

# Generalized distributed feedback design: amplification, filtering and switching

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## Abstract

A novel analytic method for analyzing hybrid DFB structures containing multiple linear and nonlinear sections is presented. The technique is illustrated by considering uniform and phase-shifted nonlinear devices that play the roles of an amplifier, a filter and an all-optical switch simultaneously.

**Keywords:** periodic structures, optical switching, nonlinear optics, amplifiers.

## 1. Introduction

Distributed feedback (DFB) devices represent one of the most important components in modern photonics design, with applications in the diverse areas of filtering, switching and amplification. In the early 1970s, DFB structures were extensively used as both linear bandpass filters and reflectors in the emerging technology of integrated optics and photonics. Following the pioneering work of Kogelnik,<sup>1,2</sup> a number of semi-analytical and numerical methods for analyzing nonuniform DFB devices were developed.<sup>3,4</sup> Nonuniform and segmented DFB design, in turn, allowed the synthesis of an arbitrary response of bandpass or reflective filters.

It was recognized early that DFB structures could be successfully utilized in the design of semiconductor laser sources.<sup>2,5</sup> In fact, the  $\lambda/4$ -shifted configuration, probably one of the best known DFB structures, led to the realization of a stable, single-mode, integrated laser source.<sup>5,6</sup> Recently, a similar type of high-Q structure has been used to design extremely narrowband tunable filters.<sup>7,8</sup>

The extraordinary transmissive properties of DFB structures fabricated in a nonlinear medium have been analyzed extensively by now.<sup>9-11</sup> Nonlinear DFB devices exhibit true bistable switching in the CW operating regime,<sup>9</sup> and a

complex dynamical behavior in the pulsed regime.<sup>11</sup> In addition to the expected switching behavior, detailed transient studies have revealed the phenomena of slow energy transport through a new kind of solitons, often referred to as the Bragg solitons.<sup>10,11</sup> Recently, we have shown that a properly designed high-resonance nonlinear DFB device exhibits switching at unusually low intensity levels.<sup>12</sup> Keeping in mind the recent progress in the fabrication of highly nonlinear optical polymers,<sup>13,14</sup> it is not difficult to predict an increasing importance of nonlinear DFB devices, especially for use in constructing ultrafast, all-optical, low-intensity switches.

While several relatively simple methods exist for analyzing the linear DFB structures, the nonlinear DFB structures often require complicated numerical simulations. The design of more complex, nonuniform and hybrid (linear/nonlinear) DFB structures, represents even a greater analytical challenge. In this paper we show that if an almost-periodic structure (DFB device with slowly varying nonuniformities) is divided into a set of strictly periodic (linear or nonlinear) segments, it is possible to obtain an analytic solution for the entire device. The effect of loss or gain can be incorporated into the linear segments, but not in nonlinear segments, since no known analytic solution exists for a lossy nonlinear DFB structure.

## 2. Generalized transfer matrix method

Consider the nonuniform DFB structure shown in Fig. 1. We assume uniformity in the transverse ( $x$ - $y$ ) direction, essentially reducing the treatment of the device to a one-dimensional transmission problem. The DFB structure can incorporate a slowly varying taper, a chirping of the grating period, and multiple phase-shifts along the  $z$ -direction. In addition, we allow the entire device or parts of it to have a nonuniform nonlinearity described by an effective nonlinear index  $n_2(z)$ . The refractive index of such device can be written as

$$n(z) = n_0 + \Delta n(z) \cos(2\pi z/\Lambda + \Omega) + n_2(z) |E(z)|^2, \quad (1)$$

where  $n_0$  represents the linear index of the device,  $\Delta n(z)$  is the amplitude of periodic perturbation,  $|E(z)|^2$  represents the intradevice optical intensity, and  $\Lambda$  is period of linear perturbation.

