Temporal Waveguiding of Optical Pulses

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Introduction

- Optical waveguides confine light spatially to a central region through total internal reflection (TIR).
- Space-time duality noted in the 1960s suggests that a temporal analog of spatial waveguides may exist.
  P. Tournois, C. R. Acad. Sci. **258**, 3839–3842 (1964);
  S. A. Akhmanov et al., Sov. Phys. JETP **28**, 748–757 (1969).
- Such waveguides make use of temporal TIR occurring at the boundary of two dispersive media with different refractive indices.
- Concept of space-time duality has found many applications in recent years.
- Temporal imaging, first discussed in 1989, is one obvious example.
  B. H. Kolner and M. Nazarathy, Opt. Lett. **14**, 630–632 (1989);
  R. Salem et al., Adv. Opt. Photon. **5**, 274–317 (2013).

What is Space–Time Duality?

- It results from a mathematical equivalence between paraxial-beam diffraction and dispersive pulse broadening.
- Diffraction in one transverse dimension is governed by

$$\frac{\partial A}{\partial z} - \frac{i}{2k}\frac{\partial^2 A}{\partial x^2} = 0.$$

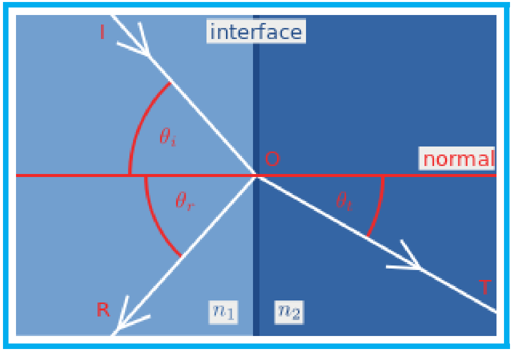
- Temporal evolution in a dispersive medium is governed by

$$\frac{\partial A}{\partial z} + \frac{i\beta_2}{2}\frac{\partial^2 A}{\partial t^2} = 0.$$

- Slit-diffraction problem is identical to a pulse propagation problem.
- The only difference is that β_2 can be positive or negative.
- Many results from diffraction theory can be used for pulses.

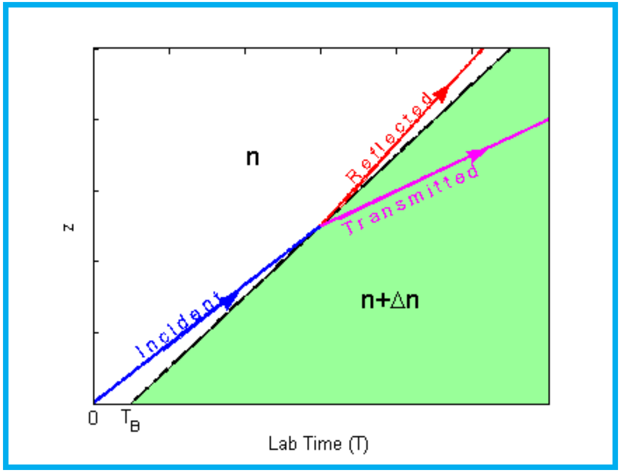
Temporal Reflection and Refraction

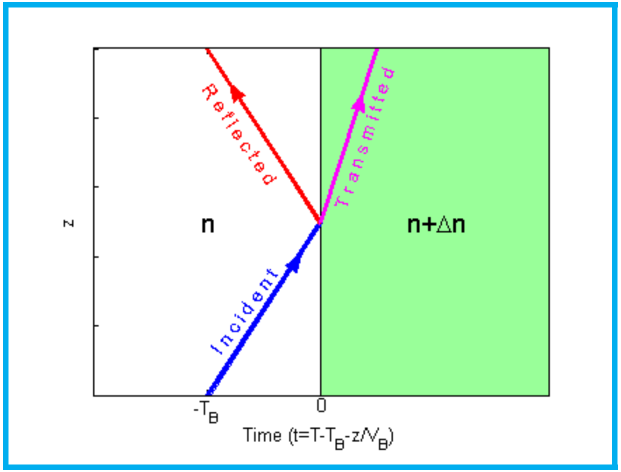
- Reflection and refraction of optical beams at a spatial boundary are well-known phenomena.
- What is the temporal analog of these two optical phenomena?



