Noise in semiconductor lasers and its impact on optical communication systems

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ABSTRACT

Spontaneous emission is a major source of noise in semiconductor lasers. The noise phenomena such as relative intensity noise, mode-partition noise, and laser linewidth are discussed by using the Langevin rate equations. Particular attention is paid to the impact of intensity and phase noise on the performance of optical communication systems.

1. INTRODUCTION

Optical communication systems almost always use semiconductor lasers as an optical source not only because such lasers are compact and efficient but also because they can be modulated directly at relatively high speeds. Indeed, the development of InGaAsP semiconductor lasers, operating in the wavelength region 1.3-1.6 μm, is fueled by the technological advances in the field of optical fiber communications. An excellent example is provided by the recent emphasis on the coherent communication techniques that has led to the development of tunable, narrow-linewidth, semiconductor lasers.[1,2] Ideally, a laser source for lightwave systems should provide an optical carrier whose amplitude and phase do not change with time under cw operation. In practice, noise inherent in the operation of semiconductor lasers introduces both amplitude and phase fluctuations which can degrade the system performance considerably.[3] It is thus essential to understand the origin of noise in semiconductor lasers and its impact on optical communication systems. In contrast with most other lasers the dominant source of noise in semiconductor lasers is the fundamental phenomenon of spontaneous emission. It is often referred to as quantum noise in order to emphasize the quantum-mechanical origin of spontaneous emission. It is however not necessary to resort to a full quantum treatment to understand the noise phenomena in semiconductor lasers. A set of stochastic rate equations, obtained by adding the appropriate Langevin noise sources, provides a theoretical framework that is often adequate for studying noise in semiconductor lasers.[3] This paper focuses on such Langevin rate equations and their solutions.

2. SPONTANEOUS EMISSION AND LASER NOISE

In this section we discuss how spontaneous emission can lead to intensity and phase fluctuations in the coherent optical field established inside the laser cavity by stimulated emission. Consider, for simplicity, a single-mode laser oscillating at the frequency ω_0. The intracavity optical field can be written as

$$E(t) = \mathrm{Re}\,[A(t)\exp(-i\omega_0 t)]\,, \tag{1}$$

where A(t) is the complex amplitude and Re stands for the real part. It is useful to represent A as a vector in the complex A-plane and normalize it in such a way that $|A|^2 = P$, where P is the number of photons inside the laser cavity. Each spontaneously emitted photon changes A to A′. Both the magnitude and the phase of A′ depend on the phase of the optical field associated with the spontaneously emitted photon. However, this phase is uncertain and can vary over the entire range 0 to 2π in a random manner. Because of phase uncertainty the amplitude and the phase of the optical field change in a random manner after each spontaneous-emission event. Furthermore, such events occur at random times with a relatively high frequency ($\sim 10^{12}$ s^{-1}). The net result is that the intensity and the phase of the laser field appear to fluctuate randomly as a result of spontaneous emission. One can estimate easily that the intensity noise level σ_p/P varies as $P^{-1/2}$, where σ_p^2 is the variance and P is the average number of intracavity photons, and decreases as the laser power is increased. The intensity noise, however, is not necessarily white. Its frequency dependence is characterized by the relative intensity noise (RIN) defined as[3]

$$\mathrm{RIN} = S_p(\omega)/P^2, \tag{2}$$

where the spectral density is related to the Fourier transform of the intensity autocorrelation and is given by

$$S_p(\omega) = \int_{-\infty}^{\infty} <\delta P(t + \tau)\, \delta P(t)> \exp(-i\omega\tau)d\tau. \tag{3}$$

The frequency dependence of RIN depends on laser dynamics and is considered in Sec. 4.

In the case of multimode lasers one must consider the intensity noise of each individual mode. Since the same supply of electrons and holes generates photons for all modes, a new phenomenon known as mode-partition noise (MPN) occurs. It refers to an anticorrelation among modes occurring in such a way that individual modes exhibit large intensity fluctuations even though the total intensity in all modes remains relatively constant. In the absence of fiber dispersion MPN would be harmless for optical communication systems as all modes would remain synchronized during transmission. However, different modes travel at slightly different speeds because of fiber dispersion and do not arrive simultaneously at the receiver. Such a desynchronization not only leads to degradation in the signal-to-noise ratio (SNR) but also creates pulse broadening and intersymbol interference. The performance of current optical communication systems is limited by the MPN. We discuss MPN in detail in Sec. 4.

Emission of spontaneously emitted photons also leads to phase fluctuations of the intracavity optical field. Such fluctuations are responsible for broadening of the linewidth associated with each longitudinal mode. Mathematically, the field spectrum is related to the Fourier transform of the field autocorrelation and is given by[3]

$$S_E(\omega) = \int_{-\infty}^{\infty} <E^*(t + \tau)E(t)> \exp(-i\omega\tau)d\tau, \tag{4}$$

where

$$E(t) = (P + \delta P)^{1/2} \exp[-(\omega_0 t + \phi + \delta\phi)] \tag{5}$$

includes both intensity and phase fluctuations given by δP and $\delta\phi$ respectively. Phase noise and the resulting laser linewidth are discussed in Sec. 5. As discussed there, the performance of coherent communication systems is severely affected by phase fluctuations of semiconductor lasers used in the system.

