Journal of Modern Optics
Publication details, including instructions for authors and subscription information:
http://www.informaworld.com/smpp/title~content=t713191304

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Online Publication Date: 01 January 1995

To cite this Article Bowden, Charles M., Singh, Suneel and Agrawal, Govind P.(1995)'Laser Instabilities and Chaos in Inhomogeneously Broadened Dense Media',Journal of Modern Optics,42:1,101 — 107

To link to this Article: DOI: 10.1080/09500349514550081

URL: http://dx.doi.org/10.1080/09500349514550081

JOURNAL OF MODERN OPTICS, 1995, VOL. 42, NO. 1, 101–107

Laser instabilities and chaos in inhomogeneously broadened dense media

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(*Received 25 November 1993; revision received 15 April 1994*)

Abstract. We study numerically the effects of local-field corrections originating from near dipole–dipole interactions on the dynamics of a single mode, inhomogeneously broadened laser. Our analysis is based on a set of generalized Maxwell–Bloch equations in which the inhomogeneous nature of gain broadening is accounted for by introducing two new dynamical variables and a single parameter that governs the extent of inhomogeneous broadening. Our results show that local-field effects occurring in a dense gain medium reduce the range of continuous wave operation and lead to instabilities and chaos at much lower pumping levels.

1. Introduction

Considerable attention has recently focused on the effects of local-field corrections in the dynamical response of two-level systems [1, 2]. Local-field corrections result from the interaction of neighbouring dipoles in a dense medium, referred to as near dipole–dipole (NDD) interaction. They become important whenever the atomic density is high enough that many atoms lie within a volume λ^3, where λ is the resonance wavelength. The inclusion of NDD effects leads to many interesting phenomena such as intrinsic optical bistability, [3], self-phase modulation in self-induced transparency [4], linear and nonlinear spectral shifts [1, 5], propagational effects in nonlinear media [6], novel inversion and ultrafast switching effects [7], and statistical effects in superfluorescence and spontaneous emission [8]. However, the atomic medium has been assumed to be homogeneously broadened in previous work. In many cases of practical interest, the atomic medium is inhomogeneously broadened. If such an atomic medium is also dense, one needs to include NDD effects within the formalism of an inhomogeneously broadened, two-level system. Such an analysis has not yet been carried out. A perturbative procedure which treats the effects of inhomogeneous broadening in the Bloch–Maxwell formulation by the introduction of new dynamical variables was introduced earlier [9]. The procedure has the advantage that the integro-differential Maxwell–Bloch equations for an inhomogeneously broadened system are reduced to a hierarchy of ordinary coupled differential equations [9]. This approach was recently formulated to include NDD effects [10]. The equations, apart from NDD contributions, were used to analyse the utility of the line width enhancement factor for semiconductor laser amplifiers operating in the subpicosecond time domain [11].

In this paper we study, for the first time to our knowledge, the dynamics of an inhomogeneously broadened, dense, atomic medium by considering the underlying Bloch–Maxwell equations. To simplify the analysis, we follow the approach, first used by Graham and Cho [9], and extended to include NDD contributions [10], in which the integro-differential Maxwell–Bloch equations are reduced to a set of ordinary differential equations by introducing new dynamical variables. Such an approximation is found to be reasonable for a study of the onset of instabilities and chaos. We focus specifically on an inhomogeneously broadened laser oscillating in a single mode and study the threshold value of the pumping level at which the steady state becomes unstable. Our results show that the NDD effects reduce the instability threshold considerably.

