COOPERATIVE MODULATION INSTABILITY IN DISPERSIVE NONLINEAR MEDIA

Govind P. Agrawal

The Institute of Optics
University of Rochester
Rochester, New York 14627

Many physical systems exhibit an instability that leads to a self-induced modulation of the steady state as a result of an interplay between the nonlinear and dispersive effects. This phenomenon is referred to as modulation instability and has been studied in such diverse fields as fluid dynamics, nonlinear optics, and plasma physics. Considerable attention has recently focussed on modulation instability occurring in optical fibers.[1-7] When an intense optical beam propagates through the fiber with its wavelength lying in the anomalous group-velocity-dispersion (GVD) regime, modulation instability leads to its breakup into a train of ultrashort pulses. This self-pulsing behavior was observed in recent experiments by using quasi-cw radiation.[2] It was necessary to perform the experiments in the infrared region beyond 1.3 μm in order to operate in the anomalous-GVD regime of the silica fiber.

This paper describes a new kind of modulation instability that can occur even in the normal-dispersion region and can be observed by using visible light in optical fibers.[3] It requires that two or more fields copropagate inside the fiber so that they can influence each other through a phenomenon known as cross-phase modulation (XPM). The two fields may correspond to the orthogonal polarization components of a single beam or to two linearly polarized beams at different wavelengths. The XPM-induced modulation instability is referred to here as the cooperative modulation instability since the two optical fields cooperate in such a way that both develop self-pulsing even when each of the fields may be inherently stable in isolation. The phenomenon is quite general and has been investigated in several different contexts.[8-14]

The propagation of two optical fields in a dispersive nonlinear optical fiber is governed by the following coupled nonlinear Schrödinger

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equations:

$$\frac{\partial A_1}{\partial z} + \frac{1}{v_{g1}} \frac{\partial A_1}{\partial t} + \frac{i}{2}\beta_1 \frac{\partial^2 A_1}{\partial t^2} = i\gamma_1(|A_1|^2 + 2|A_2|^2)A_1 , \tag{1}$$

$$\frac{\partial A_2}{\partial z} + \frac{1}{v_{g2}} \frac{\partial A_2}{\partial t} + \frac{i}{2}\beta_2 \frac{\partial^2 A_2}{\partial t^2} = i\gamma_2(|A_2|^2 + 2|A_1|^2)A_1 , \tag{2}$$

where v_{gj} (j=1 or 2) is the group velocity, β_j is the GVD coefficient, and γ_j is the nonlinear coefficient related to the nonlinear parameter n_2 of a Kerr medium. The two terms on the right-hand side of these equations are responsible for self-phase modulation (SPM) and XPM. It is the XPM-induced coupling between the two waves that leads to cooperative modulation instability in the normal-GVD regime.

The steady-state solution of Eqs. (1) and (2) is given by

$$\bar{A}_j(z) = \sqrt{P_j}\, \exp[i\gamma_j\,(P_j + 2\,P_{3-j})\,z] \quad (j=1,2), \tag{3}$$

where $P_j = |A_j(0)|^2$ is the incident power. A linear-stability analysis shows that the steady-state solution becomes unstable for perturbations occuring at certain frequencies Ω_1 and Ω_2. The weak perturbations then grow exponentially with the gain coefficient given by[7]

$$g(f_1, f_2) = \sqrt{2}\,\{[(h_1 - h_2)^2 + 4C^2]^{1/2} - (h_1 + h_2)\}^{1/2} \tag{4}$$

where

$$h_j = 4\gamma_j^2 P_j^2 f_j^{\,2}[f_j^{\,2} + \mathrm{sgn}(\beta_j)] , \tag{5}$$

$$C = 2\,\Omega_1\Omega_2(\beta_1\beta_2\gamma_1\gamma_2 P_1 P_2)^{1/2} , \tag{6}$$

$$f_j = \Omega_j/\Omega_{cj} , \quad \Omega_{cj} = (4\,\gamma_j P_j/|\beta_j|\,)^{1/2} , \tag{7}$$

with j=1 and 2. Here f_1 and f_2 are the normalized modulation frequencies. Modulation instability occurs only when f_1 and f_2 satisfy the condition

$$[f_1^{\,2} + \mathrm{sgn}(\beta_1)][f_2^{\,2} + \mathrm{sgn}(\beta_2)] < 4 . \tag{8}$$

It is evident from Eqs. (4)-(8) that the sign of the GVD coefficient for the two waves plays an important role. Figure 1 shows the domains of modulation instability in the f_1-f_2 plane for four possible choices of the signs of β_1 and β_2 depending upon whether none, one, or both waves experience anomalous GVD. The instability domain is smallest for the case in which both waves experience normal GVD ($\beta_1>0$, $\beta_2>0$). This case is of fundamental importance since each wave is stable in isolation and requires the presence of other copropagating waves to become unstable.