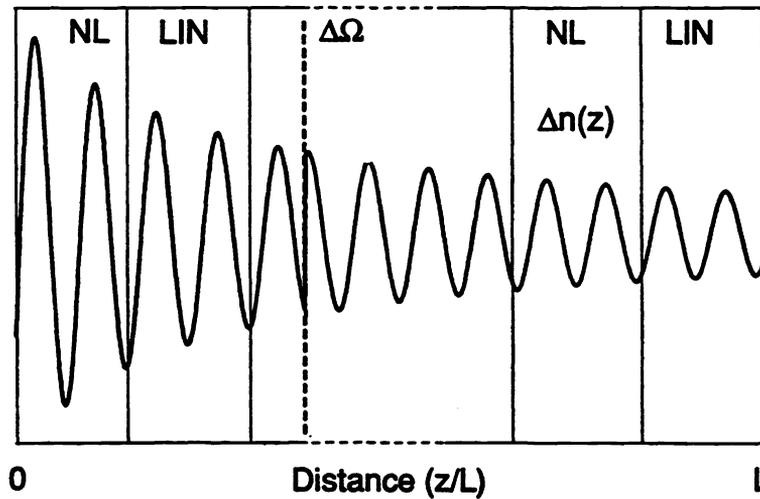


Figure 1. Nonuniform hybrid DFB structure consisting of multiple linear (LIN) and nonlinear (NL) sections. Dashed vertical lines indicate the location of phase shifts. Solid curve shows axial variation of nearly periodic  $\Delta n(z)$ .

By adopting the standard coupled-mode approximations made for describing the propagation of radiation in a DFB structure, one can obtain the generalized set of CW coupled-mode equations:

$$\frac{dE_+}{dz} = i\kappa(z)E_- \exp[-2i\Delta\beta(z)z - \Omega(z)] + i\gamma(z)[|E_+|^2 + 2|E_-|^2]E_+ + g(z)E_+, \quad (2a)$$

$$\frac{dE_-}{dz} = -i\kappa(z)E_+ \exp[2i\Delta\beta(z)z - \Omega(z)] - i\gamma(z)[2|E_-|^2 + |E_+|^2]E_- - g(z)E_-. \quad (2b)$$

In these equations, the optical field is separated into its forward and backward propagating components as  $E(z) = E_+(z)e^{i\beta z} + E_-(z)e^{-i\beta z}$ . The parameter  $\kappa(z) = \pi\Delta n(z)/\lambda_0$  defines the linear coupling strength. The nonlinear parameter  $\gamma$ , responsible for self- and cross-phase modulations, is defined as  $\gamma(z) = \pi n_2(z)/\lambda_0$ . The parameter  $\Delta\beta$  defines the detuning from the Bragg frequency  $\beta_0 = 2\pi/\lambda_0$  and is given by  $\Delta\beta = \beta - \beta_0$ . Finally,  $g(z)$  accounts for gain (or loss) within the structure.

A general solution of the coupled-mode equations (2) has to be found numerically, since no known analytic solution exists for arbitrary parameters  $\kappa(z)$ ,  $\Delta\beta(z)$ ,  $\gamma(z)$  and  $g(z)$ . Only in special cases (uniform and linear or uniform and lossless nonlinear structures) is it possible to obtain simple and elegant solutions that can be used to construct an approximate solution for a general, nonuniform DFB structure. In the case of a linear, uniform DFB structure

( $\gamma(z) = 0$  and  $\kappa$ ,  $g$  and  $\Delta\beta$  constants along the  $z$  axis ) the solutions of the Eqs. (2) can be written in the form<sup>4</sup>:

$$\begin{bmatrix} E_+(0) \\ E_-(0) \end{bmatrix} = \begin{bmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{bmatrix} \begin{bmatrix} E_+(L) \\ E_-(L) \end{bmatrix}, \quad (3)$$

where  $L$  is the device length and the transfer matrix elements are given by:

$$F_{11} = \left[ \cosh(\mu L) - i \frac{(\Delta\beta - ig)}{\mu} \sinh(\mu L) \right] \exp(i2\Delta\beta L), \quad (4a)$$

$$F_{12} = F_{21}^* = -\frac{\kappa L}{\mu L} \sinh(\mu L) \exp(-i2\Delta\beta L - \Omega), \quad (4b)$$

$$F_{11} = \left[ \cosh(\mu L) + i \frac{(\Delta\beta - ig)}{\mu} \sinh(\mu L) \right] \exp(-i2\Delta\beta L). \quad (4c)$$

The parameter  $\mu$  is defined by  $\mu^2 = \kappa^2 - (\Delta\beta - ig)^2$ . In the case of an almost-periodic, nonuniform device, we divide the structure into a set of strictly periodic segments (Fig. 1). The total transmission of the device is obtained by simply multiplying the submatrices that correspond to each uniform segment.

