Phase-Conjugate Optical Feedback in Semiconductor Lasers

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ABSTRACT

Phase-conjugate feedback affects a laser in ways which are fundamentally different than conventional feedback. Notably, when the laser oscillates in more than one longitudinal mode, phase-conjugate feedback initiates a novel mode-coupling mechanism which can even lead to mode locking behavior. This paper explores these mode-coupling effects and also summarizes the similarities and differences between the two types of feedback.

1. INTRODUCTION

The study of phase-conjugate feedback (PCF) into lasers has been experiencing significant interest recently.[1-10] The self-aligning nature of PCF makes coupling back into the laser generally easier than with conventional optical feedback (COF), which usually occurs due to unwanted reflections from various system components or from the end faces of an optical fiber. Of course, the creation of a phase-conjugate mirror (PCM) is certainly more difficult than a conventional mirror, but several experimental setups have been demonstrated. Perhaps the simplest is the use of the self-pumped photorefractive crystal.[3-4,9,11] Also demonstrated is a PCM based on degenerate four-wave mixing (FWM) in atomic Rubidium.[10] The use of non-degenerate FWM allows the possibility of a detuning between the probe and conjugate signal, a situation which doesn't exist in COF. The significance of detuning in PCF has been studied theoretically but not experimentally to our knowledge.[1,7]

Mode locking of lasers, and in particular of semiconductor lasers, continues to generate intense interest due to such potential uses as the source of ultrashort optical pulses in a soliton communication system.[12] Semiconductor lasers have been mode-locked actively,[13] passively,[14] and through hybrid schemes.[15] In active mode locking, for instance, the gain or loss of the laser is modulated at the round-trip frequency of the laser. In a frequency-domain analysis of mode locking, the active modulation serves to directly couple each longitudinal mode to its two nearest neighbors. Therefore, this type of modulator is usually referred to as a double-sideband modulator.[16] Without the modulator, the laser is of course not mode-locked, because the beat frequencies between various modes are unequal due to mode pulling and pushing effects, and because spontaneous emission and other noise sources tend to make the phases of individual modes wander independently. The active modulator induces beat-frequency locking and phase locking, even overcoming the effects of noise, and thereby creates short pulses.

We show theoretically in this paper that it is possible to achieve both frequency locking and phase locking in a multi-longitudinal mode laser through the use of phase-conjugate feedback (PCF), provided four-wave mixing (FWM) is used to generate PCF. We are considering only the case where the feedback represents a small perturbation to the laser; i.e. the output facet of the solitary laser is not antireflection-coated (the weak-feedback regime). We also point out that COF will in general *not* lead to phase and frequency locking among the modes. The primary reason is that the different modes, although coupled intrinsically through the laser medium, are only self-coupled by an ordinary reflection; that is, upon reflection, each mode only couples back onto itself and not directly to any other mode. Some of the effects of COF on multi-longitudinal mode lasers have been recently reported.[17]

In section 2 we introduce the model and summarize some of the major differences between PCF and COF. In section 3 we investigate PCF conditions which give rise to AM mode locking in the laser. In section 4 we

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consider situations in which PCF forces FM laser operation, and in section 5 we present conclusions and areas of future work.

2. MODEL

The material used for the phase-conjugate mirror (PCM) is assumed to be a highly nonlinear medium with an essentially instantaneous response. The PCM is pumped by a narrow linewidth laser operating at frequency ω_p. The phase-conjugate signal is created by a non-degenerate FWM process, so that when a frequency ω_j is incident on the PCM, the conjugate signal is shifted to $\omega_c = 2\omega_p - \omega_j$. This frequency shift stands in contrast to the case of COF. Moreover, the conjugate signal will return directly to the laser, due to the self-aligning nature of the PCF. In summarizing the differences between COF and PCF, we will first limit ourselves to a laser oscillating in only a single longitudinal mode. In this case the rate equation for the complex, slowly varying amplitude of the laser mode can be written as

$$\frac{dE}{dt} = \frac{1}{2}(1 - i\alpha)(G_L - \frac{1}{\tau_p})\,E + \zeta + F + \kappa E^*(t-\tau)\,\exp[-2i\delta(t - \frac{\tau}{2})], \qquad (1)$$

where α is the linewidth enhancement factor ($\alpha = 3$), G_L is the gain which depends linearly on the carrier density, τ_p is the photon lifetime ($\tau_p = 1.4$ ps), and F is a Langevin noise term to account for random spontaneous emission. ζ contains the nonlinear gain contribution. The last term in Eq. 1 accounts for PCF. This field component is delayed by the external cavity roundtrip time τ ($\tau = 0.67$ ns for external cavity length of 10 cm), and it appears as the complex conjugate due to the action of the PCM. The strength of the feedback is denoted by the feedback rate κ. The exponential term accounts for any additional phase delay caused by a small detuning δ of the pump laser frequency from the optimum frequency.

