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Averaged nonlinear equations for multimode fibers valid in all regimes of random linear coupling

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ABSTRACT

We develop averaged equations to model nonlinear propagation in multimode fibers that are valid in all regimes of random, linear, intermodal coupling. The propagation equations apply to the three existing regimes of linear coupling – the two previously studied all-mode (strong) and mode-group (weak) couplings and the new intermediate coupling regime. The equations are therefore general and can describe nonlinear propagation for all types of intermodal linear coupling that can exist between modes in a fiber supporting multiple spatial modes. Numerical simulations are performed to validate the new averaged propagation equations in the nonlinear regime.

1. Introduction

Space-division multiplexing (SDM) over multimode or multicore fibers is considered a promising solution to enhance the capacity limit (per fiber) of the next generation of telecommunication systems. An important challenge in such systems is to understand how linear intermodal coupling, which may arise from a variety of sources like microbending, density fluctuations, and random variations in the core shape and size, affects the nonlinear signal transmission [1,2].

Different SDM fibers or different modes within a single SDM fiber can exhibit different levels of linear coupling. Modeling nonlinear transmission including such random linear mode coupling (RLMC) effects usually is time consuming because it requires a large number of realizations of random coupling. In practice, it is important to find averaged equations that can model the effects of RLMC in an efficient manner. Such averaging can reduce the computational times by orders of magnitude. The averaged equations are referred to as generalized Manakov equations, after a similar treatment used to average random birefringence fluctuations in single-mode fibers [3,4].

Generalized Manakov equations have been derived for SDM fibers in two limiting cases when (i) all modes are strongly coupled or (ii) when groups of nearly-degenerate modes couple among each other but there is negligible inter-group coupling [5–8]. But certain multimode fibers (MMF) may exhibit some degree of inter-group coupling between these extremes. Currently, there is no theoretical framework that describes the average propagation behavior in such an intermediate coupling regime (ICR). Moreover, there are no definitive criteria to determine when the intermediate coupling region exists, or where the transition occurs between the strong and weak coupling regimes.

In this Letter we consider all regimes of linear coupling and provide a way to derive the averaged nonlinear equations by using a transfer-matrix approach to modeling RLMC. The coefficients of the averaged nonlinear terms in the generalized Manakov equations are found to depend on the fourth-order moments of the transfer matrix elements. We verify our approach by deriving the generalized Manakov equations in the all-mode (strong) coupling regime (SCR) and mode-group (weak) coupling regime (MGCR). The treatment used here also lets us identify the boundary between the different coupling regimes and the extent of the ICR, as a function of a dimensionless coupling strength parameter. We can use this treatment to study the impact of RLMC on nonlinear transmission systems in SDM fibers and we use a particular example of a few-mode fiber to show this. Finally, we perform full numerical simulations of an SDM system to study nonlinear transmission in different coupling regimes.

2. Numerical model

We consider a MMF supporting M spatial modes and $N = 2M$ total modes after accounting for both polarizations. We include the effects of RLMC through random perturbations of the refractive index of the fiber $[\epsilon(x, y, z) = n^2(x, y) + \Delta\epsilon(x, y, z)]$ in the nonlinear Helmholtz equation [9]. The procedure for deriving the N coupled nonlinear

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Schrödinger (NLS) equations is well known [10] (and references therein). The entire set of coupled NLS equations can be written in a matrix form as [7]

$$\frac{\partial \mathbf{A}}{\partial z} + \delta\mathbf{B}_1\frac{\partial \mathbf{A}}{\partial t} + \frac{i\mathbf{B}_2}{2}\frac{\partial^2 \mathbf{A}}{\partial t^2} = i\delta\mathbf{B}_0\mathbf{A} + i\mathbf{Q}(z)\mathbf{A} + \frac{i\gamma}{3}[2(\mathbf{A}^{\mathrm{H}}\mathbf{G}^{(1)}\mathbf{A})\mathbf{G}^{(1)}\mathbf{A} + (\mathbf{A}^{\mathrm{T}}\mathbf{G}^{(2)}\mathbf{A})\mathbf{G}^{(2)*}\mathbf{A}^{*}]dx\,dy, \tag{1}$$