When a nonlinearity is present, we must distinguish between the cases of a lossless and a lossy DFB device.<sup>9</sup> It has been shown earlier that an exact analytic solution in the form of Jacobian elliptic functions exists only for the case of a lossless uniform device. To obtain such a solution, it is necessary to separate the amplitude and phase of the counterpropagating field components by using  $E_{\pm} = A_{\pm}(z)\exp(\pm i\phi_{\pm}(z))$ . By substituting this form into Eqs. (2), it is possible to construct a forward-flux equation<sup>12</sup>:

$$\left( \frac{L}{2} \frac{dl}{dz} \right)^2 = (\kappa L)^2 l(1-T) - [(\kappa L)G - (\Delta\beta L + 2(1-T)l)]^2 \equiv P(l), \quad (5)$$

where

$$G = \kappa L \sqrt{l(L)(L-T)} \cos\Psi + [\Delta\beta L + 2(l(L)-T)]l(L),$$

$$\Psi(z) = 2\Delta\beta z + \phi_+(z) - \phi_-(z) - \Omega.$$

The normalized forward flux is given by  $I = A_+^2 / A_c^2$ , the transmitted flux is defined by  $T = I - A_-^2 / A_c^2$ , while the critical intensity, used in normalization, is given by  $A_c^2 = 4\lambda_0 / (3\pi n_2 L)$ . The solutions of Eq. (5) have different forms throughout the detuning region, dictated by the boundary conditions at the output of the device.<sup>12</sup> In the vicinity of the Bragg frequency ( $\Delta\beta = 0$ ), one can write the solution as:

$$I(z) = I_3 + (I_2 - I_3) \left( 1 - \frac{I_1 - I_2}{I_1 - I_3} \text{sn}^2(z) \right)^{-1}, \quad (6)$$

where  $\text{sn}(z) = \text{sn}\{\text{sn}^{-1}[u(I(L)); k] - 4z/gL; k\}$ ,  $g = [(I_1 - I_3)(I_2 - I_4)]^{-1}$  and  $k^2 = (I_1 - I_2)(I_3 - I_4)g^2$ . The parameters  $I_i$  are the zeros of the quartic polynomial  $P(I)$  appearing in Eq. (5) and can be interpreted as the turning points of a particle moving in the potential well<sup>12</sup>  $P(I)$ . In a way analogous to the linear F-matrix method, it is now possible to divide the nonuniform, lossless nonlinear device into a set of uniform, nonlinear DFB sections and find the corresponding analytic solutions similar to the one given by Eq. (6). Furthermore, the method can be combined with the conventional F-matrix method to analyze a hybrid (linear or nonlinear) structure as well. This procedure allows us to incorporate gain or loss within the device and still maintain an analytical approach with a reasonable degree of accuracy

### 3. Nonuniform passive nonlinear DFB device

We first illustrate this method for uniform and phase-shifted nonlinear DFB devices. Fig. 2a shows the change in the transmittivity of a uniform nonlinear DFB structure as the input intensity  $I_{IN}$  is increased from  $10^{-6}$  (dashed curve) to 1 (solid curve). The nonlinearity-induced changes in the photonic band structure eventually lead to full transparency at the Bragg wavelength for  $I_{IN} = 1$ . Fig. 2b shows the transmittivity of a nonlinear DFB device two  $\lambda/4$  phase shifts occurring at  $z_1 = L/3$  and  $z_2 = 2L/3$ . As we increase the input intensity from  $I_{IN} = 10^{-6}$  (dashed curve) to  $I_{IN} = 0.1$  (solid curve), each transmission peak shifts by approximately twice its own width. The highly resonant nature of this structure requires considerably lower input intensities to significantly change the transmission characteristics and perform up- or down-switching at the

designed frequency. Notice the different widths of the transmission peaks as the input intensity is increased.