Another source of noise in semiconductor lasers is shot noise associated with carrier recombination and generation. Spontaneous radiative recombination of an electron-hole pair generates shot noise and, at the same time, is the source of spontaneous emission. Carrier recombination, however, can also occur through nonradiative processes such as the Auger process or trapping of an electron by an impurity. All such processes generate shot noise and must be included. They can contribute to both intensity and phase fluctuations of the optical field. In most cases the contribution of shot noise is often negligible compared with the contribution of spontaneous emission.

3. LANGEVIN RATE EQUATIONS

Semiconductor lasers belong to a class of lasers whose dynamic response can be modeled by using a set of rate equations. These equations can be written phenomenologically simply by considering the processes through which electron and photon populations inside the laser cavity change with time. The effect of spontaneous-emission noise is included by adding a Langevin-force term. The resulting Langevin rate equations are[3]

$$\dot{P}_m = (G_m - \gamma_m)\, P_m + R_{sp}^{(m)} + F_{mP}(t) \tag{6}$$

$$\dot{\phi}_m = \frac{1}{2}\alpha_m\, (G_m - \gamma_m) + F_{m\phi}(t) \tag{7}$$

$$\dot{N} = I/q - \gamma_e N - \sum_m G_m P_m + F_N(t), \tag{8}$$

where P_m is the number of photons for the m-th longitudinal mode, ϕ_m is the corresponding phase, and N represents the total number of electrons inside the laser cavity. G_m is the photon-generation rate and γ_m is the photon-loss rate. The quantity $R_{sp}^{(m)}$ governs the rate of spontaneous emission into the m-th mode. In the phase equation (7) α_m is the phase-amplitude coupling parameter. It incorporates phase changes that occur invariably whenever the mode amplitude changes. It is also known as the linewidth enhancement factor. In general α_m can be different for different laser modes. In Eq. (8), I is the injection current and γ_e is the total carrier-recombination rate through all processes (except stimulated emission). Typically, γ_e consists of the three terms,

$$\gamma_e = A_{nr} + B_{sp}N + C_A N^2, \tag{9}$$

where A_{nr} is the rate of nonradiate recombination, B_{sp} is the rate of spontaneous electron-hole recombination, and C_A is the rate of Auger recombination.[3]

The Langevin forces $F_{mP}(t)$ and $F_{m\phi}(t)$ have their origin in the process of spontaneous emission whereas $F_N(t)$ originates through shot noise. In Eqs. (6)-(8) they are Markoffian Gaussian random processes and satisfy the general relations

$$\langle F_i(t) \rangle = 0, \tag{10}$$

$$\langle F_i(t)\, F_j(t') \rangle = 2D_{ij}\delta(t-t'), \tag{11}$$

where angle brackets denote ensemble average and D_{ij} is the diffusion coefficient associated with the corresponding noise. D_{ij}'s are generally obtained by evaluating the second moments of the dynamic variables with the help of Eqs. (6)-(8). Their explicit expressions are[4,5]

$$D_{mP,nP} = (R_{sp}^{(m)} P_m)\delta_{mn},\ D_{m\phi,n\phi} = (R_{sp}^{(m)}/4P_m)\delta_{mn},\ D_{mP,n\phi} = 0, \tag{12}$$

$$D_{N,N} = \sum_m R_{sp}^{(m)} P_m + \gamma_e N,\ \ D_{mP,N} = -R_{sp}^{(m)} P_m,\ \ D_{m\phi,N} = 0. \tag{13}$$

Here P_m and N are steady-state *average* values obtained by setting time derivatives to zero in Eqs. (6)-(8).

The Langevin rate equations are capable of describing noise phenomena in semiconductor lasers once we specify the dependence of mode gain G_m on P_m and N. In a simple approach, the gain profile is taken to be parabolic with a linear dependence of the peak gain on the electron population, i.e.,

$$G_m = G_N(N - N_0)\,[1 - (\omega_m - \omega_0)^2/\Delta\omega_g^2] - \sum_n \beta_{mn}P_n \tag{14}$$

where G_N is the gain coefficient, N_0 is the transparency value of the carrier population, ω_m is the frequency of m-th longitudinal mode, ω_0 is the frequency of the gain peak, and $\Delta\omega_g$ is the frequency spread over which the gain is positive. The last term is added phenomenologically to include the effects of gain nonlinearities which lead to a reduction in the gain as the laser power increases.[6] In the case of multimode operation the sum in Eq. (14) is over all laser modes and includes the contribution of both self- and cross-saturation. The origin of gain nonlinearities can be related to many different physical phenomena such as spectral hole-burning, spatial hole-burning, two-photon absorption, and carrier heating. An explicit expression of β_{mn} has been obtained for the case of spectral hole-burning.[6] The fundamental mechanism in that case is intraband gain saturation occurring as a result of a finite intraband relaxation time.[7]