- What happens when an optical pulse arrives at a temporal boundary across which refractive index changes suddenly?
- At a spatial boundary, frequency is preserved but momentum can change.
- At a temporal boundary, momentum is preserved but frequency can change.
- A change in angle at a spatial interface translates into a change in the frequency of incident light.

Reflection at a Moving Boundary





- Both the pulse and temporal boundary travel forward at different speeds.
- It is convenient to work in a moving frame at which the boundary is stationary ($t = T - z/V_B$).
- Momentum (or the wave number) is then conserved in the moving frame.

Simple Model of Pulse Propagation

- Let us assume that an optical pulse is propagating inside a waveguide with the dispersion relation $\beta(\omega)$.
- Temporal discontinuity at $t = T_B$ is incorporated by using

$$\beta(\omega) = \beta_0 + \Delta\beta_1(\omega - \omega_0) + \frac{\beta_2}{2}(\omega - \omega_0)^2 + \beta_B H(t - T_B).$$

- $\Delta\beta_1 = \beta_1 - 1/V_B$ is pulse's relative speed relative to the temporal boundary located at $t = T_B$; $H(x)$ is the Heaviside function.
- If refractive index changes by Δn for $t > T_B$, we can use $\beta_B = k_0 \Delta n$, where $k_0 = 2\pi/\lambda$.
- The dispersion relation can be used to investigate changes in pulse's shape and spectrum occurring when the pulse arrives at the boundary.

