Polarization Phenomena in Nonlinear Optical Fibers

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Introduction

- Nonlinear optical effects have been studied since 1962 and have found applications in many branches of optics.
- Nonlinear interaction length is limited in bulk materials because of tight focusing and diffraction of optical beams.
- Much longer interaction lengths become feasible in optical wave-guides, which confine light through total internal reflection.
- Optical fibers allow interaction lengths of meters and even > 1 km.
- The advent of Nonlinear Fiber optics during the 1970s has led to many advances, supercontinuum generation being a recent example.
- Even though most nonlinear phenomena are polarization dependent, polarization effects are often ignored.

Major Nonlinear Effects

- Self-Phase Modulation (SPM)
- Cross-Phase Modulation (XPM)
- Four-Wave Mixing (FWM)
- Stimulated Brillouin Scattering (SBS)
- Stimulated Raman Scattering (SRS)

Origin of nonlinearity in silica fibers

- Silica glass exhibits isotropic behavior ($\chi^{(2)} = 0$).
- All nonlinear effects result from third-order susceptibility $\chi^{(3)}$.
- Imaginary part of $\chi^{(3)}$, responsible for two-photon absorption, is negligible for silica fibers.

Third-order Nonlinear Susceptibility

- Maxwell's equations leads to the wave equation

$$\nabla^2\mathbf{E} - \frac{1}{c^2}\frac{\partial^2\mathbf{E}}{\partial t^2} = \mu_0\frac{\partial^2\mathbf{P}^{(1)}}{\partial t^2} + \mu_0\frac{\partial^2\mathbf{P}^{(3)}}{\partial t^2}.$$

- The general form of third-order nonlinear polarization is quite complicated. Its ith component ($i = 1,2,3$ for x, y, z directions) is

$$P_i^{(3)}(t) = \varepsilon_0 \sum_{j,k,l} \iiint_{-\infty}^{\infty} \chi_{ijkl}^{(3)}(t_1,t_2,t_3)E_j(t-t_1)E_k(t-t_2))E_l(t-t_3)dt_1dt_2dt_3.$$

- Here $\chi^{(3)}(t_1,t_2,t_3)$ is a fourth-rank tensor that vanishes for negative values of its arguments to ensure causality.
- The tensorial nature of $\chi^{(3)}$ makes the situation quite complicated.
- It can be simplified considerably when the nonlinear medium responds instantaneously. This limit is known as the Kerr nonlinearity.

Kerr nonlinearity

- Kerr nonlinearity refers to a fast-responding nonlinear medium.
- If the dominant nonlinear response comes from electrons that can respond at sub-femtosecond time scales, we obtain

$$P_i^{(3)}(t) = \varepsilon_0 \sum_{j=1}^{3} \sum_{k=1}^{3} \sum_{l=1}^{3} \chi_{ijkl}^{(3)} E_j(t) E_k(t) E_l(t).$$

- This form excludes stimulated Raman scattering that results from the response of silica molecules to the incident pump.
- This form also neglects any frequency dependence of $\chi_{ijkl}^{(3)}$.
- If a single linearly polarized field is propagating inside the fiber, only a single term survives in the triple sum:

$$P_x^{(3)}(t) = \varepsilon_0 \chi_{xxxx}^{(3)} E_x(t) E_x(t) E_x(t).$$

Kerr nonlinearity (cont.)

- The scalar form of $P_x^{(3)}(t)$ is often used in practice.
- If we use $E_x(t) = \frac{1}{2}[E(t)e^{-i\omega_0 t} + c.c.]$, we obtain

$$P_x^{(3)}(t) = \chi_{xxxx}^{(3)} \frac{\varepsilon_0}{8}\left[E^3(t)e^{-3i\omega_0 t} + 3|E(t)|^2 E(t)e^{-i\omega_0 t} + c.c.\right].$$

- Using $P_x^{(3)}(t) = \frac{1}{2}[P(t)e^{-i\omega_0 t} + c.c.]$ with $P(t) = \varepsilon_0 \varepsilon_{\mathrm{NL}} E(t)$ and neglecting the third-harmonic terms, we obtain

$$\varepsilon_{\mathrm{NL}} = \frac{3}{4}\chi_{xxxx}^{(3)}|E(t)|^2.$$

- Using $\varepsilon = \varepsilon_{\mathrm{L}} + \varepsilon_{\mathrm{NL}} = (n + \Delta n)^2$, we obtain

$$\Delta n = \frac{\varepsilon_{\mathrm{NL}}}{2n} = \frac{3}{8n}\chi_{xxxx}^{(3)}|E(t)|^2 = n_2|E(t)|^2.$$

- The Kerr coefficient n_2 plays an important role in nonlinear optics.

Self-Phase Modulation

- Since the refractive index depends on the local intensity $I(t) = |E(t)|^2$, an optical pulse modulates its own phase with propagation.
- This is the well-known phenomenon of self-phase modulation (SPM), first observed in 1967 [F. Shimizu, PRL **19**, 1097 (1967)].
- Using $\phi = k_0(n + n_2 I)L$, with $k_0 = 2\pi/\lambda$, the nonlinear part of the phase shift in a fiber of length L is given by

$$\phi_{\mathrm{NL}}(t) = (2\pi/\lambda)n_2 I(t)L = \gamma P(t)L.$$

- The nonlinear parameter $\gamma = 2\pi n_2/(\lambda A_{\mathrm{eff}})$, where A_{eff} is the effective mode area, governs the extent of SPM.
- Time dependence of ϕ_{NL} indicates that pulses become chirped inside a nonlinear fiber, resulting in SPM-induced spectral broadening.

Nonlinear Schrödinger Equation

- In the presence of dispersive effects, evolution of pulse envelope along the fiber is governed by $[\mathbf{E}(z,t) = \hat{\mathbf{x}}A(z,t)\exp(i\beta_0 z - i\omega_0 t)]$

$$i\frac{\partial A}{\partial z} - \frac{\beta_2}{2}\frac{\partial^2 A}{\partial t^2} + \gamma|A|^2 A = 0.$$

- Dispersive effects within the fiber included through the parameter β_2 that takes negative values in the case of anomalous dispersion.

- If we ignore the dispersive effects, solution can be written as

$$A(L,t) = A(0,t)\exp(i\phi_{\rm NL}), \quad \text{where} \quad \phi_{\rm NL}(t) = \gamma|A(0,t)|^2 L.$$

- Nonlinear phase shift depends on the shape of input pulses.

- In the case of anomalous dispersion ($\beta_2 < 0$), SPM leads to the formation of optical solitons.

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Polarization Effects

- When the incident field is elliptically polarized, we need to include both the x and y components:

$$\mathbf{E}(t) = \frac{1}{2}(\hat{\mathbf{x}}E_x + \hat{\mathbf{y}}E_y)e^{-i\omega_0 t} + \text{c.c.}$$

- Nonlinear polarization is again calculated using the vector relation

$$\mathbf{P}^{(3)}(t) = \varepsilon_0 \chi^{(3)} \vdots \mathbf{E}(t)\mathbf{E}(t)\mathbf{E}(t).$$

- For an isotropic medium such as silica glass $\chi^{(3)}$ has the form

$$\chi^{(3)}_{ijkl} = \chi^{(3)}_{xxyy}\delta_{ij}\delta_{kl} + \chi^{(3)}_{xyxy}\delta_{ik}\delta_{jl} + \chi^{(3)}_{xyyx}\delta_{il}\delta_{jk}.$$

- Using it, we obtain $\mathbf{P}(t) = \frac{1}{2}[(\hat{\mathbf{x}}P_x + \hat{\mathbf{y}}P_y)e^{-i\omega_0 t} + \text{c.c.}]$ with

$$P_i = \frac{3\varepsilon_0}{4} \sum_{j=x,y} \left(\chi^{(3)}_{xxyy} E_i E_j E_j^* + \chi^{(3)}_{xyxy} E_j E_i E_j^* + \chi^{(3)}_{xyyx} E_j E_j E_i^* \right).$$

Polarization Effects (cont.)