The equation for the carrier number remains unchanged by PCF and is given as usual as

$$\frac{dN}{dt} = \frac{I}{q} - \frac{N}{\tau_e} - GP, \qquad (2)$$

where N is the carrier number, I is the injection current, q is the electronic charge, τ_e is the carrier lifetime ($\tau_e = 2$ ns), and P is the photon number. P is related to the complex field by $E = \sqrt{P}\exp(-i\phi)$, where ϕ is the instantaneous phase of the mode.

2.1 Comparison of PCF and COF: Analytical Theory

We now summarize the similarities and differences between PCF and COF. When the solitary laser oscillates in a single mode, considerable analytical progress can be made for both COF[18,19] and PCF [6,7], as long as the feedback level is relatively weak. One striking difference between the two cases concerns the presence of phase locking solutions. Specifically, in the case of PCF with no detuning ($\delta = 0$), the laser phase becomes locked to a value given by

$$2\phi + \phi_{PCM} + \tan^{-1}(\alpha') = 2m\pi, \qquad (3)$$

where ϕ is the (slowly-varying) steady-state phase, m is an integer, and $\alpha' = \alpha/(1+\varepsilon P_s)$,[18] where P_s represents a saturation photon number and ε is a measure of the nonlinear gain strength. In contrast, for normal feedback the steady-state phase remains arbitrary, as in the case for no feedback. It was previously predicted from linearized theory [6] that the pinning or locking of the laser phase would lead to a laser lineshape consisting of a spike at low frequencies (since the long-time behavior of the phase is locked), superimposed on a broader pedestal (since spontaneous emission still causes the short-time behavior of the phase to wander). This was confirmed by numerical simulations in reference 7.

A second obvious difference between PCF and COF in the weak feedback regime concerns the dependence on feedback phase, usually expressed as $\omega_0\tau$, the product of the solitary laser frequency and the external roundtrip time. For COF the laser is very sensitive to the precise value of the feedback phase. The threshold gain is either increased or decreased, the laser frequency can be shifted either positively or negatively, and the laser line is either broadened or narrowed depending on the exact value of feedback phase. In contrast, for a single-mode laser with PCF and zero detuning, the feedback phase does not enter into the equations, making the results independent of feedback phase.

When small detuning (~ 100 MHz) *is* present, which of course has no analog in COF, the laser possesses the ability to adjust its frequency to cancel out the detuning. A detuning arises when the PCM pump frequency $\omega_0 + \delta$ is shifted by a small amount δ from the solitary laser frequency ω_0. The conjugate signal frequency which returns to the laser is then shifted to $\omega_0 - \delta$. (Note that a self-pumped PCM necessarily operates with zero detuning.) For very weak levels of feedback, the detuning prevents the phase-locking condition from being satisfied; rather, the laser frequency remains close to the solitary laser frequency but oscillates at 2δ due to four-wave mixing induced inside the laser. As the feedback strength increases, the feedback-induced frequency shift begins to cancel out the detuning, causing the laser to oscillate at a slower rate. Eventually the detuning becomes small enough that it falls inside the injection-locking bandwidth, and the laser frequency locks and phase locks to the PCM pump frequency.

2.2 Comparison of PCF and COF: Chaotic Regime

For both COF and PCF, once the feedback level goes above a certain point, the laser tends to go unstable and with increasing feedback, the laser follows a well-known route to chaos. We note that the levels of feedback for which this occurs are still in the weak feedback regime, that is, multiple reflections in the external cavity may still be safely neglected. When the unstable regime has been entered, analytical solutions become increasingly difficult to find. Numerical simulations then become particularly powerful for investigating the laser behavior in the chaotic regime. Two parameters which affect COF and PCF differently are the external cavity length L_{ext}, and the linewidth enhancement factor α.

A significant relationship useful for characterizing the chaotic regime is the reciprocal of the external roundtrip time (τ^{-1}) compared to the laser relaxation-oscillation frequency ν_R. For long external cavities, when $\nu_R > \tau^{-1}$, COF and PCF cause similar behavior. Notably, once the laser makes a transition to chaos, it is unlikely to return to a stable or periodic state as the feedback level continues to increase. In the short cavity limit, on the other hand, chaotic behavior is interrupted by regions of non-chaotic operation. However, this effect is much less significant for PCF. For COF, the chaotic regions become thinner and thinner as the external cavity length is reduced, eventually effectively disappearing altogether.[20] But for PCF, the chaotic regions remain prominent for similarly short external cavities. An additional fundamental difference concerns the region between chaotic attractors. For COF, these regions consist of fixed-point solutions, but for PCF the regions between chaos consist of periodic or quasi-periodic behavior.