2. Generalized Maxwell–Bloch equations

We review, here, briefly, the development of the Maxwell–Bloch formulation for inhomogeneously broadened two-level systems, which includes NDD effects [10]. In the slowly varying enveloped and mean-field approximation, the Maxwell–Bloch equations for an inhomogeneously broadened, two-level atom can be written as

$$\frac{\mathrm{d}E}{\mathrm{d}t} = -\kappa E + \mathrm{i}g\sum_k \rho(k) \tag{1}$$

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = -[\gamma_\mathrm{T} + \mathrm{i}(\omega - \omega_L)]\rho - \frac{\mathrm{i}\mu}{2\hbar}E_\mathrm{L}w \tag{2}$$

$$\frac{\mathrm{d}w}{\mathrm{d}t} = -\gamma_\mathrm{L}(w - w_\mathrm{th}) - \frac{\mathrm{i}\mu}{\hbar}(E_\mathrm{L}^*\rho - \rho^* E_\mathrm{L}) + \Lambda, \tag{3}$$

where E_L is the slowly-varying amplitude of the optical field, g represents the atom–field coupling, ω is the transition frequency and ω_L is the laser frequency, μ is the dipole moment, ρ is the off-diagonal element of the density matrix, the inversion $w = \rho_{11} - \rho_{22}$ is related to the diagonal elements of the density matrix, Λ is the pumping rate, and κ, γ_T, and γ_L are the cavity decay rate, the dipole decay rate, and the inversion decay rate, respectively. Here, ρ and w are assumed to be wave-vector, k, dependent.

Inhomogeneous broadening is included in equations (1)–(3) through the sum over ω which varies from atom to atom. It is useful to introduce $\delta \equiv (\omega - \omega_0)/\gamma_\mathrm{T}$ where ω_0 is a reference frequency normally taken with respect to the gain peak, and write [10]

$$\frac{1}{V}\sum_k \rho(k) = N_\mathrm{t}\int_{-\infty}^{\infty} \rho(\delta)\,D(\delta)\,\mathrm{d}\delta = N_\mathrm{t}\langle\rho\rangle, \tag{4}$$

where V is the volume of material, $D(\delta)$ is the distribution function over momentum states and angled brackets denote an average with respect to it. The NDD effects are included in equation (1)–(3) by eliminating the local field E_L in equations (2) and (3) in terms of the electric field E appearing in equation (1). The two are related by [2]

$$E_L = E + \frac{2\mu N_t}{3\varepsilon_0}\langle\rho\rangle, \tag{5}$$

where N_t is the total dipole number density. The relation (5) is often called the local-field correction and is known to induce linear and nonlinear spectral shifts in dense media [1, 5]. It forms the basis for Clausius–Mossotti relations in condensed matter physics.

To obtain the macroscopic Maxwell–Bloch equations, we follow the approach of Graham and Cho [9] and define average inversion and polarization as

$$\mathscr{W} = \langle w\rangle \quad \text{and} \quad p = \langle\rho\rangle. \tag{6}$$

By introducing the Rabi frequency $\Omega = \mu E/\hbar$, equations (1)–(3) can be written as

$$\frac{d\Omega}{dt} = -\kappa\Omega + i\tilde{g}p \tag{7}$$

$$\frac{dp}{dt} = -\gamma_T(1 + i\Delta)p - \gamma_T\langle i\delta\rho\rangle - \frac{i\Omega}{2}\mathscr{W} - i\varepsilon p\mathscr{W} \tag{8}$$

$$\frac{d\mathscr{W}}{dt} = \gamma_L(r - \mathscr{W}) + i(\Omega p^* - \Omega^* p), \tag{9}$$

where $\Delta = (\omega_0 - \omega_L)/\gamma_T$ is the detuning parameter and we have introduced the pump parameter r, and written the NDD interaction strength as ε,

$$\varepsilon = \frac{2\mu^2 N_t}{3\varepsilon_0\hbar}, \quad r = \frac{\Lambda}{\gamma_L} + w_{th}. \tag{10}$$