- It appears P_x and P_y depend on three different elements of $\chi^{(3)}$.
- For an isotropic medium, $\chi^{(3)}_{xxyy} = \chi^{(3)}_{xyxy} = \chi^{(3)}_{xyyx} = \frac{1}{3}\chi^{(3)}_{xxxx}$.
- Using it, P_x and P_y are found to be

$$P_x = \frac{3\varepsilon_0}{4}\chi^{(3)}_{xxxx}\left[\left(|E_x|^2 + \frac{2}{3}|E_y|^2\right)E_x + \frac{1}{3}(E_x^* E_y)E_y\right],$$

$$P_y = \frac{3\varepsilon_0}{4}\chi^{(3)}_{xxxx}\left[\left(|E_y|^2 + \frac{2}{3}|E_x|^2\right)E_y + \frac{1}{3}(E_y^* E_x)E_x\right].$$

- Writing $P_j = \varepsilon_0 \varepsilon_j^{\mathrm{NL}} E_j$ and using $\varepsilon_j = \varepsilon_j^L + \varepsilon_j^{\mathrm{NL}} = (n_j^L + \Delta n_j)^2$, we obtain the Kerr nonlinear response in the form

$$\Delta n_x = n_2\left(|E_x|^2 + \frac{2}{3}|E_y|^2\right), \qquad \Delta n_y = n_2\left(|E_y|^2 + \frac{2}{3}|E_x|^2\right).$$

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Cross-Polarization Modulation

- Nonlinear index change $\Delta n_x = n_2(|E_x|^2 + \frac{2}{3}|E_y|^2)$ has, as expected, a SPM-type contribution (first term).
- Second term indicates that Δn_x has a contribution from the polarization component in the y direction; $\Delta n_y = n_2(|E_y|^2 + \frac{2}{3}|E_x|^2)$ also has a similar contribution.
- The second contribution leads to a nonlinear phenomenon called **cross-polarization modulation**.
- If light is elliptically polarized such that $P_y \neq P_x$, the refractive indices seen by the x and y polarized components are different.
- This is the well-known phenomenon of **nonlinear birefringence**.
- It has many practical applications. For example, it can be used for pulse shaping and to make mode-locked fiber lasers.

Nonlinear Polarization Rotation

- Consider an elliptically polarized beam launched into an optical fiber:

$$\boldsymbol{E}(\boldsymbol{r},t) = F(x,y)[\hat{\boldsymbol{x}}A_x(z,t)e^{i\beta_x z} + \hat{\boldsymbol{y}}A_y(z,t)e^{i\beta_y z}]e^{-i\omega_0 t}.$$

- Two polarization components develop different nonlinear phase shifts:

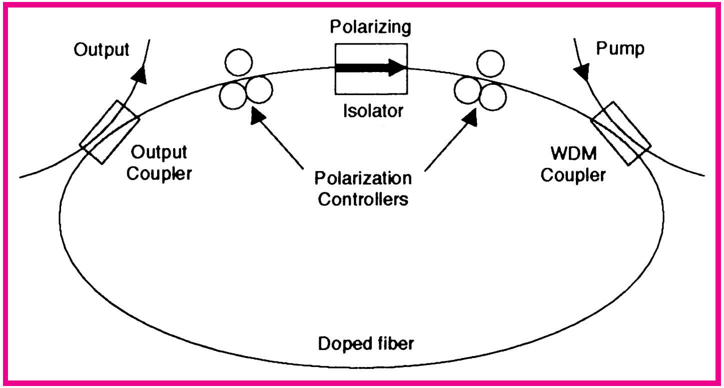
$$\phi_x = \gamma\left(|A_x|^2 + \frac{2}{3}|A_y|^2\right)L, \qquad \phi_y = \gamma\left(|A_y|^2 + \frac{2}{3}|A_x|^2\right)L.$$

- State of polarization rotates on the Pontcaré sphere if the relative phase difference between the two components is finite:

$$\Delta\phi_{\mathrm{NL}} = \phi_x - \phi_y = \frac{1}{3}\gamma L(A_x|^2 - |A_y|^2).$$

- This rotation is due to nonlinear birefringence and is known as the **Nonlinear Polarization Rotation** (NPR).

NPR-Induced Mode Locking



- A ring cavity with a polarizing isolator inside it is pumped to make a mode-locked fiber laser.
- A polarization controller after isolator ensures elliptical polarization.
- NPR changes the state of polarization such that it is linear near pulse center but remains elliptical in the wings.
- Pulse wings experience higher losses at the polarizing isolator.

Coupled NLS Equations

- In the presence of dispersive effects, evolution of pulse envelope along the fiber is governed by two coupled NLS equations:

$$\frac{\partial A_x}{\partial z} + \beta_{1x}\frac{\partial A_x}{\partial t} + \frac{i\beta_2}{2}\frac{\partial^2 A_x}{\partial t^2} + \frac{\alpha}{2}A_x$$
$$= i\gamma\left(|A_x|^2 + \frac{2}{3}|A_y|^2\right)A_x + \frac{i\gamma}{3}A_x^* A_y^2 \exp(-2i\Delta\beta z),$$
$$\frac{\partial A_y}{\partial z} + \beta_{1y}\frac{\partial A_y}{\partial t} + \frac{i\beta_2}{2}\frac{\partial^2 A_y}{\partial t^2} + \frac{\alpha}{2}A_y$$
$$= i\gamma\left(|A_y|^2 + \frac{2}{3}|A_x|^2\right)A_y + \frac{i\gamma}{3}A_y^* A_x^2 \exp(2i\Delta\beta z).$$

- Here $\Delta\beta = \beta_{0x} - \beta_{0y}$ accounts for linear birefringence of the fiber.
- The β_1 terms govern polarization-mode dispersion (PMD) resulting from slightly different speeds of the two polarization components.

Cross-Phase Modulation (XPM)

- Situation becomes more complicated when two optical pulses of different wavelengths and polarizations are launched simultaneously.

- When the two incident fields are arbitrarily polarized, we must use

$$\mathbf{E}(t) = \tfrac{1}{2}[\boldsymbol{E}_1 e^{-i\omega_1 t} + \boldsymbol{E}_2 e^{-i\omega_2 t}] + \text{c.c.},$$
$$\mathbf{P}^{(3)}(t) = \tfrac{1}{2}[\boldsymbol{P}_1 e^{-i\omega_1 t} + \boldsymbol{P}_2 e^{-i\omega_2 t}] + \text{c.c.}$$

- In the case of the Kerr nonlinearity, $\boldsymbol{P}_1$ and $\boldsymbol{P}_2$ are found to be

$$\boldsymbol{P}_j = \frac{\varepsilon_0}{4}\chi^{(3)}_{xxxx}[(\boldsymbol{E}_j \cdot \boldsymbol{E}_j)\boldsymbol{E}_j^* + 2(\boldsymbol{E}_j^* \cdot \boldsymbol{E}_j)\boldsymbol{E}_j + 2(\boldsymbol{E}_m^* \cdot \boldsymbol{E}_m)\boldsymbol{E}_j + 2(\boldsymbol{E}_m \cdot \boldsymbol{E}_j)\boldsymbol{E}_m^* + 2(\boldsymbol{E}_m^* \cdot \boldsymbol{E}_j)\boldsymbol{E}_m] \quad (j \neq m).$$