The effect of the linewidth enhancement factor α also has a dramatic on COF. In particular, as the value of α is lowered, the chaotic regions become much thinner, similar to the effect of reducing L_{ext}.[7] The dependence of chaos on α can be understood by noting that larger values of α imply larger phase changes associated with the feedback. Indeed, chaos is found to disappear for $\alpha = 0$. For a given value of α, the laser becomes more chaotic in the case of PCF compared with COF. For example, a comparison of the bifurcation diagrams for PCF and COF when $\alpha = 1.5$ and $L_{ext} = 10$ cm shows that whereas chaos has almost disappeared in the case of COF, it dominates the dynamics in the case of PCF. This difference is attributed to the fact that PCF tends to lock the laser phase (which changes because of α), and thus can destabilize the laser even for relatively small values of α.

In general, for PCF and COF with similar parameters, the dynamics are more complex for PCF.

3. PCF IN MULTI-LONGITUDINAL-MODE LASERS: AM MODE LOCKING

This section and the next consider PCF into a multimode laser. In addition to the PCM requirements mentioned in section 2, we also assume that the PCM has a broadband response, at least with respect to the longitudinal mode spacing of the solitary laser. When light from a multi-longitudinal-mode laser interacts with such a PCM based on four-wave mixing, a novel mode-coupling mechanism is realized when the PCM-pump laser is tuned correctly. Specifically, the mode at ω_j becomes shifted in frequency to $2\omega_p - \omega_j$ after reflection from the PCM. In order for the returned signal to couple into the laser, the frequency must coincide very nearly with one of the longitudinal-mode frequencies. This can only occur only if (i) ω_p nearly coincides with one of the longitudinal modes or if (ii) ω_p lies nearly in the middle of the two neighboring modes. In either case, the rate equation for the j-th longitudinal mode can be generalized from Eq. 1 to be

$$\frac{dE_j}{dt} = \frac{1}{2}(1 - i\alpha)(G_{L,j} - \frac{1}{\tau_p}) E_j + \zeta_j + F_j + \kappa_j E^*_{2p-j}(t-\tau) \exp[-2i\delta(t - \frac{\tau}{2})]. \tag{4}$$

The nonlinear gain term ζ_j is now generalized to contain both self and cross saturation as well as intramodal four-wave mixing. These terms arise from spatial and spectral hole burning and carrier heating. The explicit form of ζ_j can be found in reference 17. Note that E_j now depends explicitly on the time-delayed field of one other longitudinal mode, namely the mode at $2\omega_p - \omega_j$. Since each field rate equation now depends on one other mode, the action of the PCM might be called a type of single-sideband modulator, in contrast to ordinary active modulators.

The presence of mode locking can be studied in the usual way by considering the so-called reduced phases defined by

$$\Psi_j = (2\omega_j - \omega_{j-1} - \omega_{j+1})t + (2\phi_j - \phi_{j-1} - \phi_{j+1}). \tag{5}$$

Mode locking corresponds to Ψ_j going to a constant for all j. We note for completeness that the total laser output power is proportional to

$$P_T \propto \left| \sum_j \sqrt{P_j} \exp(-i\phi_j)\exp(-i\omega_j t) \right|^2 . \tag{6}$$

Thus the total output power of any multimode laser contains evidence of the longitudinal mode beating, even if a detector is not fast enough to respond to these oscillations. All multimode lasers are not mode-locked, of course, because the ω_j's are not equally spaced and the phases are fluctuating randomly. The high-frequency oscillations tend to wash out in this case, so that the total power reduces to $P_T \Rightarrow \Sigma P_j$, which contains only slowly-varying oscillations. When the Ψ_j as defined above are forced to become constant, the laser is both phase and frequency locked and is said to be mode locked. Generally, when the Ψ_j go to a multiple of 2π, then the laser is AM mode-locked; when the Ψ_j go to an odd multiple of π, the laser emits an FM wave.

By numerically integrating Eqs. 2 and 4, we can investigate when mode locking occurs. The numerical integration technique is based on a fourth-order Runge-Kutta algorithm. The strength of the PCF, or equivalently the reflectivity of the PCM, is conveniently characterized in terms of a dimensionless feedback parameter $\kappa\tau$. For simplicity we assume the reflectivity to be frequency independent, so that $\kappa\tau$ is the same for all the modes. For $\kappa\tau$ values between 0 and 10, we are in the weak-feedback regime. The laser behavior in the presence of PCF is generally quite complicated and the allowed parameter space is rather large. In order to facilitate the discovery of regimes where mode locking occurs, we employ the use of bifurcation diagrams, which allow immediate identification of stable, periodic, or chaotic regimes.[19] Generally, the bifurcation diagrams are constructed with the noise sources turned off, so that only deterministic rather than stochastic dynamics are involved. When the number of modes becomes large, however, we find it necessary to turn the noise sources on, in order to prevent modes from decaying to zero and not participating in the mode-locking behavior.

We have investigated the effect of PCF on lasers with 3, 4, 5, and 6 modes. In all cases stable regimes of mode locking are found. For simplicity, when the number of modes is odd, we assume the PCM pump frequency to be near the central mode; when the number of modes is even, we take the PCM pump to lie between the two central modes. In the three-mode case, Ψ_2 is constant and equal to zero for a range of $0.6 < \kappa\tau < 2.0$, indicating AM mode locking. The mode locked pulses are stable even in the presence of spontaneous-emission noise.