Also, we have used $\tilde{g} = N_t\mu g/\hbar$ in equation (7). These equations are not closed because of the presence of $\langle i\delta\rho\rangle$ in equation (8). In fact, the presence of this term leads to an infinite hierarchy. To the lowest order, this hierarchy can be truncated by defining two new dynamical variables S and U through

$$\mathfrak{P}^2 S = \langle i\delta\rho\rangle \quad \text{and} \quad \mathfrak{P}^2 U = \langle\delta w\rangle, \tag{11}$$

where the dimensionless parameter $\mathfrak{P}$ is yet to be determined. By using equation (1)–(3), S and U are found to satisfy

$$\frac{dS}{dt} = -\gamma_T(1 + i\Delta)S + \gamma_T(p - p_0) + \tfrac{1}{2}\Omega U + \varepsilon p U, \tag{12}$$

$$\frac{dU}{dt} = -\gamma_L(U - U_{th}) - (\Omega S^* + S\Omega^*) - 2\varepsilon(p^* S + pS^*), \tag{13}$$

where

$$p_0 = \langle(1 - \delta^2/\mathfrak{P}^2)\rho\rangle. \tag{14}$$

Here, U_{th} is the equilibrium value and is non-zero only if the distribution is intrinsically asymmetric [10, 11]. The set of five equations, equations (7)–(9), (12), and (13), can be closed by choosing $\mathfrak{P}^2$ such that $p_0 = 0$, i.e.,

$$\mathfrak{P}^2 = \langle\delta^2\rho\rangle/\langle\rho\rangle. \tag{15}$$

The crucial assumption of Graham and Cho is that, to the first approximation, $\mathfrak{P}^2$ can be treated as a constant by replacing ρ in equation (15) by its steady-state value. Then $\mathfrak{P}^2$ can be treated as an inhomogeneous broadening parameter since $\mathfrak{P}^2 = 0$ for homogeneous broadening and increases as the gain medium becomes more and more inhomogeneously broadened. Physically, $\mathfrak{P}^2$ is a measure of the spread of the atomic population over the inhomogeneous distribution [10, 11]. In the next section we solve the above set of five equations numerically to investigate the stability of the steady-state solution.

3. Laser instabilities and chaos

For the purpose of numerical simulations, we define a normalized time $\tau = \gamma_T t$ and write equations (7)–(9), (12), and (13) in the form

$$\frac{d\Omega}{dt} = -\frac{\kappa}{\gamma_T}\left(\Omega - \frac{i\tilde{g}}{\kappa}p\right), \tag{16}$$

$$\frac{dp}{d\tau} = -(1 + i\Delta)p - \mathfrak{P}^2 S - \frac{i\Omega}{2\gamma_T}\mathscr{W} - \frac{i\varepsilon}{\gamma_T}p\mathscr{W}, \tag{17}$$

$$\frac{d\mathscr{W}}{d\tau} = \frac{\gamma_L}{\gamma_T}(r - \mathscr{W}) + \frac{i}{\gamma_T}(\Omega p^* - \Omega^* p), \tag{18}$$

$$\frac{dS}{d\tau} = -(1 + i\Delta)S + p + \frac{1}{2}\frac{U}{\gamma_T}(\Omega + 2\varepsilon p), \tag{19}$$

$$\frac{dU}{d\tau} = -\frac{\gamma_L}{\gamma_T}(U - U_{th}) - \frac{1}{\gamma_T}(\Omega S^* + S\Omega^*) - \frac{2\varepsilon}{\gamma_T}(p^* S + pS^*). \tag{20}$$

These equations are solved numerically by taking $\gamma_L = \gamma_T$, the collisional dephasing regime, and $\kappa = 2{\cdot}8\,\gamma_T$ (bad-cavity case) together with $\tilde{g} = \kappa$. Such conditions could be controlled in a dense gas or metal vapour by adjusting the density and the loaded cavity Q. The pump parameter r is varied in the range 1–50, and is a measure of the