- If we employ the ket notation for Jones vectors, we can use

$$\boldsymbol{E}_j(\boldsymbol{r},t) = F_j(x,y)|A_j(z,t)\rangle \exp(i\beta_j z).$$

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Coupled NLS Equations

- In the case of two optical pulses of different wavelengths and polarizations, the coupled NLS equations become

$$\frac{\partial|A_1\rangle}{\partial z} + \frac{1}{v_{g1}}\frac{\partial|A_1\rangle}{\partial t} + \frac{i\beta_{21}}{2}\frac{\partial^2|A_1\rangle}{\partial t^2} + \frac{\alpha_1}{2}|A_1\rangle = \frac{i\gamma_1}{3}\Big(2\langle A_1|A_1\rangle + 2\langle A_2|A_2\rangle + 2|A_2\rangle\langle A_2| + |A_1^*\rangle\langle A_1^*| + 2|A_2^*\rangle\langle A_2^*|\Big)|A_1\rangle,$$

$$\frac{\partial|A_2\rangle}{\partial z} + \frac{1}{v_{g2}}\frac{\partial|A_2\rangle}{\partial t} + \frac{i\beta_{22}}{2}\frac{\partial^2|A_2\rangle}{\partial t^2} + \frac{\alpha_2}{2}|A_2\rangle = \frac{i\gamma_2}{3}\Big(2\langle A_2|A_2\rangle + 2\langle A_1|A_1\rangle + 2|A_1\rangle\langle A_1| + |A_2^*\rangle\langle A_2^*| + 2|A_1^*\rangle\langle A_1^*|\Big)|A_2\rangle.$$

- Here v_{g1} and v_{g2} are group velocities at the two wavelengths.
- These equations assume that fiber is without any linear birefringence. They can be generalized further to include it.

Pump–Probe Configuration

- If we neglect dispersion and assume that the probe pulse is much weaker than the pump pulse, the coupled NLS equations become

$$\frac{\partial |A_1\rangle}{\partial z} + \frac{1}{L_W}\frac{\partial |A_1\rangle}{\partial \tau} = \frac{i\gamma_1}{3}\Big(2\langle A_1|A_1\rangle + |A_1^*\rangle\langle A_1^*|\Big)|A_1\rangle,$$

$$\frac{\partial |A_2\rangle}{\partial z} = \frac{2i\gamma_2}{3}\Big(\langle A_1|A_1\rangle + |A_1\rangle\langle A_1| + |A_1^*\rangle\langle A_1^*|\Big)|A_2\rangle.$$

- Introducing the normalized Stokes vectors for the pump and probe fields as $\boldsymbol{p} = \langle A_1|\boldsymbol{\sigma}|A_1\rangle/P_0$ and $\boldsymbol{s} = \langle A_2|\boldsymbol{\sigma}|A_2\rangle/P_{20}$, we obtain

$$\frac{\partial \boldsymbol{p}}{\partial \xi} + \mu\frac{\partial \boldsymbol{p}}{\partial \tau} = \frac{2}{3}\boldsymbol{p}_3 \times \boldsymbol{p},$$

$$\frac{\partial \boldsymbol{s}}{\partial \xi} = -\frac{4\omega_2}{3\omega_1}(\boldsymbol{p} - \boldsymbol{p}_3) \times \boldsymbol{s}.$$

- Here $\xi = z/L_{\rm NL}$, $L_{\rm NL} = (\gamma_1 P_0)^{-1}$, and $\mu = L_{\rm NL}/L_W$.

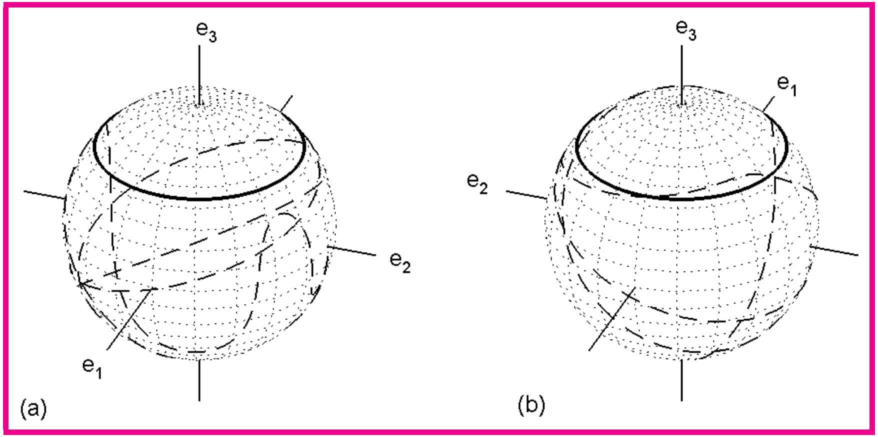
Self-polarization Changes of Pump

- Pump equation is relatively easy to solve and has the solution

$$\boldsymbol{p}(\xi,\tau) = \exp[(2\xi/3)\boldsymbol{p}_3(0,\tau-\mu\xi)\times]\boldsymbol{p}(0,\tau-\mu\xi).$$

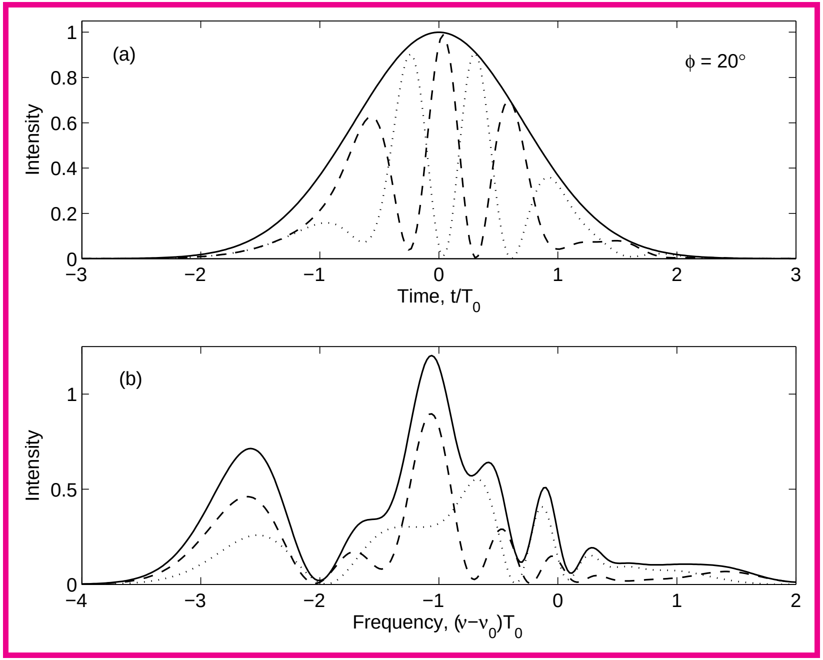
- Stokes vector $\boldsymbol{p}$ rotates on the Poincaré sphere along the vertical axis at a rate $2p_3/3$.
- This nonlinear polarization rotation is due to XPM-induced nonlinear birefringence.
- If the pump is linearly or circularly polarized initially, its state of polarization (SOP) does not change along the fiber.
- For an elliptically polarized pump, SOP changes as the pump pulse propagates through the fiber.
- Since the rotation rate depends on the optical power, different parts of a pump pulse acquire different SOPs.

Pump-Induced Probe Polarization Changes



- Evolution of pump (solid) and probe (dashed) SOP on the Poincaré sphere with time at a distance of $20L_{\rm NL}$.
- Parts (a) and (b) show the front and back of the Poincaré sphere.
- Both pulses are Gaussian in shape and have the same width.
- Pump is elliptically polarized but the probe is linearly polarized at the input end.