Many semiconductor lasers are designed to operate predominantly in a single longitudinal mode by the use of distributed feedback or coupled-cavity techniques.[1] The noise properties of such lasers can be studied by considering the single-mode rate equations. By dropping the subscript m in Eqs. (6)-(8) for notational simplicity, these equations take the form

$$\dot{P} = (G - \gamma)P + R_{sp} + F_P(t) \quad (15)$$

$$\dot{\phi} = \frac{1}{2}\alpha(G - \gamma) + F_\phi(t) \quad (16)$$

$$\dot{N} = I/q - \gamma_e N - GP + F_N(t), \quad (17)$$

where the gain G from Eq. (14) is given by

$$G = G_N(N - N_0) - \beta P \quad (18)$$

if we assume that the mode coincides with the gain peak. Detuning from the gain peak reduces G_N by a constant amount; Eqs. (15)-(17) remain applicable even in that case. The cavity decay rate γ is inversely related to the photon lifetime τ_p and is given by

$$\gamma = \frac{1}{\tau_p} = v_g(\alpha_{mir} + \alpha_{int}), \quad (19)$$

where v_g is the group velocity, α_{mir} is the mirror loss, and α_{int} is the internal loss due to free-carrier absorption, scattering, etc. It is also customary to introduce a carrier lifetime τ_e by the relation $\tau_e = 1/\gamma_e$. Typically τ_e = 2-3 ns whereas τ_p = 1-2 ps. The gain coefficient G_N is related to device parameters by the relation

$$G_N = \Gamma v_g\, a/V, \quad (20)$$

where V is the active volume, Γ is the confinement factor, and $a = 2\text{-}3 \times 10^{-16}$ cm^2 in most lasers.

4. INTENSITY NOISE

4.1 Relative Intensity Noise (RIN)

Consider a single-mode laser operating continuously at a current $I > I_{th}$, where I_{th} is the threshold current. The steady-state average values P, ϕ, and N can be obtained by neglecting the time derivatives and taking the ensemble average in Eqs. (15)-(17). Fluctuations from the steady state can be obtained by linearizing the rate equations under the assumption of relatively small fluctuations. We then obtain

$$\delta\dot{P} = -\Gamma_P \delta P + (G_N P)\, \delta N + F_P(t)\ , \quad (21)$$

$$\delta\dot{N} = -\Gamma_N \delta N - (G + G_P P)\, \delta P + F_N(t)\ , \quad (22)$$

where $G_P = \partial G/\partial P$. The linear equations (21) and (22) are easily solved in the frequency domain by using the Fourier transform

$$\delta\tilde{P}(\omega) = \int_{-\infty}^{\infty} \delta P(t) \exp(-i\omega t)\, dt \quad (23)$$

together with similar relations for δN, F_P, and F_N. The final result is

$$\delta\tilde{P}(\omega) = \frac{(\Gamma_N + i\omega)\ \tilde{F}_P + (G_N P)\tilde{F}_N}{(\Omega_R + \omega - i\Gamma_R)\ (\Omega_R - \omega + i\Gamma_R)}\ , \tag{24}$$

where

$$\Omega_R = [(G + G_P P)\ G_N P - (\Gamma_N - \Gamma_P)^2/4]^{1/2} \tag{25}$$

$$\Gamma_R = (\Gamma_N + \Gamma_P)/2 \tag{26}$$

$$\Gamma_N = \gamma_e + N(\partial\ \gamma_e/\partial N) + G_N P \tag{27}$$

$$\Gamma_P = R_{sp}/P - G_P P. \tag{28}$$

Physically Ω_R and Γ_R represent the frequency and the damping rate of relaxation oscillations. Their presence in Eq. (24) indicates that intensity fluctuations occurring as a result of spontaneous emission are affected by relaxation oscillations whose role is to establish the steady state. In particular, the intensity noise does not remain white even though the Langevin forces have a white spectrum. Indeed, if we use Eq. (24) in Eqs. (2) and (3) and perform the averaging procedure by using Eq. (11)-(13), the RIN is found to be given by

$$\mathrm{RIN} = \frac{2R_{sp}\ [(\Gamma_N{}^2 + \omega^2) + G_N{}^2P^2\ (1 + \gamma_e N/R_{sp}P)]}{P\ [(\Omega_R{}^2 - \omega^2)^2 + (2\omega\Gamma_R)^2]}. \tag{29}$$