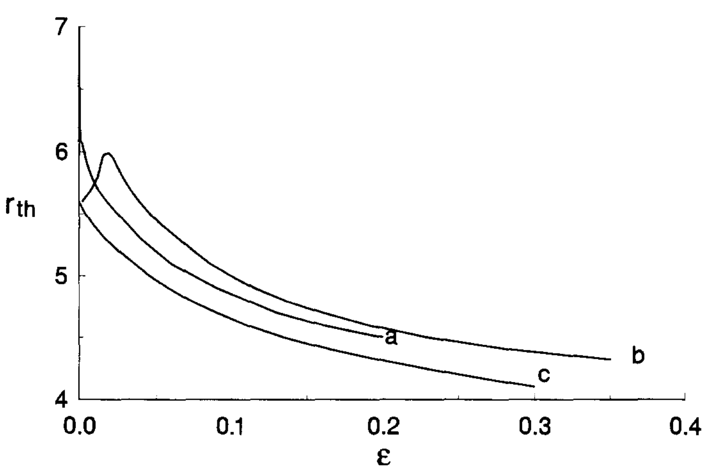

Figure 1. Illustration of the behaviour of instability threshold r_{th} as a function of the NDD parameter ε for various values of detuning Δ. Curves a, b, and c are for values of Δ equal to 0, $+0{\cdot}1$, and $-0{\cdot}1$, respectively. All parameters are scaled in terms of the homogeneous width γ_T.

ratio of the pumping rate to the spontaneous emission rate, equation (10). The steady state becomes unstable at a threshold value r_{th} of r. The instability threshold r_{th} depends on the parameters $\mathfrak{P}^2$, ε and Δ. Consider first the case of a laser oscillating at the gain peak so that $\Delta = 0$. For $\mathfrak{P}^2 = 0$, we recover the results obtained for a homogeneously broadened gain medium, where the threshold for instability occurs at $r_{th} = 44$. Figure 1 shows the variation of r_{th} as a function of the NDD parameter ε for a value of $\mathfrak{P}^2$ equal to 2, and values for Δ above and below resonance. In terms of our choice of scaling in equations (16)–(20), ε is a measure of the NDD strength in units of the homogeneous line width determined by γ_T. Typically, for $\mu \approx 1$ Debye and $\gamma_T \approx 10^8\,s^{-1}$, a value $\varepsilon = 0{\cdot}1$ corresponds to a density $N_t \approx 10^{15}\,cm^{-3}$, which, for optical wavelengths, gives approximately 100 atoms, on average, within a cubic wavelength. Obviously, the instability threshold decreases as $\mathfrak{P}^2$ increases. It also decreases as the NDD parameter ε increases. Thus, the main effect of NDD interaction is to lower the instability threshold for both homogeneously and

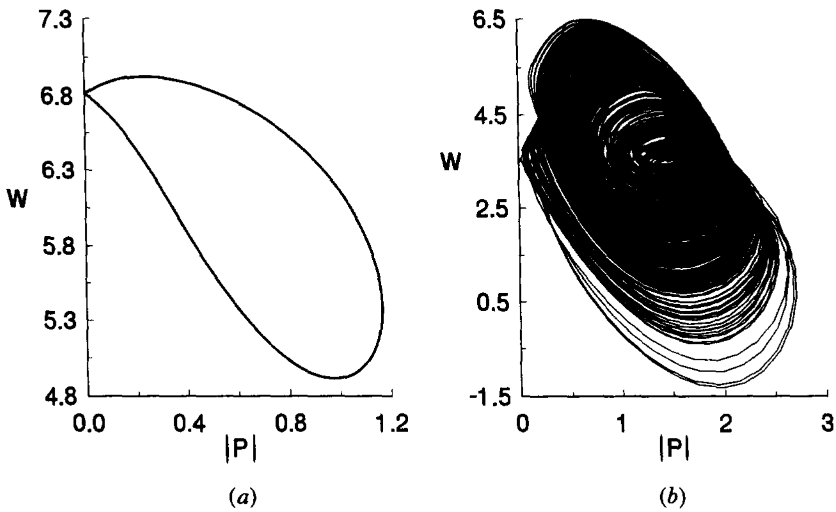

Figure 2. Effect of ε on the laser instabilities at the threshold value $r_{th} = 7$. (*a*) $\varepsilon = 0$ and (*b*) $\varepsilon = 0{\cdot}4$. All parameters are scaled in terms of γ_T.