Polarization Dependence of the Probe

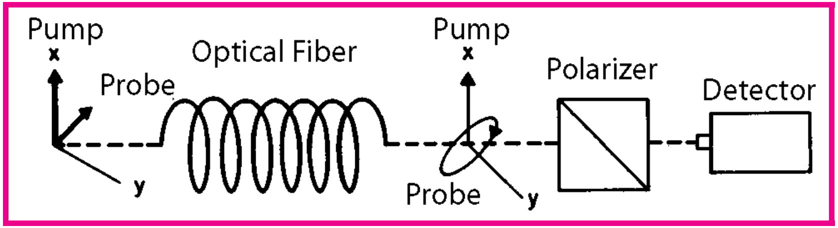


- (a) Shapes and (b) spectra at $z = 20L_{\mathrm{NL}}$ for the x (dashed) and y (dotted) components of the probe pulse. Total intensity is shown by a solid line.

Probe Polarization Dependence (cont.)

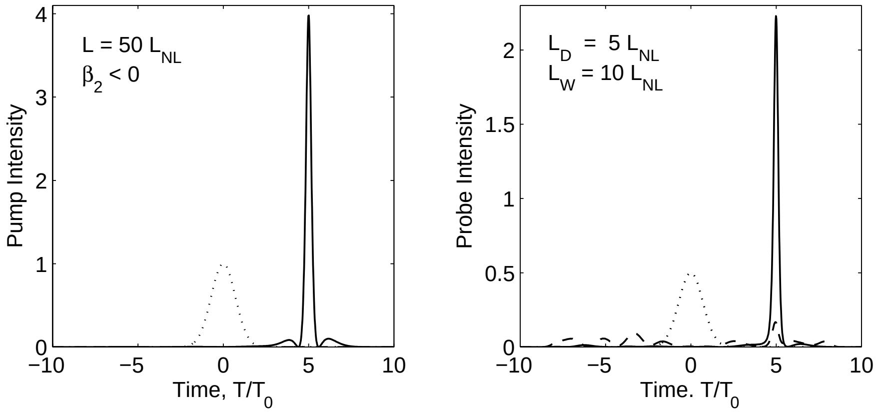
- Probe pulse is linearly polarized at 45° and the pump pulse is elliptically polarized at $z = 0$.
- Different spectral broadening is expected from different XPM-induced phase shifts. They occur even for all input SOPs of the pump.
- Different shapes of the x and y components of the probe pulse occur only for elliptically polarized pump pulses.
- They are related to changes in the SOP of the the pump pulse.
- As the pump SOP evolves, the probe SOP changes across the pulse in a complex manner.
- These results show that the nonlinear interaction of two pulses of different wavelengths exhibits quite complex polarization dynamics inside optical fibers.

Simple Application: Kerr Shutter



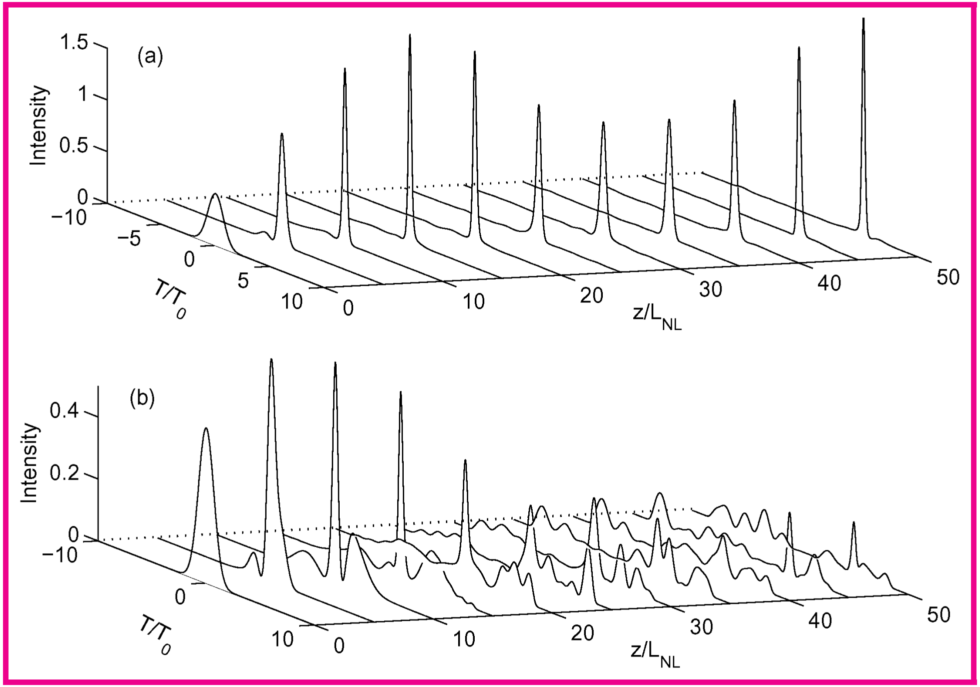
- A cross polarizer at the fiber output blocks probe transmission (polarized at 45°) in the absence of the pump beam.
- When pump is turned on, $n_x > n_y$ for the probe because of pump-induced cross-phase modulation.
- Probe transmissivity depends on the pump intensity and can be controlled simply by changing it.
- In particular, a pulse of suitable energy at the pump wavelength opens the Kerr shutter only during its passage through the fiber.

Dispersion and Soliton Effects



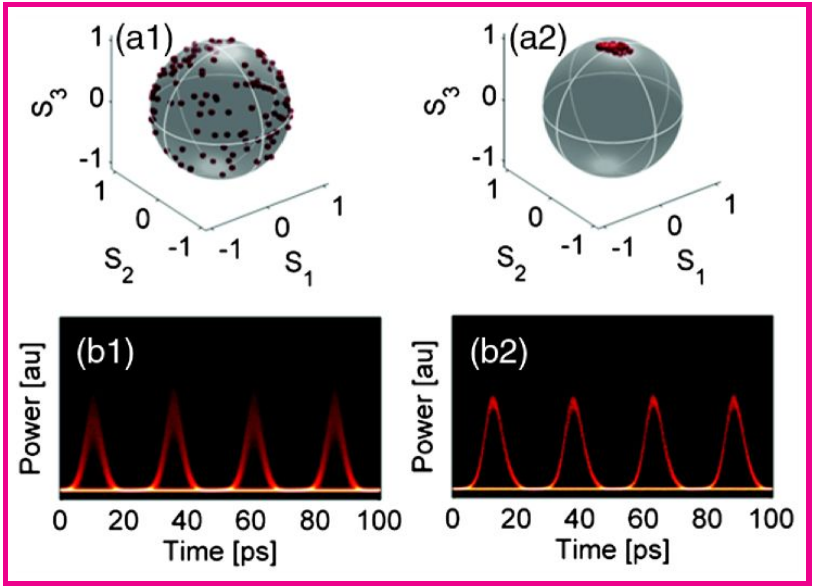
- In the case of anomalous dispersion, pump pulse forms an optical soliton. Probe pulse is trapped by the pump and travels with it.
- Temporal shapes of pump (left) and probe (right) pulses at $z = 0$ (dotted) and $50L_D$ for the x (solid) and y (dashed) components.
- Pump pulse is linearly polarized along the x axis, while the probe is oriented at 45° with respect to it.