A representative bifurcation diagram for the slowly-varying total power vs. $\kappa\tau$ is shown in Figure 1 for the 4-mode case. Also shown on the bifurcation diagram is the total standard deviation (square root of the sum of the two variances) of the Ψ's (Ψ_2 and Ψ_3 for four modes), with and without the presence of noise. The PCM pump laser frequency is tuned exactly between the two central modes (modes 2 and 3). This explicitly couples modes 2 and 3 together and modes 1 and 4 together. The relative injection current is $I/I_{th} = 1.4$. The nonlinear gain terms are adjusted so that the modes are weakly coupled; i.e. with no feedback, all modes oscillate simultaneously with a certain power distribution rather than in the bistable fashion which is characteristic of strong coupling.[21] In these simulations of AM mode locking, the intrinsic FWM terms are neglected.

The bifurcation diagram indicates that the laser in the presence of PCF exhibits complex dynamical behavior. For different values of $\kappa\tau$, the laser exhibits quasi-periodicity, period-doubling bifurcations, and chaos. Of particular interest is the region corresponding to $\kappa\tau$ values between about 1.6 and 2.4. In this region the average power output is relatively well-behaved, exhibiting small-amplitude quasi-periodic behavior. However, over this entire region, the total standard deviation of the two Ψ values is practically zero without noise and about $\pi/10$ even with noise; that is, the Ψ's are constant in this range and the laser is mode locked. Interestingly, the Ψ values are nearly constant even in the feedback range $0.8 < \kappa\tau < 1.6$, although the modes exhibit a somewhat complicated (period 4) quasi-periodic behavior. The best mode locking, however, occurs in the range: $1.6 < \kappa\tau < 2.4$. Several points need to be made regarding the mode locking.

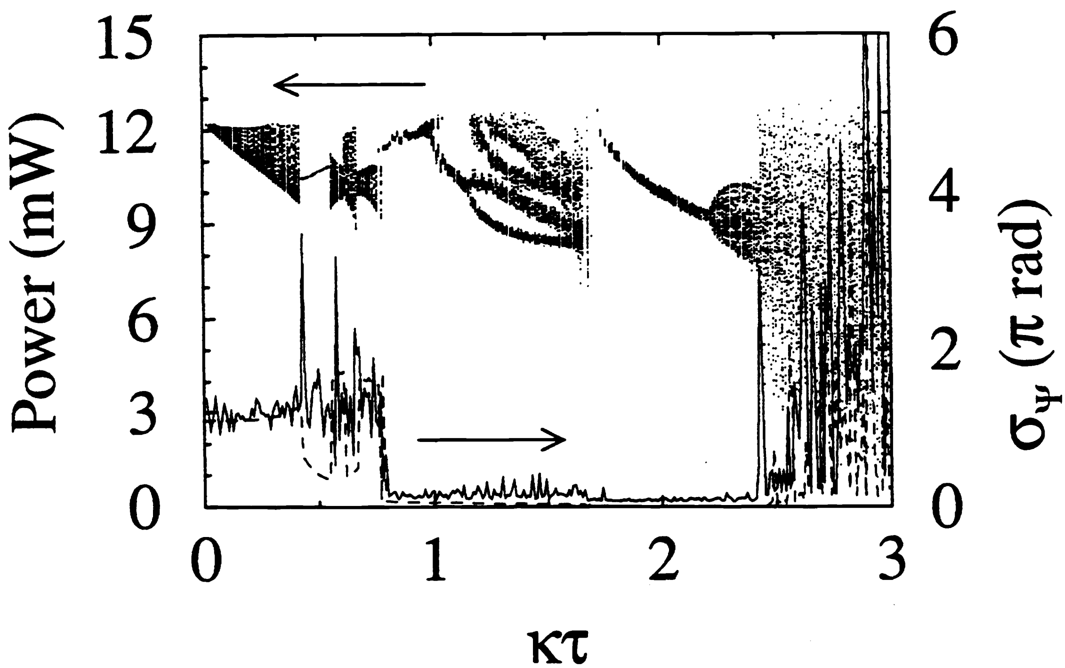

Figure 1. Bifurcation diagram of the slowly-varying total power vs. $\kappa\tau$. Also shown are the total standard deviations of the reduced phases without noise (-------) and with noise (———).

1. Usually mode-locked solutions are restricted such that the mode amplitudes as well as the reduced phases are constant. In the presence of PCF, the mode amplitudes are rarely constant, even in the single mode case, yet this only weakly affects the purity of the mode-locked pulses, as shown below.