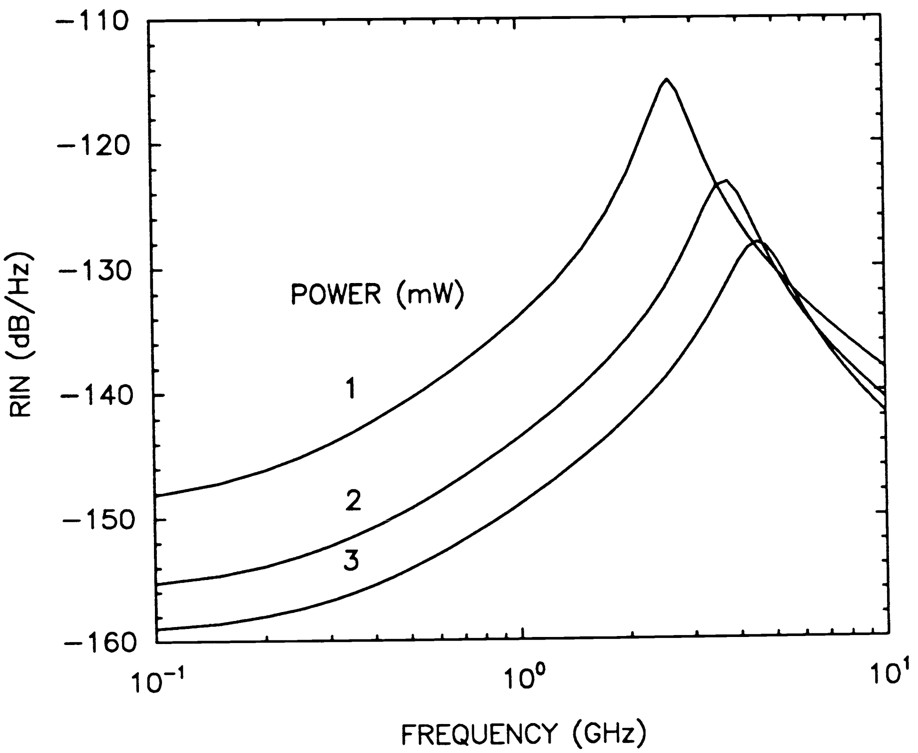

Fig. 1. Intensity-noise spectra at several power levels.

Figure 1 shows the RIN spectra at several output powers by using typical parameter values for a 1.55-μm InGaAsP laser.[8] At a given power, intensity noise is significantly enhanced near Ω_R because of an internal resonance but decreases rapidly for $\omega >> \Omega_R$ as the laser is not able to respond to fluctuations at such high frequencies. Indeed, the semiconductor laser acts as a bandpass filter of bandwidth $\sim \Omega_R$ to spontaneous-emission fluctuations. At a given frequency, RIN decreases with an increase in the laser power. In the low-frequency regime $\omega << \Omega_R$, RIN varies as P^{-3} at low power levels but changes to P^{-1} dependence at high powers.

The solution (24) can be used to calculate the intensity autocorrelation function defined in the normalized form by

$$C_{pp}(\tau) = \langle\delta P(t+\tau)\,\delta P(t)\rangle / P^2, \qquad (30)$$

and related to $\delta\tilde{P}(\omega)$ by the relation

$$C_{pp}(\tau) = \frac{1}{2\pi P^2}\int_{-\infty}^{\infty} \langle|\delta\tilde{P}(\omega)|^2\rangle \exp(i\omega\tau)d\omega. \qquad (31)$$

The integration in Eq. (31) can be performed by using the method of contour integration. A straightforward calculation yields the following expression:[8]

$$C_{pp}(\tau) = \frac{R_{sp}}{P}\left[\frac{\exp(-\Gamma_R\tau)}{2\Gamma_R}\,\mathrm{Re}\left(\frac{\Omega_R{}^2 + \Gamma_e{}^2 - \Gamma_R{}^2 + 2i\Omega_R\Gamma_R}{\Omega_R\,(\Omega_R + i\Gamma_R)}\exp(i\Omega_R\,\tau)\right)\right]. \qquad (32)$$

where $\Gamma_e^2 = \Gamma_N^2 + G_N^2\ P^2\,(1 + \gamma_e N/R_{sp}P)$. This equation shows that the time dependence of $C_{pp}(\tau)$ is governed by relaxation oscillations. In particular, the intensity autocorrelation function exhibit oscillations at the frequency Ω_R and decays to zero with a time constant Γ_R. As seen from Eq. (30), $C_{pp}(0)$ is a measure of the noise variance $\sigma_p{}^2$. More explicitly,

$$\frac{\sigma_p{}^2}{P^2} = C_{pp}(0) = \frac{R_{sp}}{2\Gamma_R P}\left(1 + \frac{\Gamma_e^2}{\Omega_R{}^2 + \Gamma_R{}^2}\right) = \frac{R_{sp}}{2\Gamma_R P} \qquad (33)$$