Pulse Trapping and Compression



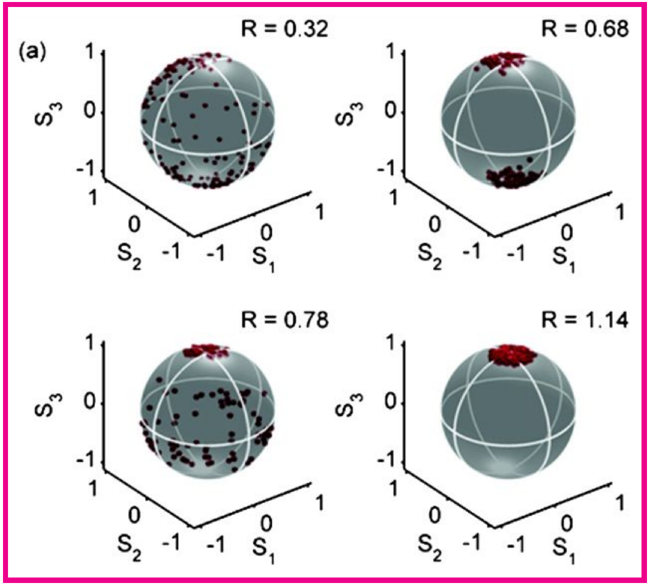
- (a) copolarized and (b) orthogonally polarized components of a probe pulse, polarized at 45° from a linearly polarized pump pulse.

Pump-Induced Polarization Pulling



- A counter-propagating pump can pull probe polarization toward its own SOP [review by Millot and Wabnitz (JOSA B, Nov. 2014)].
- Scrambled SOP of a 40-Gb/s signal became uniformly polarized in the presence of a CW pump inside a 6.2 km long fiber.

Polarization Pulling (cont.)



- Signal can itself act as the pump if a part of the output is reflected and amplified before sending it back into the fiber.
- Figure shows how the SOP of the signal changes in four cases (Millot and Wabnitz, JOSA B, Nov. 2014).

Four-Wave Mixing (FWM)

- FWM involves four waves such that $\omega_1 + \omega_2 = \omega_3 + \omega_4$. In the case of a single pump, $\omega_1 = \omega_2$.

- Total electric field and third-order polarization now have the form

$$\mathbf{E} = \mathrm{Re}\left[\sum_{j=1}^{4} \boldsymbol{E}_j \exp(-i\omega_j t)\right], \qquad \mathbf{P}_{\mathrm{NL}} = \mathrm{Re}\left[\sum_{j=1}^{4} \boldsymbol{P}_j \exp(-i\omega_j t)\right].$$

- In the case of Kerr nonlinearity, $\boldsymbol{P}_1$ and $\boldsymbol{P}_2$ related to the two pumps are found to be (assuming much weaker signal and idler)

$$\boldsymbol{P}_j(\omega_j) = \frac{\varepsilon_0}{4}\chi^{(3)}_{xxxx}\big[(\boldsymbol{E}_j\cdot\boldsymbol{E}_j)\boldsymbol{E}_j^* + 2(\boldsymbol{E}_j^*\cdot\boldsymbol{E}_j)\boldsymbol{E}_j + 2(\boldsymbol{E}_m^*\cdot\boldsymbol{E}_m)\boldsymbol{E}_j + 2(\boldsymbol{E}_m\cdot\boldsymbol{E}_j)\boldsymbol{E}_m^* + 2(\boldsymbol{E}_m^*\cdot\boldsymbol{E}_j)\boldsymbol{E}_m\big] \quad (j \neq m).$$

- First term governs SPM of the two pumps. The remaining terms govern the interaction between two pump waves through XPM.

Four-Wave Mixing (cont.)

- Using the same procedure, $\boldsymbol{P}_3$ and $\boldsymbol{P}_4$ are found to be

$$\boldsymbol{P}_j(\omega_j) = \frac{\epsilon_0}{2}\chi^{(3)}_{xxxx}\Big[(\boldsymbol{E}_1^*\cdot\boldsymbol{E}_1)\boldsymbol{E}_j + (\boldsymbol{E}_1\cdot\boldsymbol{E}_j)\boldsymbol{E}_1^* + (\boldsymbol{E}_1^*\cdot\boldsymbol{E}_j)\boldsymbol{E}_1 + (\boldsymbol{E}_2^*\cdot\boldsymbol{E}_2)\boldsymbol{E}_j + (\boldsymbol{E}_2\cdot\boldsymbol{E}_j)\boldsymbol{E}_2^* + (\boldsymbol{E}_2^*\cdot\boldsymbol{E}_j)\boldsymbol{E}_2 + (\boldsymbol{E}_m^*\cdot\boldsymbol{E}_1)\boldsymbol{E}_2 + (\boldsymbol{E}_m^*\cdot\boldsymbol{E}_2)\boldsymbol{E}_1 + (\boldsymbol{E}_1\cdot\boldsymbol{E}_2)\boldsymbol{E}_m^*\Big].$$

- If we employ the ket notation for Jones vectors, the evolution of signal and idler waves inside an optical fiber is governed by

$$\frac{d|A_3\rangle}{dz} = \frac{2i\gamma}{3}\Big(\langle A_1|A_1\rangle + |A_1\rangle\langle A_1| + |A_1^*\rangle\langle A_1^*| + \langle A_2|A_2\rangle + |A_2\rangle\langle A_2| + |A_2^*\rangle\langle A_2^*|\Big)|A_3\rangle + \frac{2i\gamma}{3}\Big(|A_2\rangle\langle A_1^*| + |A_1\rangle\langle A_2^*| + \langle A_1^*|A_2\rangle\Big)|A_4^*\rangle e^{-i\Delta kz},$$

$$\frac{d|A_4\rangle}{dz} = \frac{2i\gamma}{3}\Big(\langle A_1|A_1\rangle + |A_1\rangle\langle A_1| + |A_1^*\rangle\langle A_1^*| + \langle A_2|A_2\rangle + |A_2\rangle\langle A_2| + |A_2^*\rangle\langle A_2^*|\Big)|A_4\rangle + \frac{2i\gamma}{3}\Big(|A_2\rangle\langle A_1^*| + |A_1\rangle\langle A_2^*| + \langle A_1^*|A_2\rangle\Big)|A_3^*\rangle e^{-i\Delta kz}.$$

- Here $\Delta k = \beta(\omega_3) + \beta(\omega_4) - \beta(\omega_1) - \beta(\omega_2)$ is the phase mismatch.

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Conservation of Angular Momentum

- In addition to the conservation of linear momentum (phase matching), FWM also requires conservation of angular momentum.
- The LCP and RCP states represent photons with angular momenta of $+\hbar$ and $-\hbar$, respectively.
- To describe FWM among arbitrarily polarized optical fields, we decompose the Jones vector of each field as

$$|A_j\rangle = \mathscr{U}_j|\uparrow\rangle + \mathscr{D}_j|\downarrow\rangle.$$

- Creation of idler photons is then governed by the following two equations (assuming perfect phase matching):

$$\frac{d\mathscr{U}_4}{dz} = \frac{4i\gamma}{3}\left[\mathscr{U}_1\mathscr{U}_2\mathscr{U}_3^* + (\mathscr{U}_1\mathscr{D}_2 + \mathscr{D}_1\mathscr{U}_2)\mathscr{D}_3^*\right],$$

$$\frac{d\mathscr{D}_4}{dz} = \frac{4i\gamma}{3}\left[\mathscr{D}_1\mathscr{D}_2\mathscr{D}_3^* + (\mathscr{U}_1\mathscr{D}_2 + \mathscr{D}_1\mathscr{U}_2)\mathscr{U}_3^*\right].$$

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Angular Momentum (cont.)

- Consider one of the two equations:

$$\frac{d\mathscr{U}_4}{dz} = \frac{4i\gamma}{3}\left[\mathscr{U}_1\mathscr{U}_2\mathscr{U}_3^* + \mathscr{U}_1\mathscr{D}_2\mathscr{D}_3^* + \mathscr{D}_1\mathscr{U}_2\mathscr{D}_3^*\right].$$

- Three terms on the right represent three FWM processes that conserve angular momentum.
- First term: Both pumps and the signal are in $\mathscr{U}$ state and produce the idler in the same state.
- Second and third terms: Two pump photons are orthogonally polarized $\mathscr{U}$ and $\mathscr{D}$ such that their total angular momentum is zero.
- To conserve this value, the signal and idlers must also be orthogonally polarized.
- Thus, only signal photons in $\mathscr{D}$ state can produce $\mathscr{U}$ idler photons.