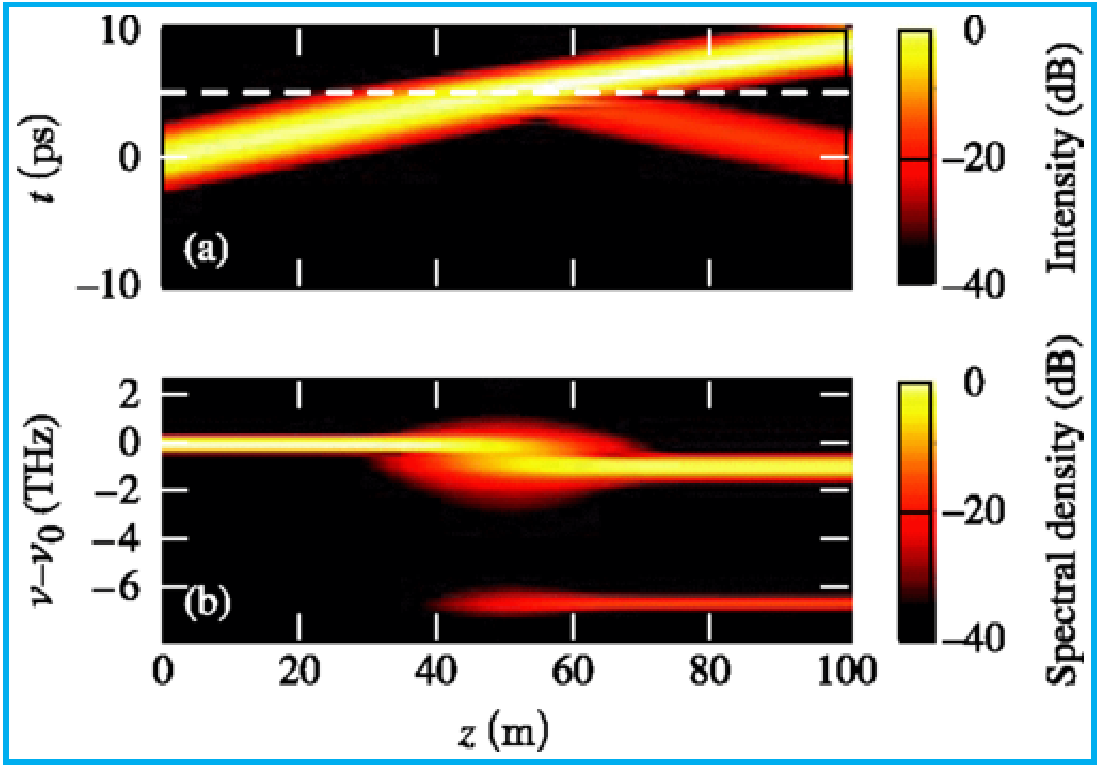
Numerical Simulations

- Slowly varying envelope of the pulse satisfies

$$\frac{\partial A}{\partial z} + \Delta\beta_1 \frac{\partial A}{\partial t} + \frac{i\beta_2}{2}\frac{\partial^2 A}{\partial t^2} = i\beta_B H(t - T_B)A.$$

- Numerical results are for Gaussian pulses using $A(0,t) = A_0 \exp(-t^2/2T_0^2)$ with $T_0 = 1$ ps. Temporal boundary is located at $T_B = 5$ ps.
- Other parameters: $\Delta\beta_1 = 0.1$ ps/m, $\beta_2 = 5$ ps^2/km, and $\beta_B = 0.5$ m^{-1}.
- Using $\beta_B = k_0 \Delta n$, this value corresponds to an index change of only $\Delta n = 2 \times 10^{-7}$ at a wavelength near 1 μm.
- Temporal evolution of the pulse shows a clear evidence of both the reflection and refraction at the boundary.
- Spectrum shows the spectral shifts associated with reflection and refraction.

Temporal Reflection and Refraction



Plansinis et al., PRL **115**, 183901 (2015).

Total Internal Reflection

- Total internal reflection (TIR) occurs in the spatial case when an optical beam enters from the high-index side to the low-index side ($\Delta n < 0$).
- The condition for TIR is obtained from Snell's law: $\theta > \theta_c$, where $\sin\theta_c = (n+\Delta n)/n$.
- In the case of temporal TIR, the condition is related to frequency shifts.
- One way to find the condition for TIR is to see when the spectral shift of the transmitted pulse becomes unphysical:

$$\omega_t = \omega_0 + \frac{\Delta\beta_1}{\beta_2}\left(\sqrt{1 - \frac{2\beta_B\beta_2}{(\Delta\beta_1)^2}} - 1\right).$$

- This condition is clearly $2\beta_B\beta_2 > (\Delta\beta_1)^2$. Temporal TIR occurs only if β_B and β_2 have the same signs.