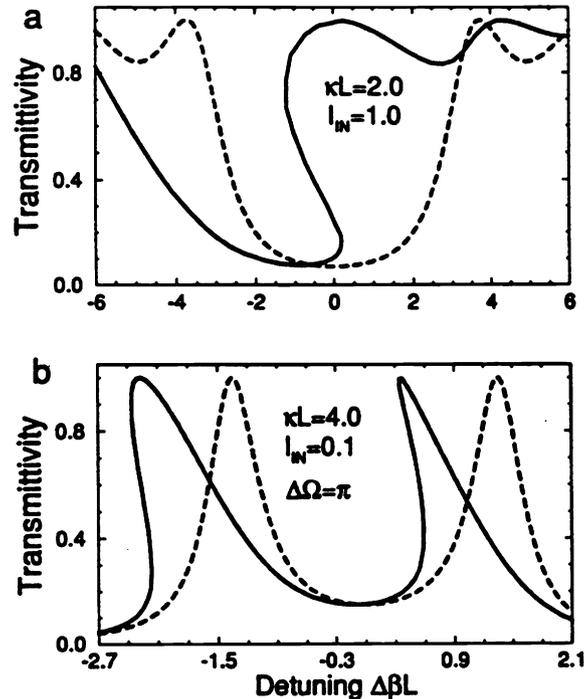


Figure 2. Transmittivity of (a) a uniform nonlinear DFB structure and (b) a multiple phase-shifted nonlinear DFB structure. In both cases the dashed curves show the low-intensity transmittivity of the structure.

#### 4. Hybrid nonuniform passive DFB device

Consider a nonlinear DFB structure that has a linear DFB section of length  $\Delta L$  inserted and centered at  $L_c$ . Fig. 3 shows the intradvice forward intensity for a fixed output in two different cases in which the linear DFB section is  $0.2L$  long and is centered (a) at  $L_c = 0.7L$  and (b) at  $L_c = 0.2L$ . In the first case, when the linear section is placed close to the device output, the intradvice intensity profile (thin solid curve) does not differ much from the intensity profile of a fully nonlinear device (thick solid curve). The average intensity close to the output is relatively low and the accumulated nonlinear shift does not change the solution appreciably. In contrast, by placing the linear section close to the input of the device, the optical field is changed dramatically (thick dashed curve), since the average intensity is an order of magnitude higher in this case. As a consequence, the transmittivity of the device decreases significantly, making the structure almost an opaque one.

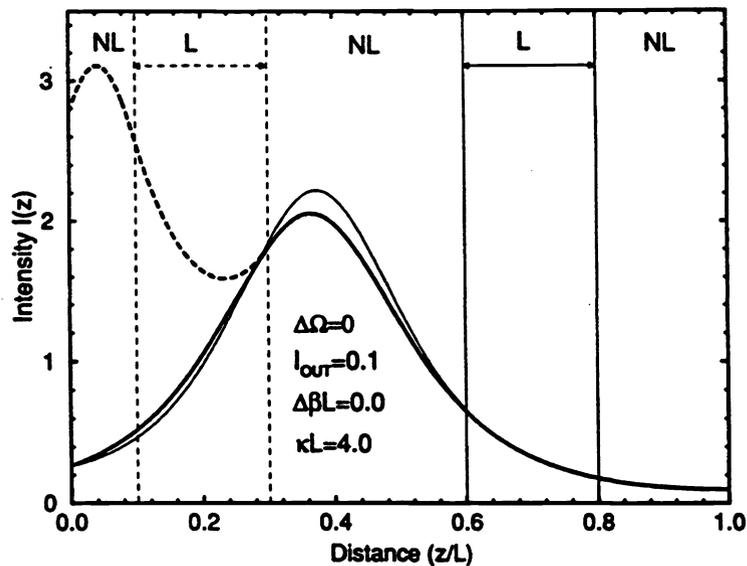


Figure 3. Intensity variation in nonlinear DFB structure with a linear DFB section inserted at  $0.7L$  (thin solid curve) and  $0.2L$  (dashed curve). The thick solid curve shows for comparison the case of a uniform nonlinear device.