Polarization Dependence of Parametric Gain

- Conservation of angular momentum affects considerably the parametric gain and the efficiency of underlying FWM process.
- Consider again the generation of idler photons in the $\mathscr{U}$ state:

$$\frac{d\mathscr{U}_4}{dz} = \frac{4i\gamma}{3}\left[\mathscr{U}_1\mathscr{U}_2\mathscr{U}_3^* + \mathscr{U}_1\mathscr{D}_2\mathscr{D}_3^* + \mathscr{D}_1\mathscr{U}_2\mathscr{D}_3^*\right].$$

- In the case of single-pump configuration, the two pump photons have the same SOP as they are indistinguishable.
- If the pump is circularly polarized, only the first term can produce idler photons.
- In the case of a linearly polarized pump, all terms can produce idler photons as long as the selection rules are satisfied.
- It is easy to conclude that the parametric gain for a single-pump configuration is always polarization-dependent.

Dual-Pump Configuration

- The use of two distinct pumps makes it possible to make a fiber-optic parametric amplifier (FOPA) whose gain does not depend on the signal's SOP.
- In the dual-pump configuration, the two pumps photons are distinguishable and can be chosen to be orthogonally polarized.
- If the two pumps are in the LCP and RCP states, the FWM process becomes independent of the signal's SOP.
- The FWM process does not depend on the signal's SOP when the two pumps are linearly polarized with orthogonal SOPs.
- The important question to ask is which configuration produces a better FOPA from the standpoint of practical applications.

Dual-Pump Configuration

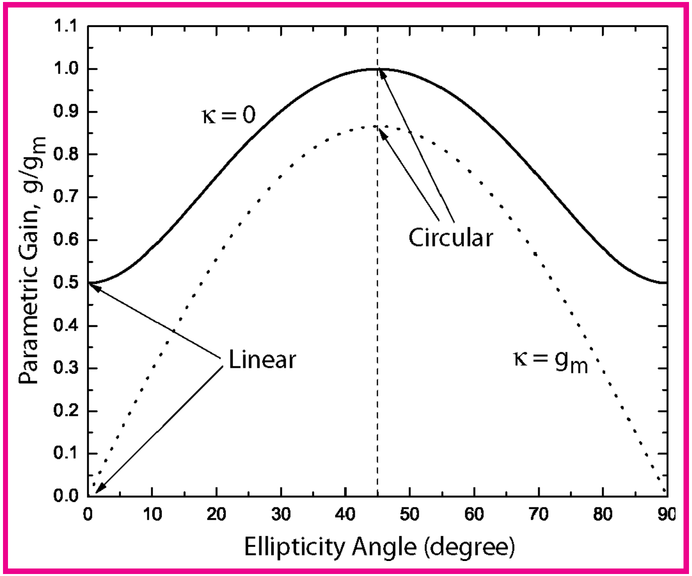
- Vector FWM theory can be used to answer this question..
- For two elliptically but orthogonally polarized pumps, we obtain

$$\frac{d|A_3\rangle}{dz} = \frac{2i\gamma}{3}\sqrt{P_1P_2}e^{-i\Delta kz}(\cos 2\theta\sigma_2 + 2i\sin 2\theta\sigma_0)|A_4^*\rangle,$$
$$\frac{d|A_4\rangle}{dz} = \frac{2i\gamma}{3}\sqrt{P_1P_2}e^{-i\Delta kz}(\cos 2\theta\sigma_2 + 2i\sin 2\theta\sigma_0)|A_3^*\rangle.$$

- Here θ is the ellipticity angle of the pump, σ_0 is a unit matrix, and σ_2 is one of the Pauli matrices.
- Assuming $A_4(0) = 0$ initially, the parametric gain g is found to be

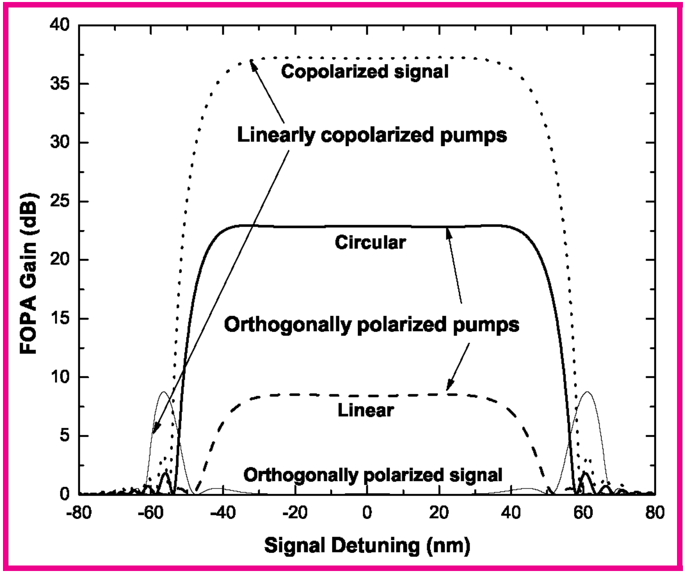
$$g(\theta) = (2\gamma/3)\sqrt{P_1P_2(1+3\sin^2 2\theta) - (3\Delta k/4\gamma)^2}.$$

Polarization Dependence of Parametric Gain



- Parametric gain versus θ plotted for two elliptically polarized pumps with orthogonal SOPs. It peaks for circularly polarized pumps.
- The maximum value used for normalization is $g_m = 4\gamma\sqrt{P_1 P_2}/3$.
- Solid curve assumes perfect phase matching.

Linear versus Circular Polarization



- Gain spectra for four different pumping schemes for a dual-pump FOPA pumped with $P_1 = P_2 = 0.5$ W at 1535 and 1628 nm.
- Circularly polarized pumps provide 23-dB gain that does not depend on signal's SOP.

Stimulated Raman Scattering

- Raman Scattering involves delayed molecular response EM fields.
- One must go beyond the instantaneous Kerr response to have a suitable mathematical model of stimulated Raman scattering (SRS).
- The slowly varying part of third-order polarization now has the form

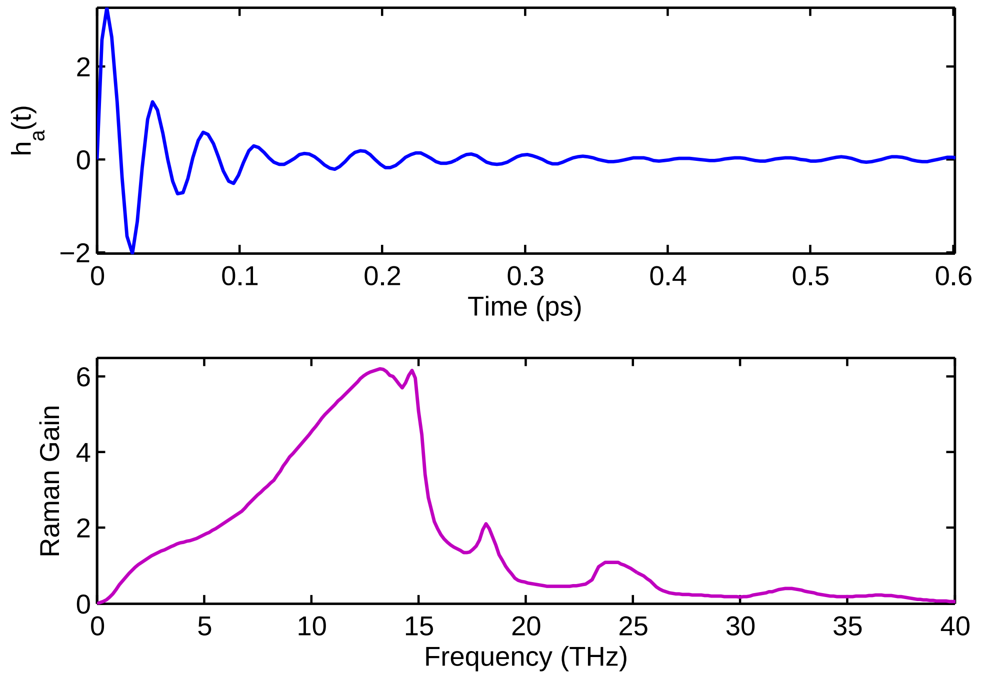
$$P_i^{(3)}(t) = \frac{\varepsilon_0}{4}\sum_j\sum_k\sum_l \chi_{ijkl}^{(3)} E_j(t)\int_{-\infty}^{t} R(t-t_1)E_k^*(t_1)E_l(t_1)\,dt_1.$$