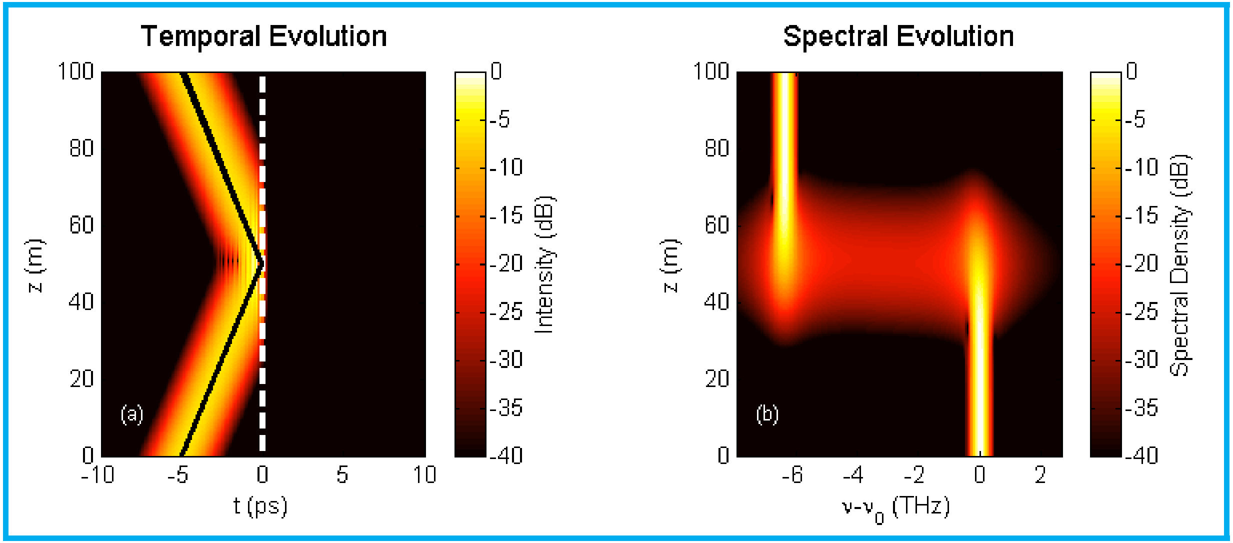
Temporal TIR

- Temporal TIR is not restricted to the situation $\Delta n < 0$.
- Using $\beta_B = k_0 \Delta n$, we can write the condition for TIR as

$$\beta_2 \Delta n > (\Delta\beta_1)^2 / 2k_0.$$

- When $\Delta n > 0$, the pulse needs to propagate in the normal-dispersion region.
- In contrast, the pulse must propagate in the anomalous-dispersion region when $\Delta n < 0$.
- This freedom is a consequence of the fact that dispersion term can be positive or negative whereas the diffraction term has only one sign.
- The requirement $\Delta n > (\Delta\beta_1)^2/(2k_0\beta_2)$ can be satisfied in practice even for a relatively small change in the refractive index across the boundary ($\Delta n < 10^{-5}$).

TIR of Gaussian Pulses

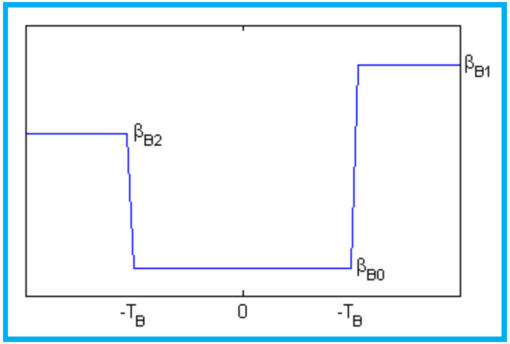


Plansinis et al., PRL **115**, 183901 (2015).

- Index change was large enough ($\Delta n \sim 10^{-6}$) to satisfy the TIR condition.
- Entire pulse energy gets reflected at the temporal boundary
- Spectrum shifted toward the red side during TIR if $\Delta n > 0$.

Temporal Waveguides

- TIR can be used to make temporal waveguides that trap optical pulses.
- Two temporal boundaries are needed.
- Refractive index of the central region can be lower or higher.



- This technique has the potential of creating pulses that remain confined to a time window.
- A pulse is trapped inside the waveguide if it undergoes TIR at both temporal boundaries.
- Temporal waveguide support modes just as spatial waveguides do.

Temporal Waveguide Modes

- In view of the space–time duality, a temporal waveguide should have modes whose shape does not change during propagation.
- This is indeed the case with the major difference that a spectral shift occurs in the temporal case.
- Assuming $A(z,t) = M(t)e^{i(Kz-\Omega t)}$, the spectral shift is found to be $\Omega = -\Delta\beta_1/\beta_2$ and the mode shape is governed by

$$\frac{d^2M}{dt^2} + \frac{2}{\beta_2}\left[K + \frac{1}{2}\beta_2\Omega^2 - \beta_{\mathrm{B}}\right]M = 0.$$