2. Not only are the mode amplitudes not constant but their behavior can be rather complex, as indicated by the bifurcation diagram in the $\kappa\tau$ range from 1 to 2.4. Although the bifurcation diagram shows only total power, the individual mode powers behave in a similar fashion. For $\kappa\tau = 1.3$, for example, the slowly-varying laser output is quasi-periodic with a period-4 behavior, yet the phases are locked to a very good degree, the total standard deviation being about 0.2π.

3. We find empirically that in each case, it is the sum of the Ψ's, rather than the constant Ψ values themselves, that approach a multiple of 2π for AM mode locking.

As can be seen from Eq. 5, the fact that both Ψ_2 and Ψ_3 are constant implies that all three beat frequencies are the same. Moreover, $\Psi_2 + \Psi_3 \equiv (\phi_2 - \phi_1) - (\phi_4 - \phi_3) = 0$ in the locked region which is somewhat different than the usual phase condition for AM mode locking. In Fig. 2 we show the mode locked pulses themselves, with the noise sources turned on. The quasi-periodic behavior of the mode amplitudes mentioned above is responsible for the slow variation of the pulse heights. This variation occurs at a frequency of about 3 GHz and is close to the relaxation-oscillation frequency of the solitary laser. The spontaneous-emission noise has a minimal effect. Also shown in Fig. 2 is a zoomed-in version of the mode-locked pulses. The pulses are spaced, as usual, by the laser roundtrip time, or the reciprocal of the longitudinal mode spacing [9.3 psec = $(107\ \text{GHz})^{-1}$]. A simulation time-step of 0.1-ps allows the well-known secondary maxima to be clearly discernible.

We have also found mode locking to occur for five modes and six modes, although the proper mode coupling becomes more difficult as the number of modes grows. Representative pulse trains, showing both low and high resolution, are shown for both the five and six-mode cases, in Fig. 3 and 4, respectively. Note that in each case, there are the expected M-2 secondary maxima, where M is the number of modes.

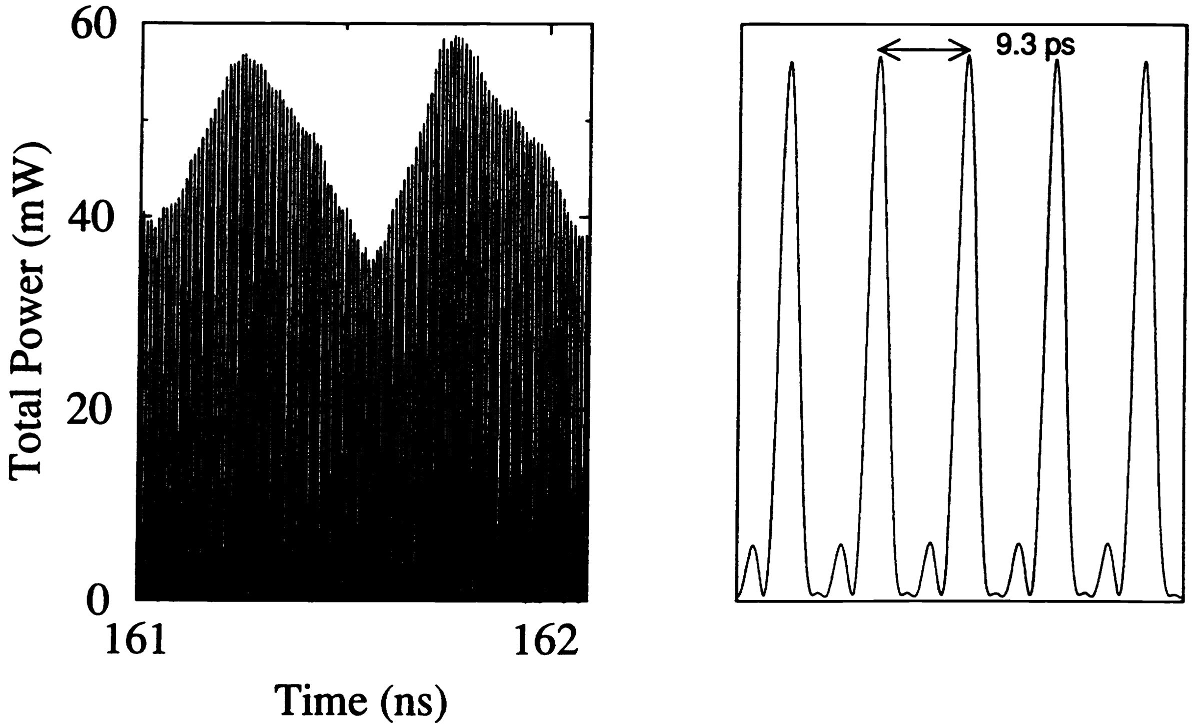

Figure 2. (a) Representative mode-locked pulse train for $\kappa\tau = 2.0$. The intensity modulation is discussed in the text. (b) A zoomed-in version reveals the spacing between pulses as well as the secondary pulse structure.