- Nonlinear response function $R(t)$ has the general form

$$R(t) = (1-f_R)\delta(t) + f_a h_a(t) + f_b h_b(t).$$

- $f_R = f_a + f_b$ is the fractional contribution of delayed response.
- Functions $h_a(t)$ and $h_b(t)$ represent the isotropic and anisotropic parts of this time-dependent response.

Rman response Function



- Time dependence of $h_a(t)$ and corresponding frequency dependence of the Raman gain.

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Tensor Nature of Nonlinear Response

- For an isotropic medium such as silica glass $\chi^{(3)}$ has the form

$$\chi^{(3)}_{ijkl} = \chi^{(3)}_{xxyy}\delta_{ij}\delta_{kl} + \chi^{(3)}_{xyxy}\delta_{ik}\delta_{jl} + \chi^{(3)}_{xyyx}\delta_{il}\delta_{jk}.$$

- Using this form and $R(t)$, we find [Hellwarth, PQE **5**, 1, (1979)]

$$\chi^{(3)}_{ijkl}R(t) = \chi^{(3)}_{xxxx}\Big[\frac{1-f_R}{3}(\delta_{ij}\delta_{kl} + \delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk})\delta(t) + f_a h_a(t)\delta_{ij}\delta_{kl} + \tfrac{1}{2}f_b h_b(t)(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk})\Big].$$

- In the case of SRS in fibers, total field and its response is of the form

$$\mathbf{E}(t) = \mathrm{Re}[\boldsymbol{E}_p \exp(-i\omega_p t) + \boldsymbol{E}_s \exp(-i\omega_s t)],$$
$$\mathbf{P}^{(3)}(t) = \mathrm{Re}[\boldsymbol{P}_p \exp(-i\omega_p t) + \boldsymbol{P}_s \exp(-i\omega_s t)].$$

Coupled Pump and Stokes Equations

- Third-order polarization at pump and Stokes frequencies is

$$\boldsymbol{P}_j = \frac{3\varepsilon_0}{4}\chi^{(3)}_{xxxx}\Big[c_0(\boldsymbol{E}_j\cdot\boldsymbol{E}_j)\boldsymbol{E}_j^* + c_1(\boldsymbol{E}_j^*\cdot\boldsymbol{E}_j)\boldsymbol{E}_j$$
$$+ c_2(\boldsymbol{E}_m^*\cdot\boldsymbol{E}_m)\boldsymbol{E}_j + c_3(\boldsymbol{E}_m\cdot\boldsymbol{E}_j)\boldsymbol{E}_m^* + c_4(\boldsymbol{E}_m^*\cdot\boldsymbol{E}_j)\boldsymbol{E}_m\Big],$$

- Here c_0 to c_4 depend on the two Raman response functions.

- Introducing Jones vectors $|A_p\rangle$ and $|A_s\rangle$, we obtain

$$\frac{d|A_p\rangle}{dz} + \frac{\alpha_p}{2}|A_p\rangle = i\gamma_p\Big(c_1\langle A_p|A_p\rangle + c_0|A_p^*\rangle\langle A_p^*|$$
$$+ c_2\langle A_s|A_s\rangle + c_3|A_s\rangle\langle A_s| + c_4|A_s^*\rangle\langle A_s^*|\Big)|A_p\rangle,$$

$$\frac{d|A_s\rangle}{dz} + \frac{\alpha_s}{2}|A_s\rangle = i\gamma_s\Big(c_1\langle A_s|A_s\rangle + c_0|A_s^*\rangle\langle A_s^*|$$
$$+ c_2\langle A_p|A_p\rangle + c_3|A_p\rangle\langle A_p| + c_4|A_p^*\rangle\langle A_p^*|\Big)|A_s\rangle.$$

Evolution of Stokes Vectors

- Polarization effects can be studied by using the Stokes vectors

$$\boldsymbol{P} = \langle A_p|\boldsymbol{\sigma}|A_p\rangle, \qquad \boldsymbol{S} = \langle A_s|\boldsymbol{\sigma}|A_s\rangle.$$

- Evolution of $\boldsymbol{P}$ and $\boldsymbol{S}$ on the Poincaré sphere is governed by

$$\frac{d\boldsymbol{P}}{dz} + \alpha_p\boldsymbol{P} = -\frac{\omega_p}{2\omega_s}[(g_a+3g_b)P_s\boldsymbol{P} + (g_a+g_b)P_p\boldsymbol{S} - 2g_bP_p\boldsymbol{S}_3] + \boldsymbol{W}_p \times \boldsymbol{P}$$

$$\frac{d\boldsymbol{S}}{dz} + \alpha_s\boldsymbol{S} = \frac{1}{2}[(g_a+3g_b)P_p\boldsymbol{S} + (g_a+g_b)P_s\boldsymbol{P} - 2g_bP_s\boldsymbol{P}_3] + \boldsymbol{W}_s \times \boldsymbol{S}.$$

- Here P_p and P_s are the pump and Signal powers and

$$\boldsymbol{W}_p = \frac{2\gamma_p}{3}[\boldsymbol{P}_3 + 2(1+\delta_b)\boldsymbol{S}_3 - (2+\delta_a+\delta_b)\boldsymbol{S}],$$

$$\boldsymbol{W}_s = \frac{2\gamma_s}{3}[\boldsymbol{S}_3 + 2(1+\delta_b)\boldsymbol{P}_3 - (2+\delta_a+\delta_b)\boldsymbol{P}].$$

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Linear and Circular Polarizations

- For a linearly polarized pump ($P_3 = 0$), the signal evolves as

$$\frac{d\boldsymbol{S}}{dz} + \alpha_s \boldsymbol{S} = \frac{1}{2}[(g_a + 3g_b)P_p \boldsymbol{S} + (g_a + g_b)P_s \boldsymbol{P}].$$

- If signal is also linear polarized, both P and S maintain their initial SOPs with propagation.
- When the pump and signal are co-polarized, two gain terms add in phase, and Raman gain is maximum with a value $g_\parallel = g_a + 2g_b$.
- When the two are orthogonally polarized, two gain terms add out of phase, and the Raman gain is minimum with the value $g_\perp = g_b$.
- In the case of circular polarization, $g_\parallel = g_a + g_b$ and $g_\perp = 2g_b$.
- Raman gain clearly depends on the relative SOPs of the pump and signal.