- This equation can be used to find an eigenvalue equation in the form

$$2\Omega_0 T_{\mathrm{B}} = m\pi + \tan^{-1}\left(\frac{\Omega_1}{\Omega_0}\right) + \tan^{-1}\left(\frac{\Omega_2}{\Omega_0}\right).$$

Symmetric Temporal Waveguides

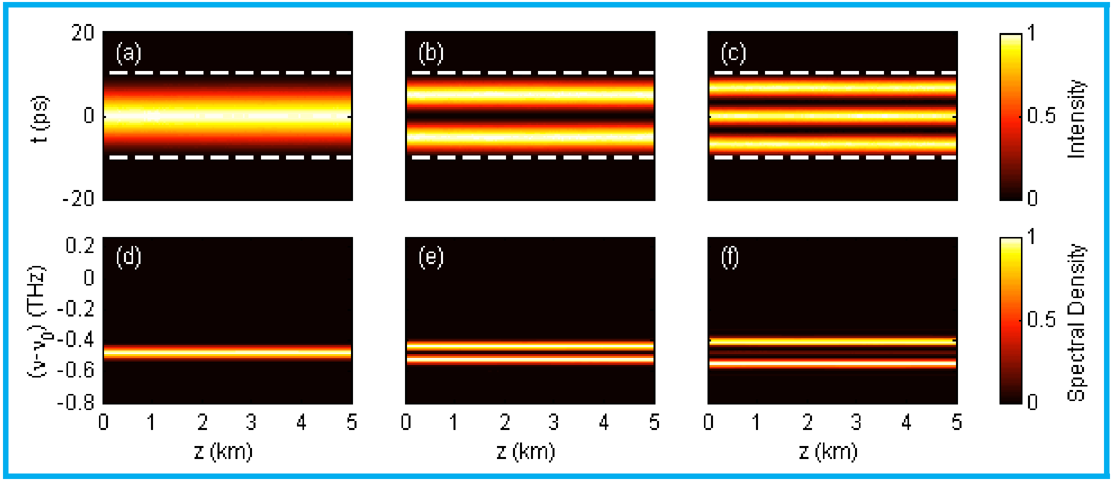
- A symmetric waveguide is designed to have the same refractive index change at the two temporal boundaries.
- In this case, the eigenvalue equation takes a much simpler form

$$\Omega_1 = \Omega_0 \tan(\Omega_0 T_\mathrm{B} + m\pi/2),$$

$$\Omega_0^2 = 2(K - \beta_{B0})/\beta_2 + \Omega^2,$$
$$\Omega_1^2 = 2(\beta_{B1} - K)/\beta_2 - \Omega^2.$$

- Introducing a dimensionless parameter as $V = T_\mathrm{B}\sqrt{\Omega_0^2 + \Omega_1^2}$, the waveguide supports m modes when $V < (m+1)\pi/2$.
- A temporal waveguide supports only the fundamental $m = 0$ mode if it is designed such that $V < \pi/2$.