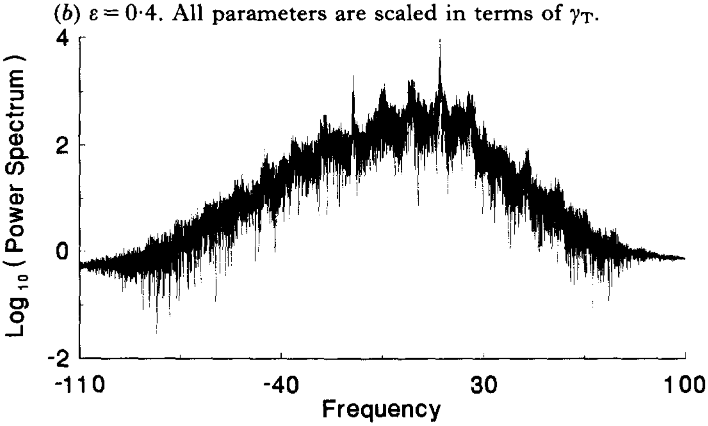

Figure 3. Power spectrum for the case $r = 7$ and $\varepsilon = 0{\cdot}4$. The y axis represents the $\log_{10}$ of the absolute value of the Fourier transform of p. All parameters are scaled in terms of γ_T.

inhomogeneously broadened atomic media, although we find that in the former case, the effect of ε is not so drastic. For homogeneously broadened systems, saturation of the threshold occurs at $r_{th} = 42$ for $\varepsilon = 0{\cdot}2$. The decrease can be quite substantial (by a factor of 1·7) in an inhomogeneously broadened medium. The effect of detuning on the instability threshold is also shown in figure 1. The effect of detuning is to further lower threshold. In a dense medium however, the behaviour is markedly different depending upon the detuning.

When r exceeds r_{th} the laser output becomes chaotic following a period-doubling or quasi-periodic route. Figure 2 shows the phase diagram (in $\mathscr{W} - |p|$ space) for $r = 7$, $\mathfrak{P}^2 = 2$, $\Delta = 0$, and $\varepsilon = 0$ (figure 2(*a*)) and $\varepsilon = 0{\cdot}4$ (figure 2(*b*)). The chaotic nature of the laser output for $\varepsilon \neq 0$ is evident. It is important to note that the laser would operate continuously at this pumping level if the NDD effects were absent. The chaotic output is solely due to the NDD interaction in dense media for the operating conditions shown in figure 2. Figure 3 shows the power spectrum corresponding to the chaotic attractor which appears in figure 2. The asymmetry seen in figure 3 has its origin in NDD interaction since the power spectrum is symmetric when $\varepsilon = 0$. This is due to frequency renormalization of atoms in a dense medium. Thus, even when $\Delta = 0$, there is an inversion-dependent term that acts like a detuning term in equation (17) which gives rise to asymmetry of the power spectrum.

4. Conclusions

In conclusion, we have studied the effects of local field corrections on the dynamics of a single-mode, inhomogeneously broadened laser system. The local field effects that arise due to near dipole–dipole interactions in a dense medium are found to reduce the range of continuous wave operation and lead to instabilities and chaos at much lower pumping levels. It is noted, however, that Meziane [12] has obtained a better matching of the dynamical evolution of equations (16)–(17), under conditions of self-pulsing, with Casperson's experimental and numerical results [13] by replacing the parameter $\mathfrak{P}$ with an intensity-dependent saturable parameter. Qualitative results are obtained from the original equations, as compared with numerical integration of the modified equations, and the conditions which lower the threshold of instability are not expected to change. We have sought, in this paper, to examine the effects of NDD on the equations (16)–(20) which were obtained self-consistently.

Acknowledgments

Support of this project for G. P. Agrawal was provided, in part, by the US Army Research Office.

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