if we assume that $\Gamma_e << \Omega_R$. This is a remarkably simple expression for the noise variance. For a constant Γ_R, σ_p/P decreases as $P^{-1/2}$ as expected from the discussion in Sec. 2. However, Eqs. (26)-(28) show that Γ_R is not constant and changes significantly with the photon population. When this intensity dependence of Γ_R is taken into account, the relative noise level σ_p/P is given by

$$\frac{\sigma_p}{P} = \left[1 + \frac{\gamma_e P}{R_{sp}} + \frac{(G_N - G_p)P^2}{R_{sp}}\right]^{-1/2}, \qquad (34)$$

where we assumed $\partial\gamma_e/\partial N = 0$. Near or below threshold ($P < 100$), $\sigma_p/P = 1$ indicating that the noise variance is equal to the mean. This is expected since a laser near or below threshold is like a thermal source with an exponential distribution for the mode intensity. In the above-threshold regime intensity fluctuations decrease and $\sigma_p/P < 1$. For a semiconductor laser operating at few milliwatts ($P > 10^5$) the last term in Eq. (34) dominates, and intensity noise decreases as $1/P$ according to the relation

$$\frac{\sigma_p}{P} = \left(\frac{R_{sp}}{G_N + \beta}\right)^{1/2}\frac{1}{P} = \left(\frac{R_{sp}}{\beta}\right)^{1/2}\frac{1}{P} \qquad (35)$$

where we used Eq. (18) to obtain $G_p = \partial G/\partial P = -\beta$ together with $\beta >> G_N$, a condition satisfied in practice. Equation (35) stresses the role of gain nonlinearities in stabilizing the intensity fluctuations. However, it is valid as long as Eq.

(18) is valid. Recent work on intraband gain saturation indicates that a more appropriate form of the nonlinear gain is given by[7,9]

$$G = \frac{G_N (N - N_0)}{\sqrt{1 + P/P_s}}, \tag{36}$$

where P_s is the saturation photon number with typical values in the range 10^6 to 10^7. Equation (35) is then replaced by

$$\frac{\sigma_p}{P} = \left(\frac{2n_{sp}}{P_s}\right)^{1/2} \frac{(1 + P/P_s)^{3/4}}{P/P_s}, \tag{37}$$

where we used $R_{sp} = n_{sp}\gamma$. By using $n_{sp} = 2$ and $P_s = 4 \times 10^6$ as representative values we find that $\sigma_p/P \sim 10^{-3}$ at high intensities ($P \sim P_s$) but is expected to be ~1% under typical operating conditions.

4.2 Mode-Partition Noise (MPN)

In the case of multimode semiconductor lasers, different modes may have large intensity fluctuations in such a way that the total intensity is relatively constant. This phenomenon is called MPN and has its origin in the fact that the same supply of electrons provides gain for all modes. Since the analysis is quite complicated in the general case, we focus our attention on the two-mode case. This case is of practical interest as many single-mode semiconductor lasers operate in such a way that the main mode carries most of its output power while a small fraction is shared by a side mode. By using Eqs. (6)-(8) with the notation $P = P_1$ and $S = P_2$ for the main and side-mode intensities, we obtain

$$\dot{P} = (G_1 - \gamma_1)P + R_{sp} + F_p(t) \tag{38}$$

$$\dot{S} = (G_2 - \gamma_2)S + R_{sp} + F_s(t) \tag{39}$$

$$\dot{N} = I/q - \gamma_e N - (G_1 P + G_2 S) + F_N(t), \tag{40}$$

where G_1 and G_2 are obtained from Eq. (14) and are given by

$$G_1 = G_N (N - N_0) - \beta_{11}P - \beta_{12}S \tag{41}$$

$$G_2 = G_N (N - N_0) - \beta_{22}S - \beta_{21}P - \delta G. \tag{42}$$

Here δG is the reduction in the side-mode gain due to gain rolloff. The rate of spontaneous emission R_{sp} is assumed to be the same for both modes. This is justified in practice since the mode spacing is much smaller than the width of the spontaneous-emission spectrum.

Equations (38)-(40) can be linearized around the steady state and the intensity noise for the main mode and the side mode can be obtained by following the procedure of Sec. 4.1. The RIN spectrum depends on the relative mode intensities S/P. Figure 2 shows the RIN spectra for the main mode[8] for three value of S/P in the range 0.1 to 10^{-5}. The new feature is the dramatic increase in the RIN for low frequencies ($\omega << \Omega_R$) when $S/P \geq 0.01$. The RIN for the total intensity P + S, however, remains small. The RIN spectrum for the total intensity nearly coincides with the curve for $S/P = 10^{-5}$ in Fig. 2 indicating that total intensity fluctuations are well described by the single-mode theory of Sec. 4.1. This behavior can be understood only if fluctuations in the two modes are anti-correlated. For this purpose, we need to evaluate the cross-correlation function defined by