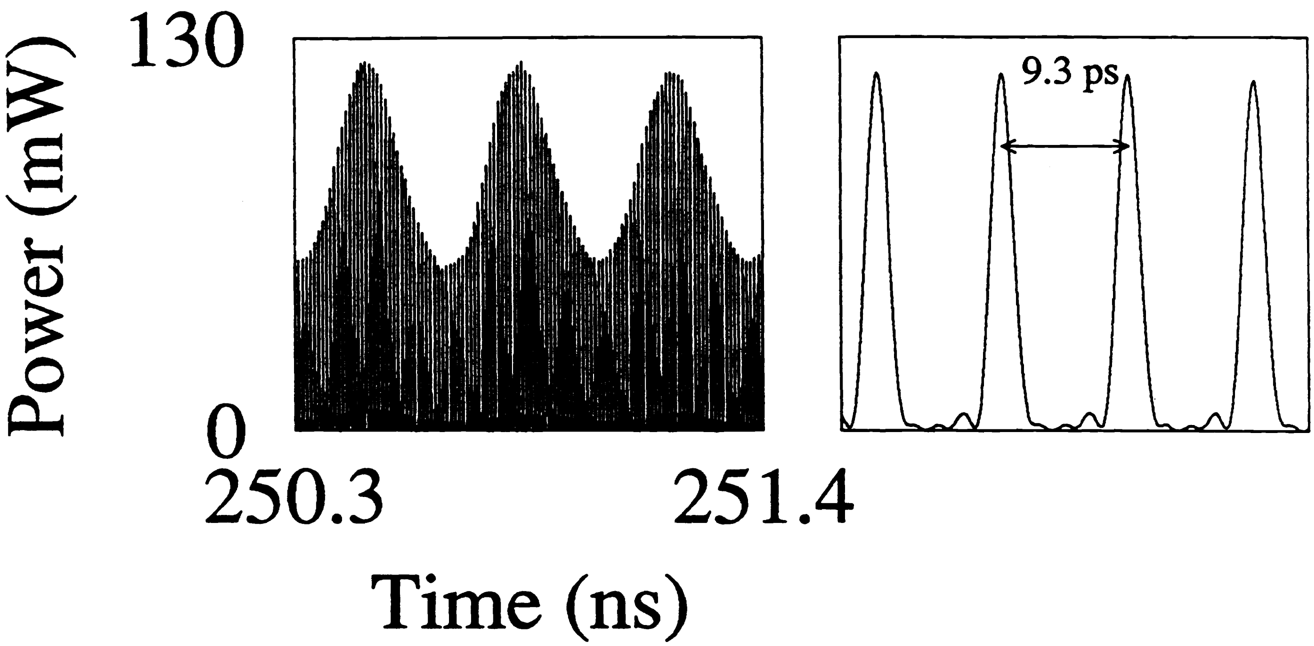

Figure 3. Mode locked pulse train (a) and individual pulses (b) for a five-mode laser operating at 60% above threshold and $\kappa\tau = 1.0$.