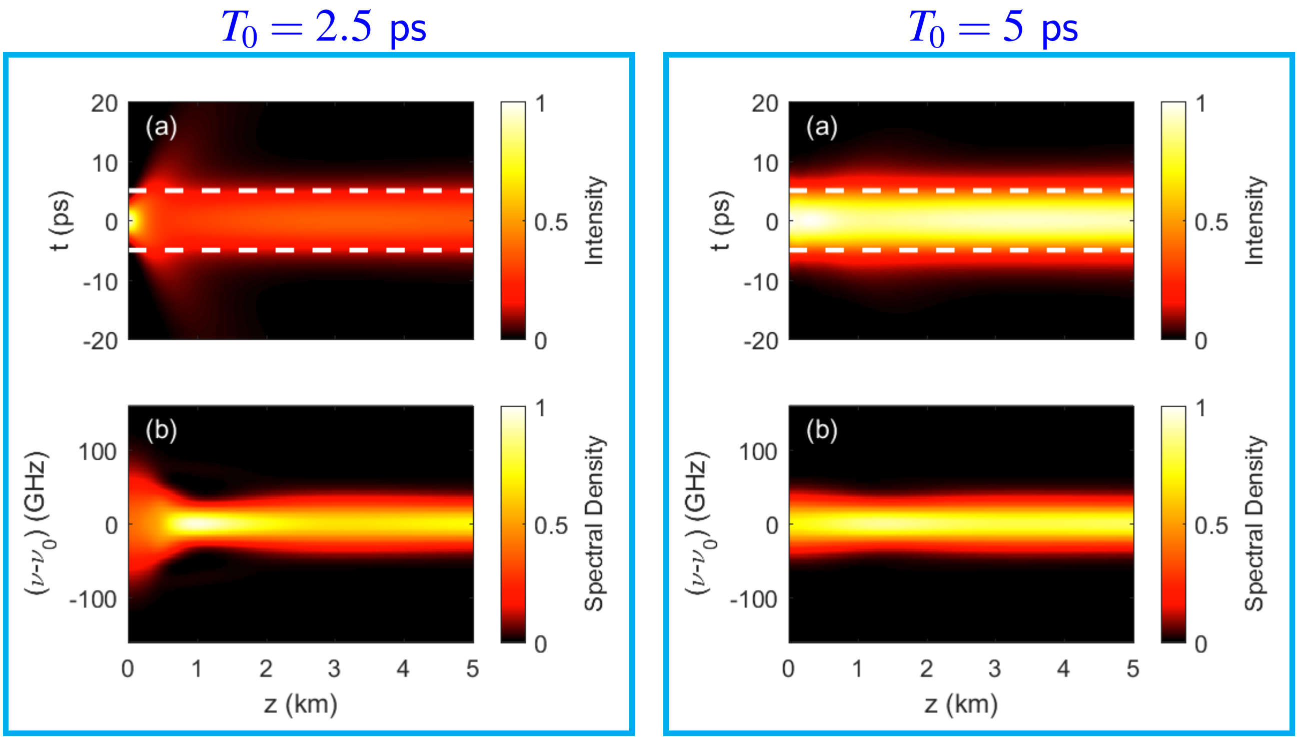
Temporal Waveguide Modes



Plansinis et al., JOSA B **33**, 1112 (2016).

- Modes of a temporal waveguide can be found analytically.
- The number of modes depends on $V = 2\sqrt{k_0 \Delta n T_B^2/\beta_2}$.
- Figure shows the first 3 modes for a 20-ps wide waveguide with $V = 55$.

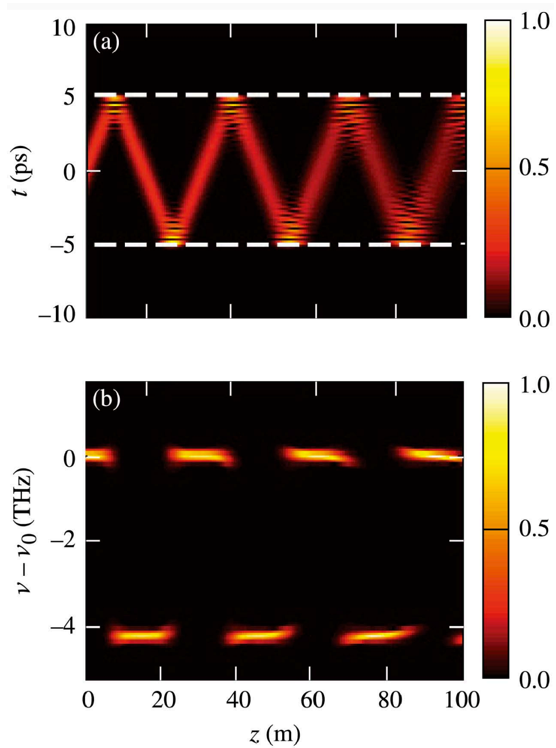
Single-Mode Waveguides



Plansinis et al., JOSA B **33**, 1112 (2016).

