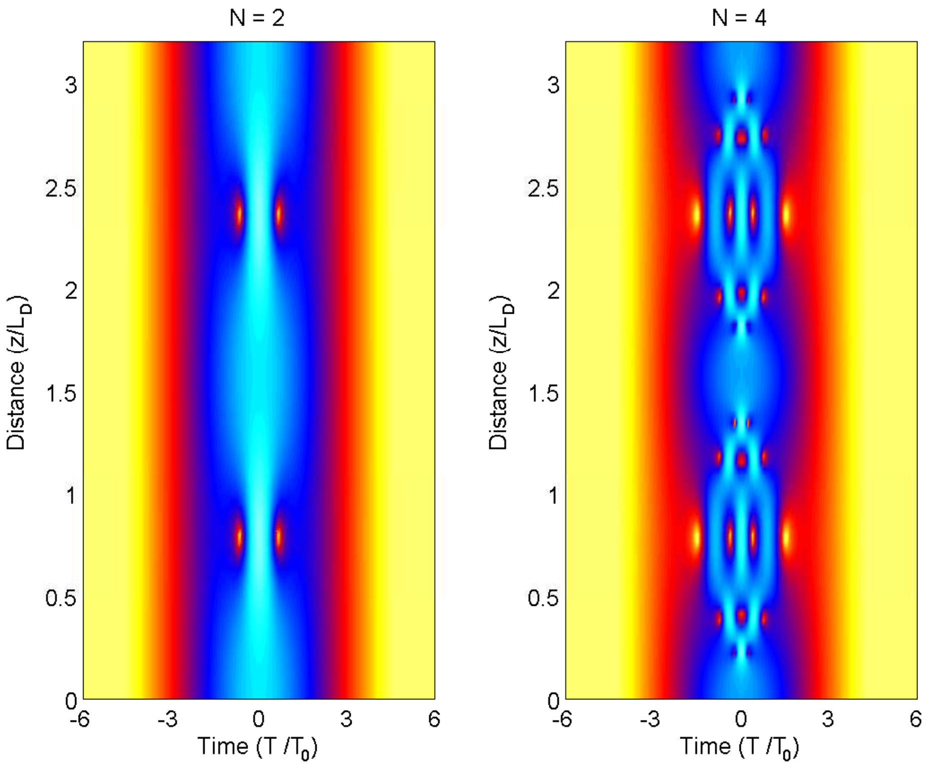
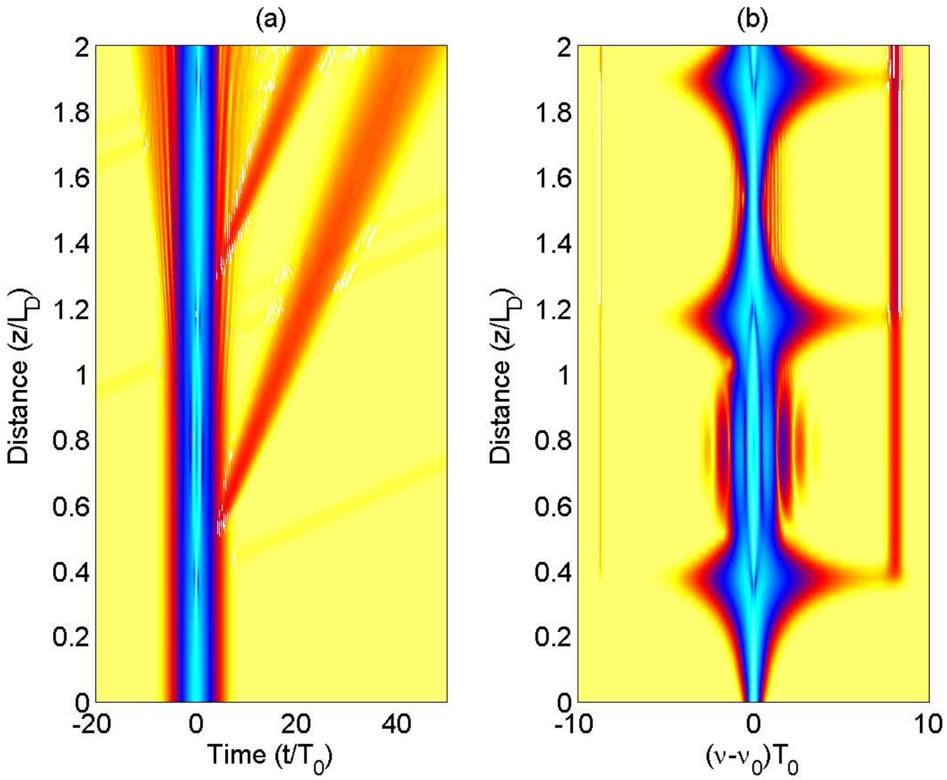
Higher-Order Solitons