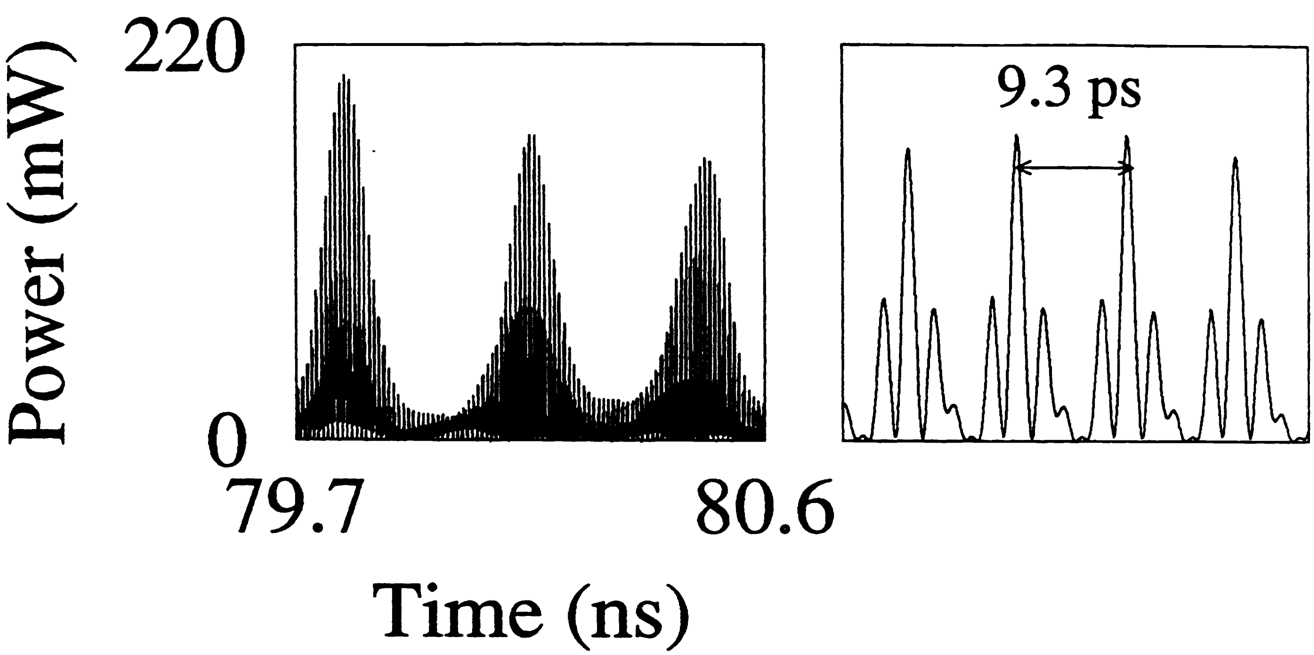

Figure 4. Mode locked pulse train (a) and individual pulses (b) for a six-mode laser operating at 60% above threshold and $\kappa\tau = 4.9$.

Why does PCF lead to mode locking? A possible interpretation is that PCF provides a frequency shift which can counteract the mode-pulling and pushing (frequency shifts induced by nonlinear gain) which ordinarily render the various beat frequencies unequal. For very weak feedback, the PCF-induced frequency shift is too small to accomplish the locking. Beyond a certain feedback strength, however, locking occurs over a wide region. At large values of feedback, mode locking is lost because of the onset of chaos.

We comment briefly on the assumption of an extremely fast responding PCM. In reality, of course, any material has a finite response time, leading to a frequency-dependent response. However, our assumption is reasonably valid as long as the PCM bandwidth is much larger than the longitudinal mode spacing. In our simulations we assumed a mode spacing of about 100 GHz, which is a typical value for solitary semiconductor lasers. For this case, a semiconductor laser amplifier could provide frequency-independent PCF, since its bandwidth has been measured to be greater than 1 THz. More generally, a PCM based on a Kerr-type nonlinearity would have an adequately wide bandwidth. Alternatively, in order to use a PCM with a smaller bandwidth, one could construct an external cavity laser, with any desired longitudinal mode spacing and number of modes.

4. FM LASER OPERATION

A contrasting solution to the mode-locking equations exists when the reduced phases achieve an odd multiple of π instead of a multiple of 2π. The alternate solution has been deemed the FM laser operation, as the optical wave appears to be a type of frequency-modulated signal. This time-domain signal usually changes too rapidly to be detected, so one often looks for evidence of the FM operation in the frequency domain. The simplest frequency-domain solution corresponds to the mode amplitudes being related as Bessel functions.[22,16]

Under certain circumstances, the mode amplitudes of semiconductor lasers have been shown to be closely related to the FM Bessel function solution.[23] This can be explained by invoking intramodal four-wave mixing, which provides a phase-sensitive coupling mechanism among the modes. Simulations employing the four-wave mixing coupling have demonstrated laser operation which approximates the FM solution.[24]

Our numerical model also includes the effects of intramodal four-wave mixing, which have been neglected for the AM mode locking solutions. However, we find that when we include the effects of the four-wave mixing, PCF can then force the laser to emit a pure FM wave, with the corresponding reduced phase becoming locked to a value of π. Figure 5 shows the relative phase with and without noise for a three-mode laser operating at 60% above threshold, for a PCF value of $\kappa\tau = 2.0$. When the noise sources are turned on, the reduced phase obtains a slow modulation in addition to the high-frequency oscillation. The slow modulation disappears when noise is absent. In either case, the mean value of the reduced phase is π, although the phase oscillates periodically about this value.

An advantage of the computer simulations is that high-frequency oscillations can be detected. Also the reduced phases can be ascertained directly, so that the FM laser operation can be seen directly rather than inferred. An example of the emitted FM wave is shown in Figure 6, with again both low and high resolution time traces. We expect for a pure FM wave that the total power will not oscillate between 0 and a maximum value, as in the AM locking case. Rather, because of the phase relationships among the modes, cancellation occurs so that the total power remains relatively constant. The high-resolution power trajectory gives the appearance of the so-called frequency-swept operation.

5. CONCLUSIONS

Phase-conjugate feedback can lead to a wide variety of dynamics in semiconductor lasers, similar to conventional optical feedback. But there are clear differences between the two types of feedback, and these differences have been summarized in this paper. We have also shown that, in stark contrast to conventional feedback, phase-conjugate feedback can couple the longitudinal modes of the laser in such a way that mode-locked pulses are emitted. Finally, we have also demonstrated that phase-conjugate feedback, in conjuction with intrinsic four-wave mixing coupling, can force the laser into FM operation.