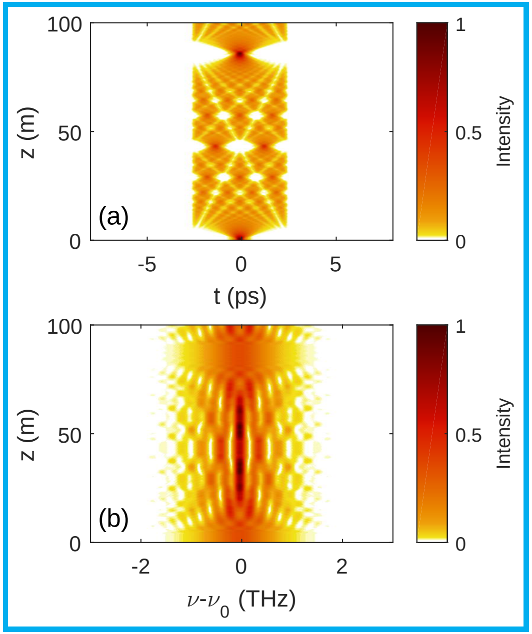
Multimode Waveguides

- A 1-ps Gaussian pulse trapped inside a 10-ps wide waveguide: $\beta_B = 5.6\ \text{m}^{-1}$, $\Delta\beta_1 = 66.7$ ps/km, $\beta_2 = 50$ ps^2/km
- Pulse undergoes TIR and its spectrum shifts after each reflection.
- Pulse broadening eventually creates distortions and pulse excites multiple waveguide modes ($V = 26.8$).
- This approach can work when the propagation length is a fraction of the dispersion length ($z < L_D$).



Temporal Talbot effect

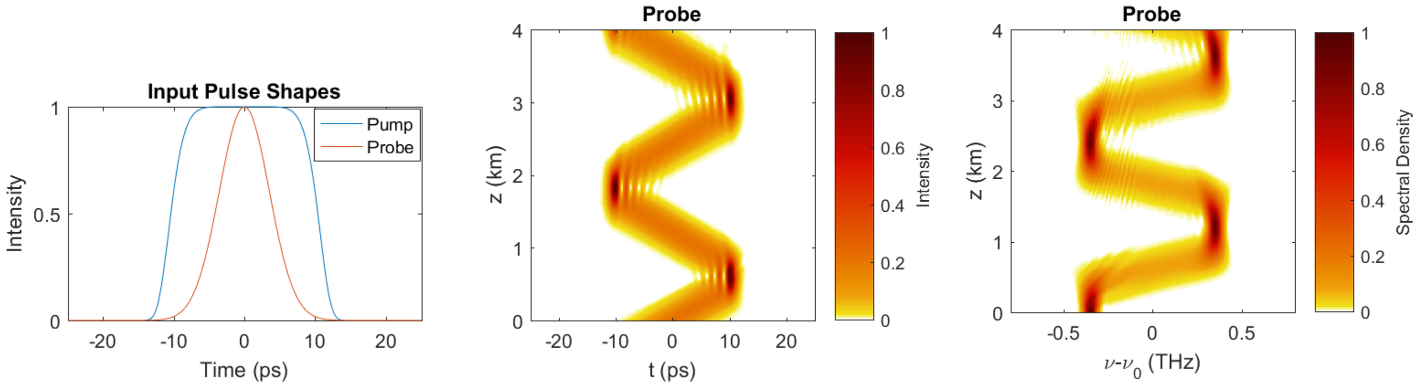
- Spatial waveguides exhibit self-imaging and power splitting owing to multi-mode interference.
- This is related to the Talbot effect, studied first in 1836 with diffraction gratings.
- Temporal analog of this phenomenon is the temporal Talbot effect.
- Figure shows self-imaging and pulse splitting inside a temporal waveguide.