$$C_{ps}(\tau) = <\delta P(t+\tau)\,\delta S(t)>/PS. \qquad (43)$$

It is possible to obtain a closed-form expression for $C_{ps}(\tau)$ in terms of the relaxation-oscillation parameters.[8] It is found to be negative for all values of τ as expected for anticorrelated modes. It is useful to define the mode-partition coefficient k by using

$$k^2 = -C_{ps}(0). \qquad (44)$$

For $\Omega_R >> \Gamma_R$, k^2 is approximately given by

$$k^2 = \frac{R_{sp}}{P}\left(\frac{1}{\Gamma_s} - \frac{1}{\Gamma_p}\right), \qquad (45)$$

where

$$\Gamma_s = R_{sp}/S + \beta_{22}S,\ \ \Gamma_p = R_{sp}/P + \beta_{11}P. \qquad (46)$$

For a weak side mode, $k^2 = S/P$. Typically, $k < 0.1$ for a laser with at least 20-dB mode-suppression ratio. However, k can approach 1 for multimode lasers with nearly equal mode powers.

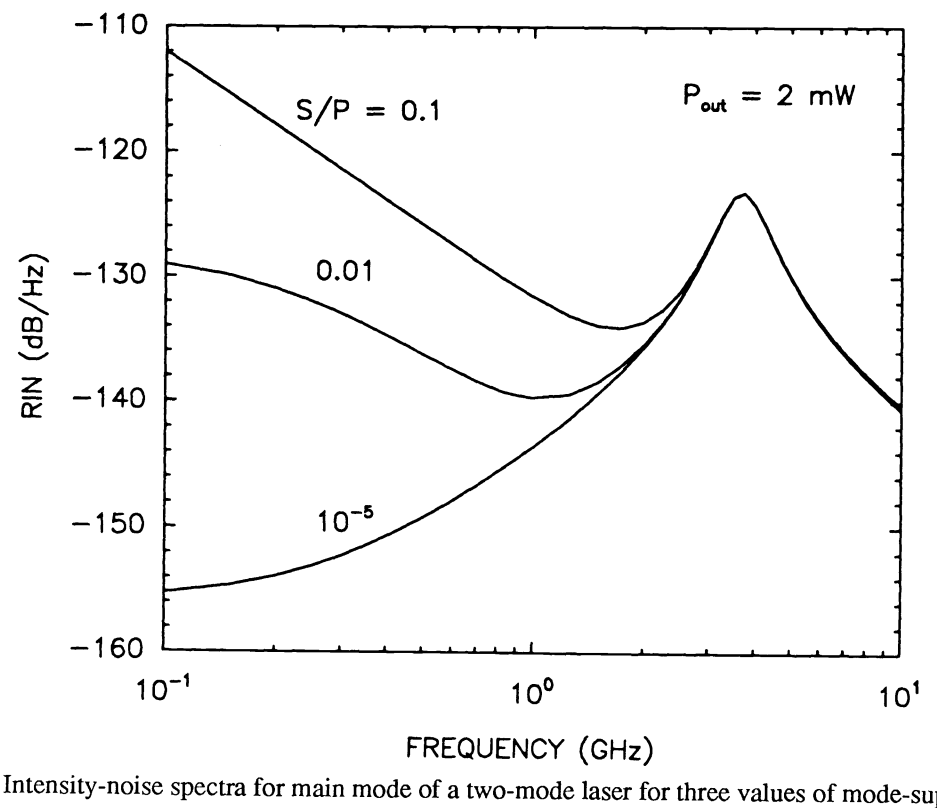

Fig. 2 Intensity-noise spectra for main mode of a two-mode laser for three values of mode-suppression ratio.

4.3 Impact on Optical Communication Systems

The performance of optical communication systems depends on the signal-to-noise ratio (SNR) of the received signal. Many noise sources contribute to the SNR. Some of them originate at the receiver (thermal noise, shot noise, and amplifier noise) while others originate at the transmitter or during propagation inside the fiber. Both RIN and MPN contribute to the SNR. For digital systems the RIN of semiconductor lasers is not a limiting factor since its contribution is well below those of other noise sources. It can, however, effect the performance of subcarrier-multiplexed analog systems, particularly in the presence of reflection feedback that can increase RIN by 10-20 dB.[10]