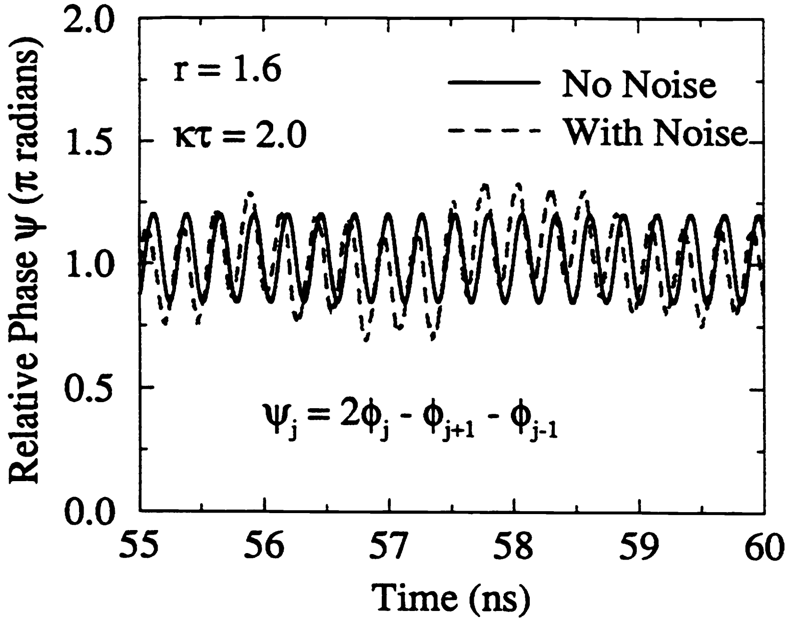

Figure 5. Reduced phase with noise (-------) and without noise (———) for three-mode laser operating at 60% above threshold and $\kappa\tau = 2$.

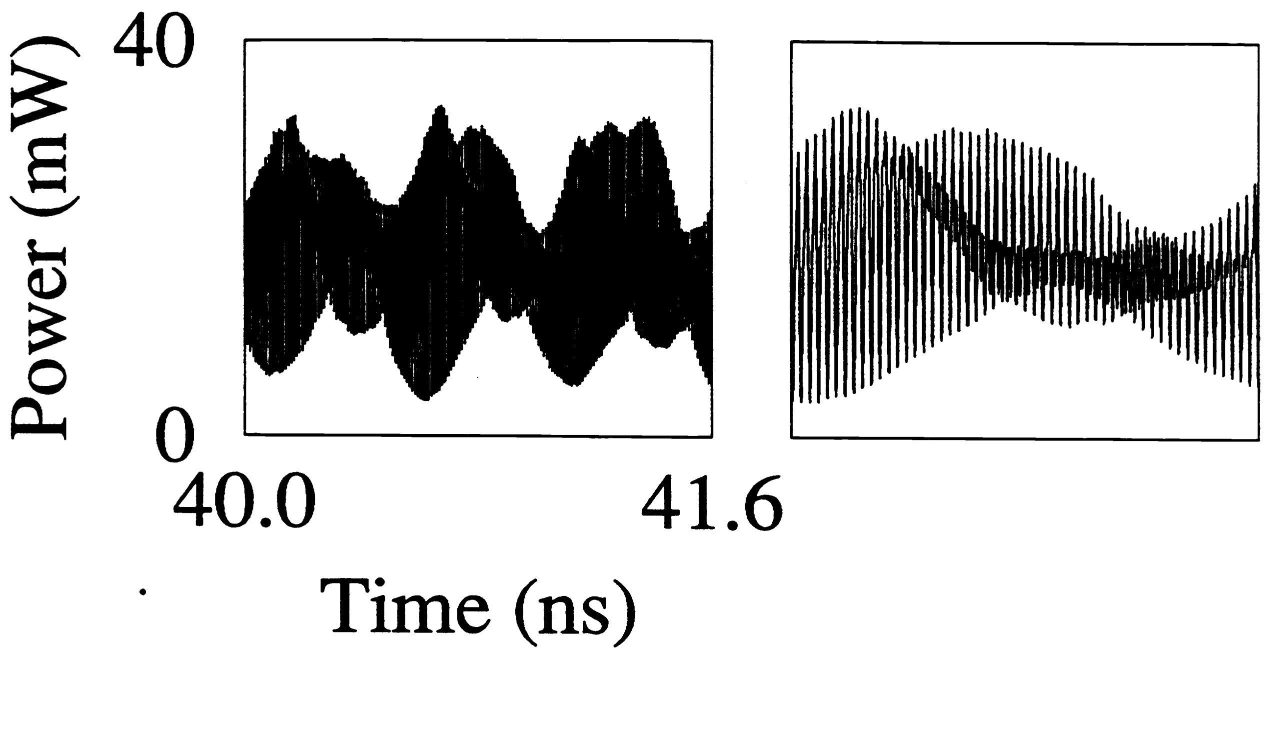

Figure 6. FM wave emitted by three-mode laser operating at 60% above threshold and $\kappa\tau = 4$. A close-up version of 0.5 ns is also shown.

6. ACKNOWLEDGMENTS

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