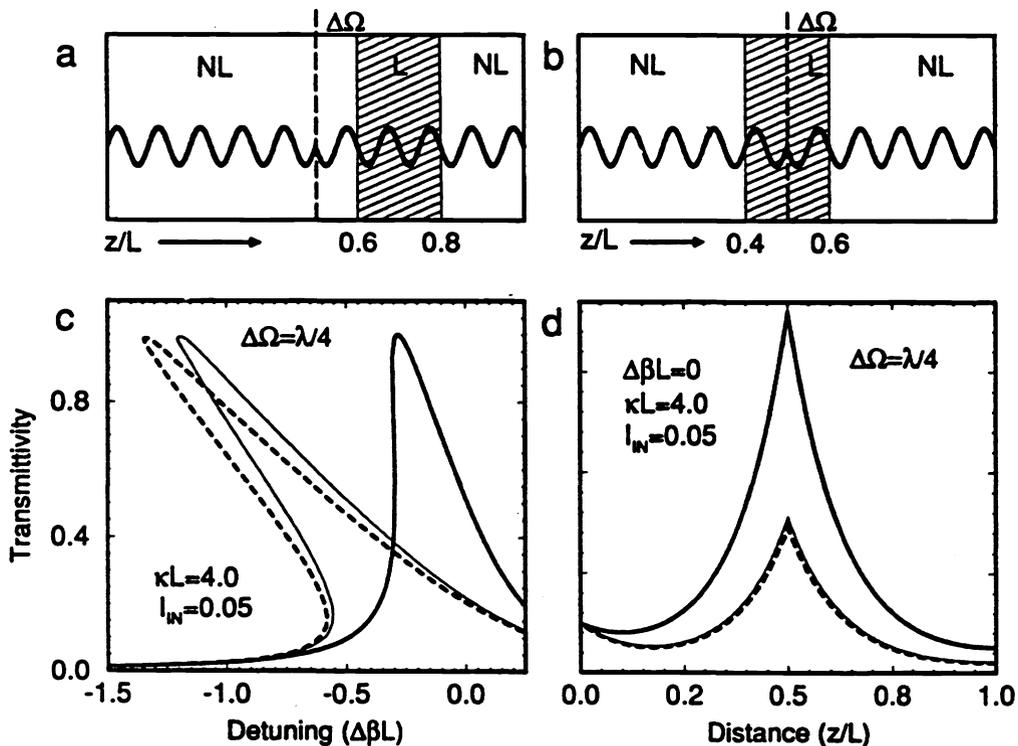


Figure 4. Effect of the linear region on (c) the transmittivity and (d) axial intensity profile for two hybrid  $\lambda/4$ -shifted DFB structures. Thick and thin solid curves correspond to structures a and b, respectively. Dashed curves correspond to the case of a uniform device.

The position of the linear section is even more important for the phase-shifted structures shown in Figs. 4a and 4b. The internal intensity of this device peaks at the phase-shift position  $z = L/2$ . By positioning the linear section close to the phase-shifting region (Fig. 4a), one expects to change the transmittivity of the device shown in Fig. 4c. However the thick solid curve shows that the change is much more dramatic if the linear section encompasses the phase-shifting region itself. In this case, the bistable behavior of the DFB structure nearly disappears. The central phase-shifting region plays the role of a variable phase-retarding plate controlled by the field intensity at the center of the device. When the phase-shifting region is linear, this feature is removed, thus destroying low-intensity switching capabilities of the device.