The most severe impact of intensity noise on lightwave systems is through MPN. MPN would be harmless in the absence of fiber dispersion since all modes would then remain synchronized, and fluctuations in individual mode intensities would not be seen by the receiver. In practice, modes are dispersed because of their different group velocities inside the fiber. Such desynchronization leads to intersymbol interference and a decrease in the SNR at the receiver. MPN is a major limiting factor for high-performance lightwave systems operating at bit rates exceeding 1 Gb/s. For a multimode semiconductor laser with a Gaussian spectrum of rms width σ, the MPN-induced noise is given by[11]

$$\frac{\sigma_{MPN}}{P_{TOT}} = \frac{k}{\sqrt{2}} \{ 1 - \exp[-(\pi BLD\sigma)^2] \} , \tag{47}$$

where B is the bit rate, L is the fiber length, D is the dispersion parameter, and k is given by Eq. (44). For k in the range 0.5-1, D must be reduced below 1 ps/(km.nm) for B = 1.7 Gb/s and L = 50 km in order to achieve a bit-error rate of less than 10^{-9}. This can be achieved by operating the lightwave system near the zero-dispersion wavelength. For communication systems operating near 1.55 μm, D is generally quite large [D = 15-18 ps/(km.nm)] for conventional fibers. It is then necessary to use semiconductor lasers which operate predominantly in a single longitudinal mode. MPN, of course, vanishes when the laser is strictly single mode. In practice, such lasers have one or more side modes whose power is lower by 30 dB or more compared with the main mode under cw operation. MPN is found to be a limiting factor even for such lasers. The reason is that side modes, although suppressed under cw operation, can occasionally turn on to significant power levels under direct modulation.

5. PHASE NOISE

As discussed in Sec. 2 spontaneous emission also leads to phase fluctuations in semiconductor lasers. Phase noise is not important for direct-detection lightwave systems since only intensity is used for demodulation of the received signal. However, it plays a critical role for coherent communication systems.

5.1 Phase Variance

Consider a single-mode laser whose dynamical response is governed by Eqs. (15)-(17). We can obtain $\delta\tilde{\phi}(\omega)$ by following the procedure outlined in Sec. 4.1, where $\delta\tilde{\phi}(\omega)$ is the Fourier transform of phase fluctuation $\delta\tilde{\phi}(t)$. If we define the phase difference

$$\Delta\phi(\tau) = \delta\phi(t+\tau) - \delta\phi(t), \tag{48}$$

the variance of $\Delta\phi(\tau)$ is obtained by using[3]

$$<\Delta\phi^2(\tau)> = \frac{1}{\pi} \int_{-\infty}^{\infty} <|\delta\tilde{\phi}(\omega)|^2> (1 - \cos\omega\tau)\, d\omega. \tag{49}$$

A straightforward calculation yields the following expression for the phase variance:[5]

$$\langle\Delta\phi^2(\tau)\rangle = \frac{R_{sp}}{2P}\left[(1+\alpha^2 b)\tau + \frac{\alpha^2 b[\cos(3\delta) - e^{-\Gamma_R\tau}\cos(\Omega_R\tau - 3\delta)]}{2\Gamma_R\cos\delta}\right], \tag{50}$$

where

$$b = \Omega_R/(\Omega_R^2 + \Gamma_R^2)^{1/2}, \;\; \delta = \tan^{-1}(\Gamma_R/\Omega_R), \tag{51}$$

and Ω_R and Γ_R are defined by Eqs. (25) and (26). The phase variance consists of a linear term and a damped oscillatory term that has its origin in relaxation oscillations. Typically $\langle\Delta\phi^2\rangle > 1$ for $\tau > 1$ ns, indicating that phase fluctuations can be detrimental to low bit-rate lightwave systems operating below 1 Gb/s. This is opposite to MPN which is critical for high bit-rate lightwave systems.

5.2 Laser Linewidth

Phase fluctuations manifest through a broadening of the field spectrum defined by Eq. (4). If we substitute Eq. (5) in Eq. (4) and neglect intensity fluctuations, we obtain

$$S_E(\omega) = P\int_{-\infty}^{\infty} \langle\exp(i\Delta\phi)\rangle \exp[-i(\omega-\omega_0)\tau]d\tau. \tag{52}$$

If $\Delta\phi$ is assumed to be a Gaussian process, $\langle\exp(i\Delta\phi)\rangle = \exp(-\langle\Delta\phi^2\rangle/2)$, where $\langle\Delta\phi^2\rangle$ is given by Eq. (50). The field spectrum is then found to consist of a dominant central peak located at ω_0 and multiple satellite peaks at $\omega = \omega_0 \pm m\Omega_R$, where m is an integer. The amplitude of satellite peaks is typically below 1% of the main peak. The laser linewidth $\Delta\nu$ is generally defined as the full-width at half maximum (FWHM) of the main peak and is given by[5]

$$\Delta\nu = \frac{R_{sp}}{4\pi P}(1+\alpha^2). \tag{53}$$

The term proportional to α^2 results from a coupling between the intensity and phase fluctuations. Its presence enhances the linewidth by a factor $1 + \alpha^2$ compared with the value expected from phase fluctuations alone. For this reason α is referred to as the linewidth enhancement factor.[5]