Fission of a Third-Order Soliton



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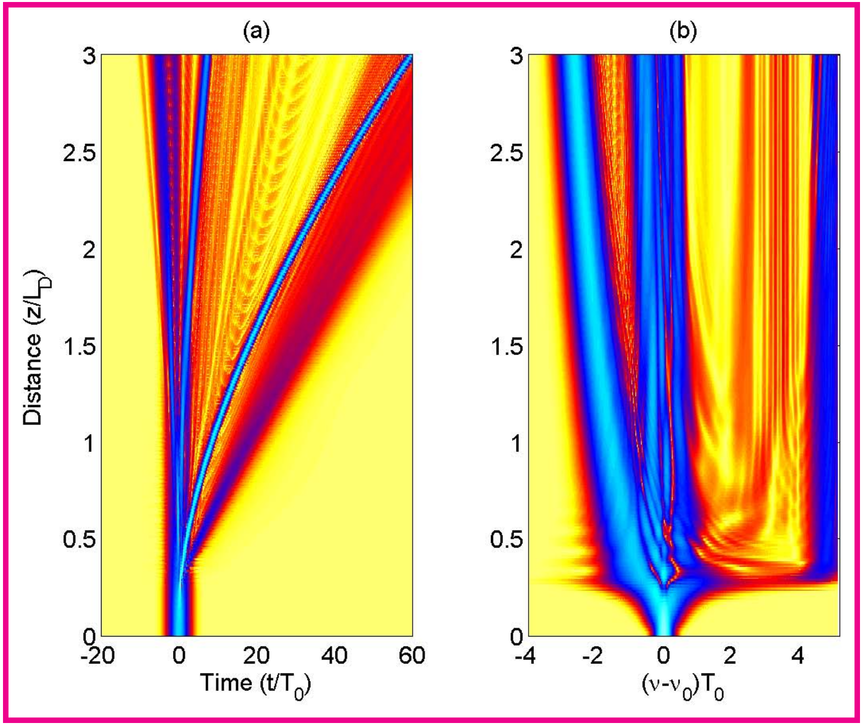
Supercontinuum Formation

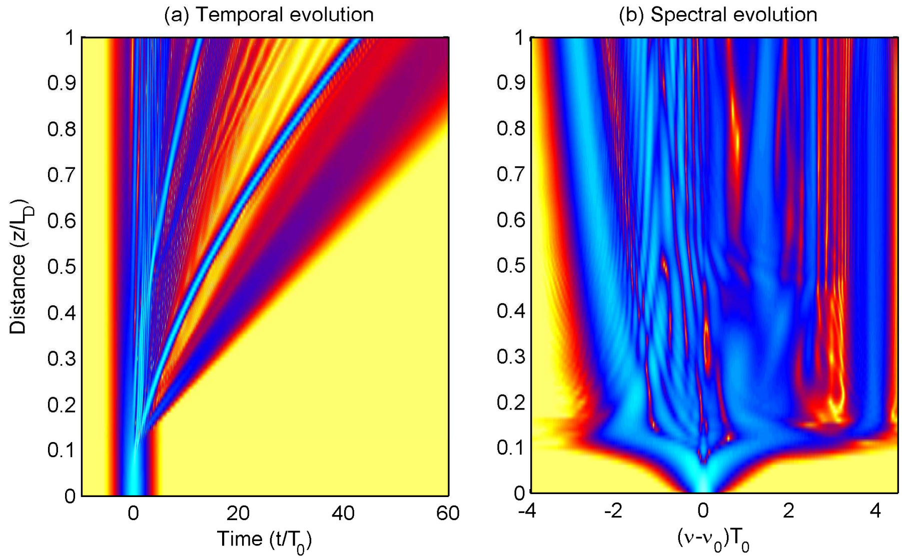


Fission of a fourth-order soliton in the presence of third-order dispersion and intrapulse Raman scattering.

Multiphoton Processes Involved

- Initial spectral broadening through intrapulse FWM (SPM)
- Soliton fission induced by third-order dispersion creates multiple fundamental solitons of different widths and peak powers.
- It also forces solitons to emit radiation at blue-shifted frequencies.
- Spectrum of each soliton shifts by different ammounts toward the red side through intrapulse Raman scattering.
- Each soliton also slows down as its spectrum shifts toward the red.
- FWM and cross-phase modulation among different spectral bands create additional spectral contents.
- The net result is the formation of a supercontinuum for $N > 6$.

Supercontinuum Generation



- Fission of a $N = 8$ soliton inside a silica fiber.
- Multiple solitons and dispersive waves produce new frequencies.
- Supercontinuum formed after one dispersion length.

Concluding Remarks

- Optical waveguides enhance nonlinear effects by confining light to narrow cores and maintaining high intensities over long distances.
- Many multiphoton processes can occur under such conditions at relatively modest power levels.
- The use of short pulses further enhances peak intensities and allows soliton formation in the anomalous dispersion region.
- Self-phase modulation can be viewed as intrapulse FWM.
- Intrapulse Raman scattering transfers energy from blue components of a pulse to the red ones (larger red shifts for shorter solitons).
- New kinds of fibers have been developed for enhancing nonlinear effects (photonic crystal and other microstructured fibers).
- New applications in fields such as biomedical imaging.