## 5. Hybrid nonuniform active device

Consider the phase-shifted linear DFB structure shown in Fig. 5 with two active (gain) regions centered at  $Z_c$  and  $L-Z_c$ . The active regions are deliberately placed away from the phase-shifting region in order to allow for an eventual nonlinearity at the center of the device. The threshold characteristics of such a device are plotted in Fig. 5 for different center positions and lengths of the active regions. We are primarily interested in the amplification and filtering properties of the device. Fig. 6 shows the transmission gain when the device is operated as both an amplifier and a narrowband filter, achieving the signal gain of  $\sim 50$  dB before it eventually starts to lase.

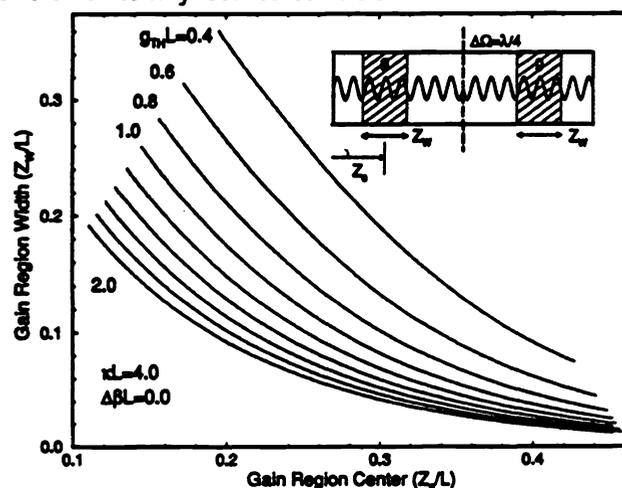


Figure 5. Threshold characteristics of an active  $\lambda/4$ -shifted DFB device with gain in the shaded regions. The threshold gain depends on both the location and the width of the gain section.

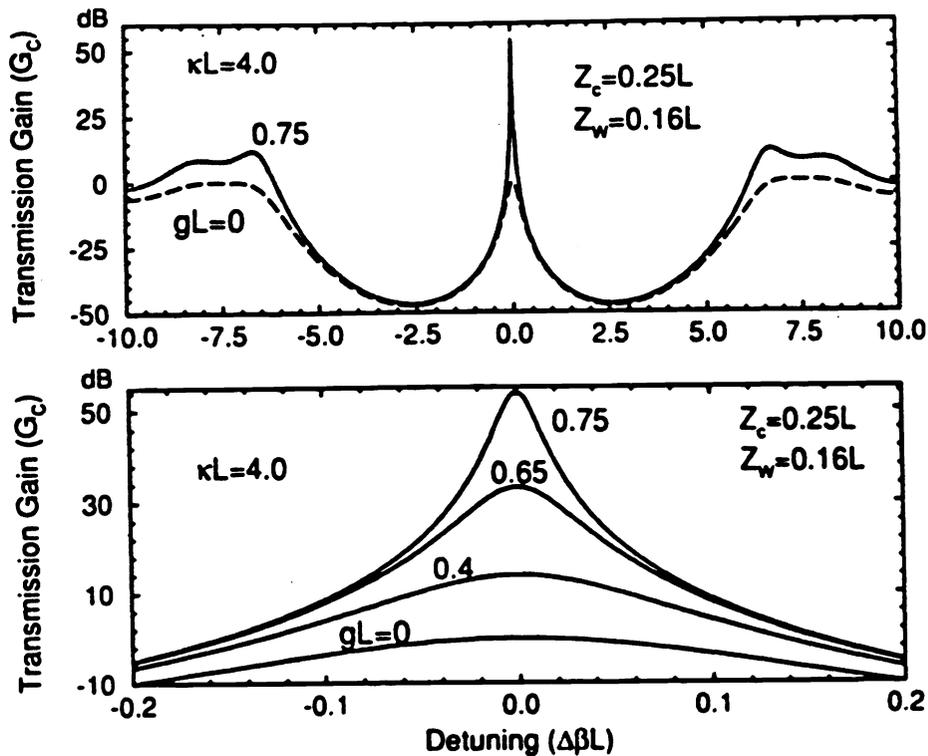


Figure 6. (a) The transmission characteristic of a  $\lambda/4$ -shifted DFB optical amplifier. Dashed curve corresponds to an unpumped device. (b) Blow up of the central peak for several values of gain.