Eq. (53) predicts that $\Delta\nu$ should decrease as 1/P with an increase in the output power. Although such an inverse dependence on P has been observed experimentally for low powers, the linewidth saturates to a constant value at high power levels and may even exhibit a rebroadening. This behavior is referred to as linewidth saturation and mechanisms such as 1/f noise, spatial hole-burning, and spectral hole-burning have been proposed to explain it. A fundamental mechanism is intraband gain saturation. It results in a reduction in the optical gain at high operating powers as seen in Eq. (36). By following the above procedure with this form of the nonlinear gain the linewidth $\Delta\nu$ is found to be saturated to a value $\Delta\nu_s$ in the range 1-10 MHz for typical semiconductor lasers.[12] It can also exhibit rebroadening when the operating wavelength does not coincide with the gain peak. Figure 3 shows $\Delta\nu/\Delta\nu_s$ as a function of P/P_s. The parameter β is a measure of detuning from the gain peak and is negative for lasers operating on the low-frequency side of the gain spectrum. Both saturation and rebroadening of the laser linewidth in Fig. 3 occur as a result of self saturation of the mode gain at high operating powers. Cross saturation can also lead to a qualitatively similar behavior if the main mode is accompanied by a weak side mode.[13,14] Recent calculations[14] show that the laser noise is very sensitive to the relative strengths of self and cross saturation in such a two-mode semiconductor laser.[8] In particular, the main-mode linewidth can increase by two orders of magnitude even for lasers whose side modes are suppressed by 20 dB or more.

5.3 Impact on Optical Communication Systems

Phase noise is a limiting factor for coherent communication systems employing homodyne or heterodyne detection.[15] For a system operating at the bit rate B, the ratio $\Delta\nu/B$ should be <<1 in order to ensure an acceptable bit error rate (typically $<10^{-9}$). The maximum tolerable value of $\Delta\nu/B$ depends on the modulation format. For the case of

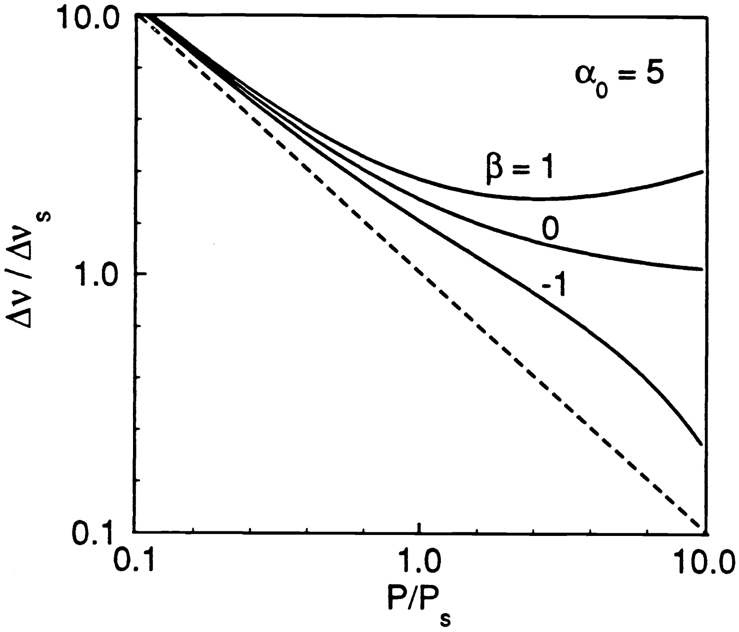

Fig. 3 Variation of laser linewidth with output power. Dashed line shows the expected behavior in the absence of intraband gain saturation.

phase-shift keying (PSK) $\Delta\nu/B$ is required to be typically below 5×10^{-3}. On the other hand, $\Delta\nu/B$ can be as large as 0.2 for amplitude or frequency-shift keying if asynchronous detection is used. Since $\Delta\nu$ is in the range 10-100 MHz under typical operating conditions, it is generally too large for PSK applications particularly at low bit rates below 1 Gb/s. Considerable effort has been directed to design semiconductor lasers whose linewidth is below 1 MHz. The linewidth $\Delta\nu$ is known to be sensitive to external feedback and can be reduced by several orders of magnitude by this technique. Such lasers are often called external-cavity semiconductor lasers. Multisection semiconductor lasers with distributed bragg reflectors[2] also can be designed to have linewidths ~1 MHz.

6. CONCLUDING REMARKS

This paper has discussed intensity and phase fluctuations in the output of a semiconductor laser and their effect on the performance of optical communication systems. The attention was mainly focused on the case of cw operation of the laser. In practice, the laser output is directly modulated by varying the injection current. Many other noise issues then become important. Some of them are the timing jitter in switching of the optical pulse,[16,17] transient intensity and phase fluctuations,[18] and occasional turn-on of a side mode in a distributed feedback laser designed to operate in a single-longitudinal mode under cw operation.[19,20] The last issue is of critical importance since it leads to an unacceptable bit-error rate of many lightwave systems. Its effect can be reduced by biasing the laser above threshold so that spontaneous emission becomes less important between successive optical pulses. An analysis of such transient effects often requires a numerical solution of the stochastic rate equations of Sec. 3.

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