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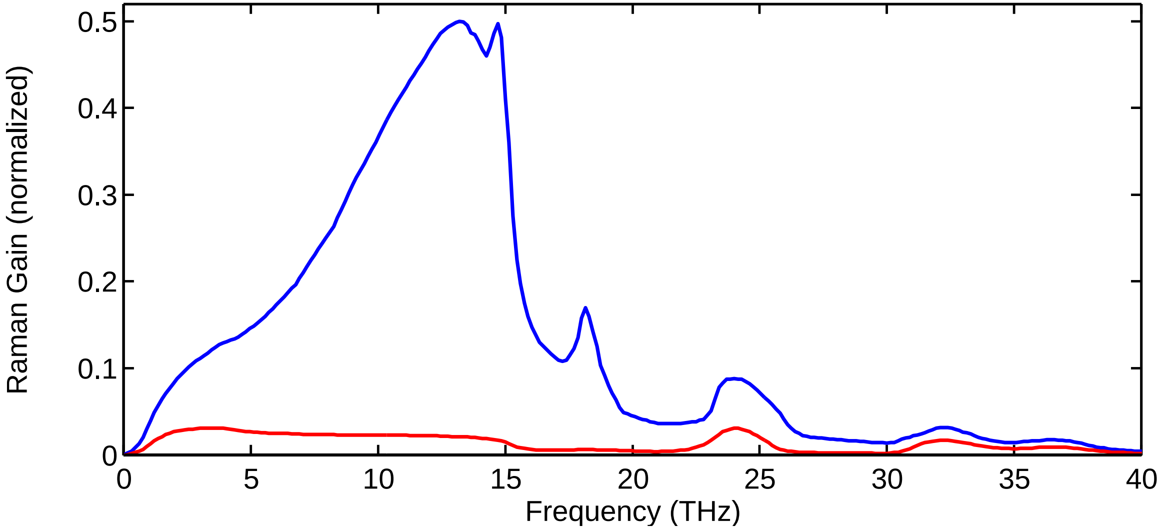
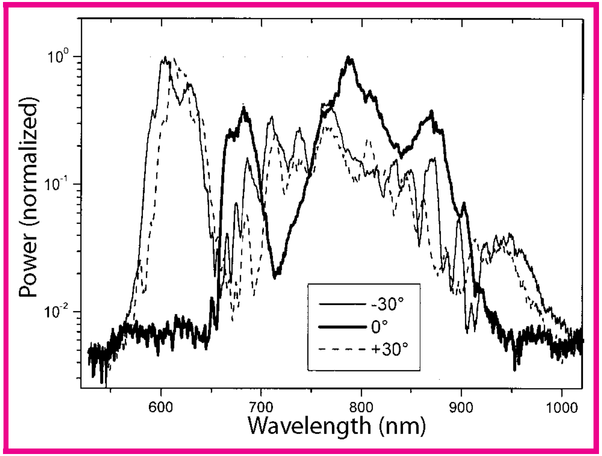
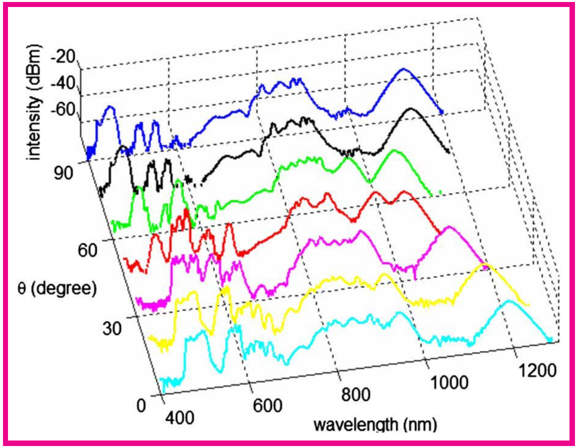
Polarization Dependence of Raman Gain



- Raman gain when the Stokes and the pump are co-polarized (blue curve) and orthogonally polarized (red curve).
- Raman gain coefficient is highly polarization dependent for silica fibers.

Supercontinuum Generation





Apolonski et al., JOSA B (2002) Choi et al., PRA (2008)

- Suppercontinuum generation depends on the SOP of input pulses.
- Two examples of this behavior are shown in the figures above.
- Even at a given input SOP, output may exhibit complicated polarization properties.

Vector NLS Equation for isotropic Fibers

- This case was studied in 2004 [Lu et al., PRL **93**, 183901 (2004)].
- Mathematically, we need to solve the vector NLS equation:

$$\frac{\partial |A\rangle}{\partial z} + \frac{1}{2}\left(\alpha + i\alpha_1 \frac{\partial}{\partial t}\right)|A\rangle + \sum_{m=2}^{M} i^{m-1} \frac{\beta_m}{m!}\frac{\partial^m |A\rangle}{\partial t^m}$$
$$= \sigma_1 |A\rangle + i\left(\gamma + i\gamma_1 \frac{\partial}{\partial t}\right)|Q(z,t)\rangle$$

- Polarization dependent nonlinear effects are included through

$$|Q(z,t)\rangle = \frac{2}{3}(1-f_R)\left[\langle A|A\rangle\right]|A\rangle + \frac{1}{3}(1-f_R)\left[\langle A^*|A\rangle\right]|A^*\rangle$$
$$+ f_R|A(z,t)\rangle \int_{-\infty}^{t} h_R(t-t')\langle A(z,t')|A(z,t')\rangle\, dt'.$$

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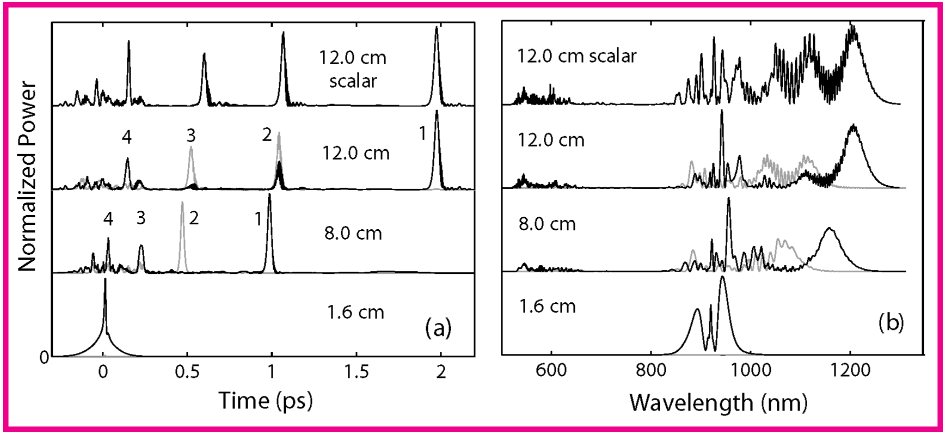
Vector Nature of Soliton Fission



Lu et al., PRL **93**, 183901 (2004)

- Propagation in ideal isotropic fiber (no birefringence).
- Input SOP slightly elliptical (32 dB extinction ratio).
- A 150-fs pulse with $N \approx 12$ is launched into a tapered fiber.
- Different solitons exhibit different SOPs.

Concluding Remarks

- Nonlinear phenomena in optical fibers exhibit a rich variety of polarization-dependent effects.
- In the case of a single pulse, cross-polarization modulation leads to nonlinear polarization rotation with a multitude of applications.
- In the case of two different wavelengths, cross-phase modulation can be used to control polarization of one pulse using the other.
- XPM applications include polarization pulling, ultrafast optical switching, pump-induced probe compression, and soliton trapping.
- In the case of four-wave mixing, circularly polarized pumps can provide larger polarization-independent parametric gain.
- Polarization effects play an important role during supercontinuum generation inside optical fibers.

Further Reading

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- F. Lu, Q. Lin, W. H. Knox, and G. P. Agrawal, "Vector Soliton Fission," Phys. Rev. Lett. **93**, 183901 (2004).