A much more interesting behavior is obtained when we introduce the nonlinearity in the center of the device, thus combining the three functions in one DFB structure: amplification, filtering, and switching. There are at least two practical configurations that can be accurately described by this model. One is a multi-segment DFB semiconductor amplifier with at least three injection contacts. Gain regions shown in Fig. 5 correspond to two contacts that provide injection current that is higher than the transparency current, but lower than the threshold current. The central, nonlinear region corresponds to a third contact that provides the pumping just above the material transparency. High field intensity in this region leads to a gain saturation, which in turn results in high nonlinear index such that  $n_2$  exceed  $10^{-11}$  cm<sup>2</sup>/W in GaAs material.<sup>15,16</sup> Even if we bias the central section at exactly the transparency level, the carrier heating can lead to high index nonlinearities.<sup>17,18</sup> The second, more complicated configuration would be a planar waveguide structure with the alternating gain and nonlinear sections. However its fabrication would be a rather involved process requiring the integration of two dissimilar materials on same substrate (silica/nonlinear polymer/silica).

Fig. 7a shows the transmission of such a device for different gain and input intensity levels. It is important to notice that increased gain not only increases the intensity level of the signal, but also alters the width of the hysteresis, thus fundamentally altering its switching properties. The input-output characteristics shown in Fig. 7b emphasize this point. For a fixed detuning (wavelength) the up-switching intensity level is almost halved and the hysteresis width is quadrupled by increasing the device gain to about a  $g_{TH}/2$ . Finally, Fig. 8 shows the effect of asymmetric pumping of this device. The shown characteristics vary relatively little as we alternatively turn off and on the injection currents in each gain region, maintaining the total device gain constant in the process.