Experimental Considerations

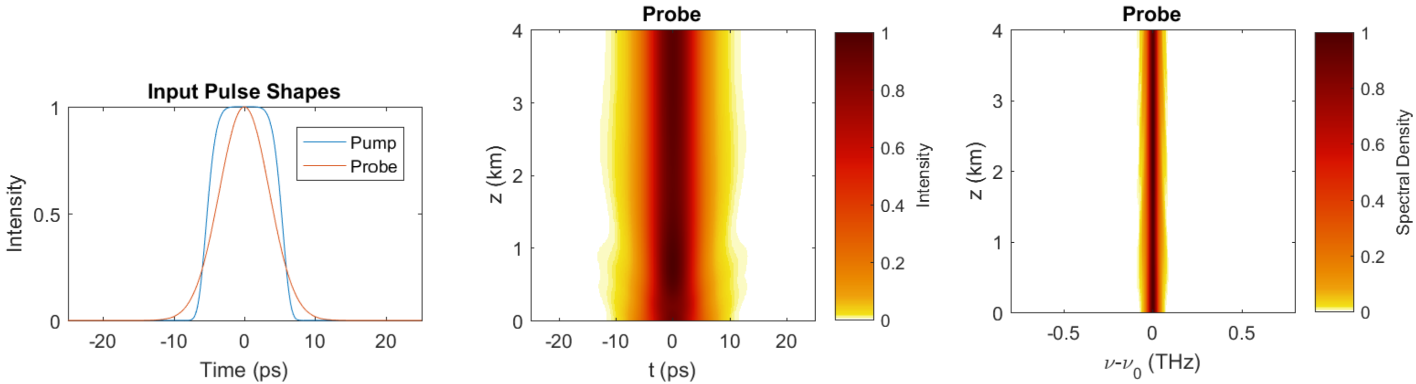
- Experimental realization of temporal waveguides requires two moving boundaries. Two techniques can be used for this purpose.
- A microwave signal can be used to create refractive index changes inside an electro-optic modulator.
- Cross-phase modulation can be employed inside a nonlinear waveguide through an intense pump pulse.
- The spreading of pump pulse can be controlled if pump wavelength lies near the zero-dispersion wavelength of an optical fiber.
- The width of the pump pulse controls the temporal window inside which refractive index is enhanced through the Kerr nonlinearity.
- Numerical simulations show that shorter probe pulses can be confined within a temporal window realized using super-Gaussian pump pulses.

XPM-induced Multimode Waveguide



- Super-Gaussian 20-ps pump pulse with 10 W peak power ($\beta_2 = 0$).
- Gaussian 8-ps probe pulse with 3 nm wavelength shift ($\beta_2 = -10$ ps^2/km).
- (left) Shapes of input pump and probe pulses compared.
- (middle) Temporal evolution of probe pulse over 4 km of optical fiber.
- (right) Spectral evolution of the same probe pulse over 4 km.

XPM-induced Single-Mode Waveguides



- Super-Gaussian 10-ps pump pulse with 1 W peak power ($\beta_2 = 0$).
- Gaussian 8-ps probe pulse moving at the same speed ($\beta_2 = -30$ ps^2/km).
- (left) Shapes of input pump and probe pulses compared.
- (middle) Temporal evolution of probe pulse over 4 km of optical fiber.
- (right) Spectral evolution of the same probe pulse over 4 km.

Conclusions

- Space–time duality is a simple concept with many applications.
- It can be used for temporal imaging and for making time microscopes.
- Temporal equivalent of total internal reflection occurs for optical pulses at a moving temporal boundary.
- In the temporal case, frequency shifts play the role of angles.
- We have identified conditions under which an optical pulse can be confined within a temporal waveguide.
- Modes of such waveguides were found analytically and a single-mode condition was identified.
- A pump–probe configuration can be used to make such waveguides through the nonlinear phenomenon of cross-phase modulation.