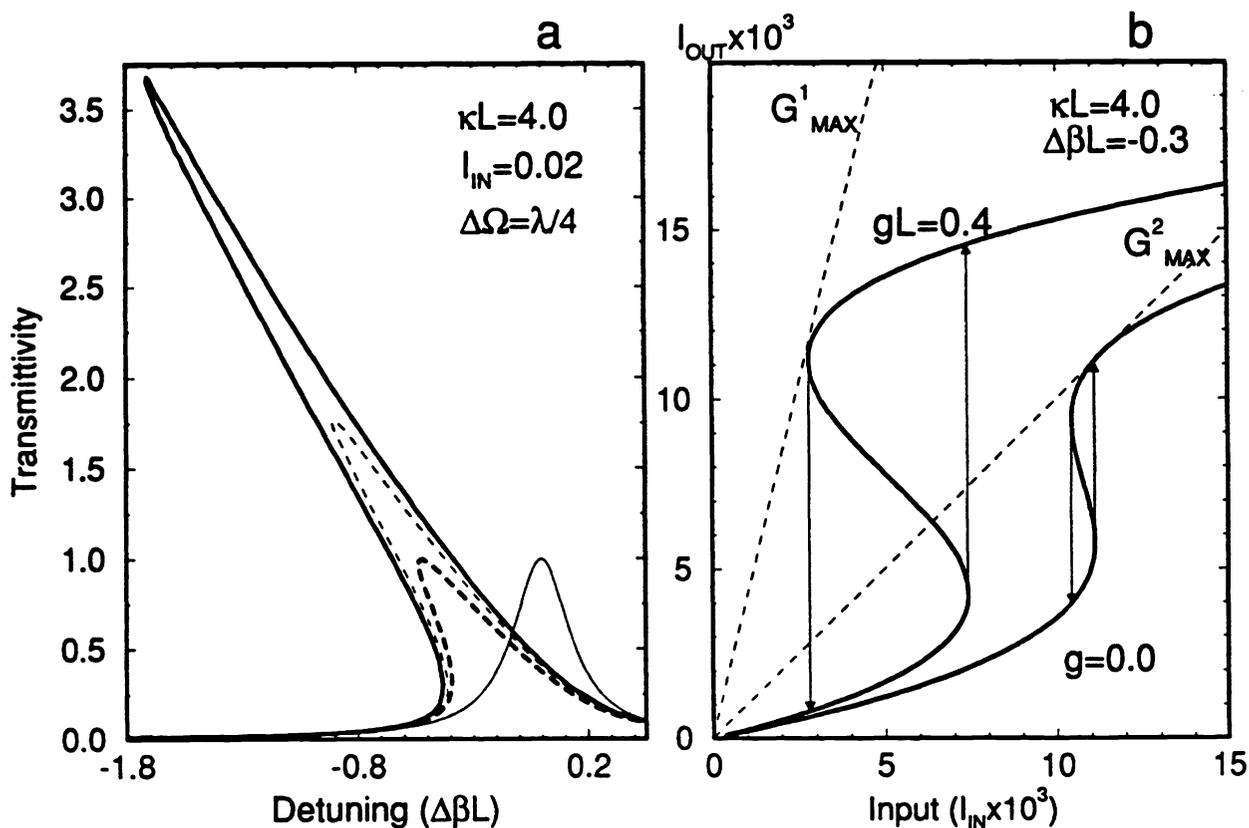


Figure 7. (a) Spectral transmission characteristics of the device from Fig. 5 with the inclusion of nonlinearity in the passive sections for  $gL = 0$  (thick dashed curve), 0.2 (thin dashed curve), and 0.4 (thick solid curve). The thin solid curve shows for comparison the expected behavior for a passive linear device. (b) Comparison of bistable input-output characteristics for  $gL = 0$  and 0.4.

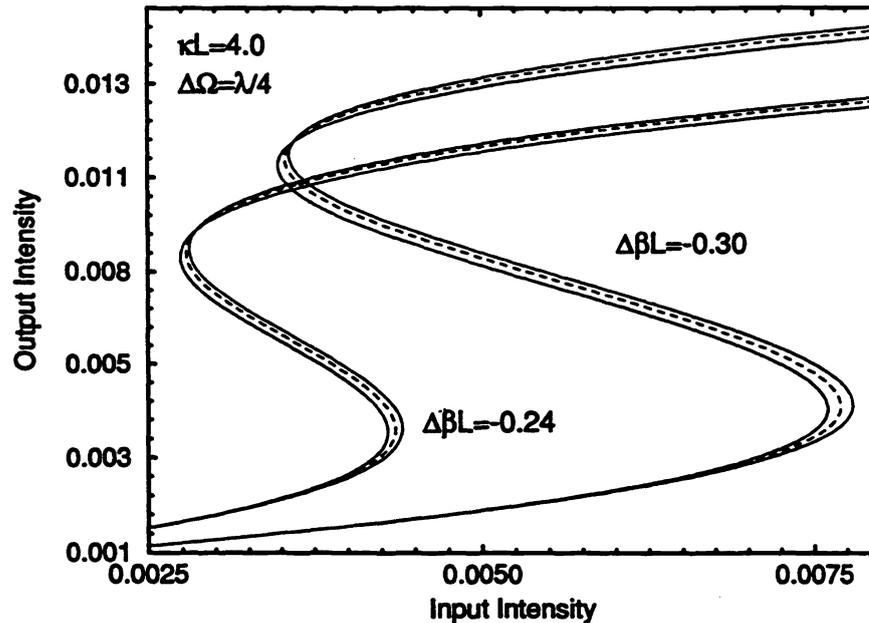


Figure 8. Bistable input-output characteristic of the hybrid DFB device of Fig. 7. Dashed curve corresponds to symmetrical pumping (equal gain in each active section), while the solid curves indicate variations due to asymmetric pumping (gain in only one of the active sections).

## 6. Conclusion

We have demonstrated a novel method for analyzing hybrid (linear/nonlinear) DFB optical structures. The method can be described as an approximate, analytic, single-sweep technique that requires comparatively low computational effort. We have illustrated the use of this technique in a design of DFB device that plays simultaneous role of highly discriminative filter, optical amplifier and all-optical switch. It is shown that even a highly localized optical nonlinearity plays an important role in switching performance of a phase-shifted DFB device.

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## 8. References

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