where $\mathbf{A}$ is an N-element column vector containing the field envelopes of each mode and $\mathbf{B}_0$, $\mathbf{B}_1$, $\mathbf{B}_2$ are $N \times N$ diagonal matrices containing, respectively, the propagation constant (β_0), inverse group velocity (β_1) and dispersion parameter (β_2) of various modes along their diagonal. The matrices $\delta\mathbf{B}_0 = \mathbf{B}_0 - \beta_{01}$ and $\delta\mathbf{B}_1 = \mathbf{B}_1 - 1/v_{g_1}$ use β_{01} and group velocity $1/v_{g1}$ of the fundamental mode as reference values, and γ ($=n_2\omega_0/cA^{\mathrm{eff}}$) is the nonlinear parameter of this mode. The effects of linear and nonlinear couplings are included through the matrices $\mathbf{Q}$, $\mathbf{G}^{(1)}$, and $\mathbf{G}^{(2)}$, whose elements are defined as

$$q_{ij}(z) = \frac{k_0}{2n_{\mathrm{eff}}^{i}}\iint \Delta\epsilon\left(x, y, z\right)F_i\left(x, y\right)F_j^{*}\left(x, y\right)dx\,dy, \tag{2a}$$

$$g_{ij}^{(1)}\left(x, y\right) = F_i^{*}\left(x, y\right)F_j\left(x, y\right), \quad g_{ij}^{(2)}\left(x, y\right) = F_i\left(x, y\right)F_j\left(x, y\right). \tag{2b}$$

Here $F_k(x, y)$ is the transverse field distribution of the k^{th} mode of the fiber. The orthogonality condition for spatial modes is given by

$$\iint F_p^{*}\left(x, y\right)F_m\left(x, y\right)dx\,dy = \frac{n_{\mathrm{eff}}^{1}}{n_{\mathrm{eff}}^{m}}\delta_{pm}, \tag{3}$$

where n_{eff}^{m} is the effective propagation constant of the m^{th} mode.

To study the impact of linear coupling on nonlinear transmission, we introduce the concept of a random transfer matrix $\mathbf{T}(z)$, which tracks the random linear coupling effects and is given by

$$\frac{\partial \mathbf{T}}{\partial z} = i\delta\mathbf{B}_0\mathbf{T} + i\mathbf{Q}\mathbf{T}; \qquad \mathbf{T}(0) = \mathbf{I}_{\mathrm{N}}, \tag{4}$$

where $\mathbf{I}_{\mathrm{N}}$ is a $N \times N$ identity matrix. We make a transformation $\mathbf{A} = \mathbf{T}\bar{\mathbf{A}}$ to obtain the following matrix NLS equation

$$\frac{\partial \bar{\mathbf{A}}}{\partial z} + \left(\mathbf{T}^{\mathrm{H}}\mathbf{B}_1\mathbf{T} - \frac{1}{v_{g_1}}\right)\frac{\partial \bar{\mathbf{A}}}{\partial t} + \frac{i}{2}\mathbf{T}^{\mathrm{H}}\mathbf{B}_2\mathbf{T}\frac{\partial^2 \bar{\mathbf{A}}}{\partial t^2} = \mathcal{N} \tag{5}$$

where the nonlinear term $\mathcal{N}$ is given by

$$\mathcal{N} = \frac{i\gamma}{3}\iint\left[2\left(\bar{\mathbf{A}}^{\mathrm{H}}\mathbf{T}^{\mathrm{H}}\mathbf{G}^{(1)}\mathbf{T}\bar{\mathbf{A}}\right)\mathbf{T}^{\mathrm{H}}\mathbf{G}^{(1)}\mathbf{T}\bar{\mathbf{A}} + \left(\bar{\mathbf{A}}^{\mathrm{T}}\mathbf{T}^{\mathrm{T}}\mathbf{G}^{(2)}\mathbf{T}\bar{\mathbf{A}}\right)\mathbf{T}^{\mathrm{H}}\mathbf{G}^{(2)*}\mathbf{T}^{*}\bar{\mathbf{A}}^{*}\right]dx\,dy. \tag{6}$$

A discretized model used to calculate the transfer matrix is shown in Fig. 1. We divide the fiber into K segments of length l_d, which corresponds to the decorrelation length associated with the fluctuations of $\Delta\epsilon(x, y, z)$. We assume $\Delta\epsilon$ to be normally distributed with zero mean and a pre-specified standard deviation σ_ϵ. To account for the fast random birefringence fluctuations, after every fiber section we multiply the transfer matrix by a block-diagonal matrix $\mathbf{R}_k$ comprising of M 2×2 random unitary sub-matrices along the diagonal, the total transfer matrix of a fiber of length $L = Kl_d$ is then given by

$$\mathbf{T} = \prod_{k=1}^{K}\mathbf{R}_k\exp[i(\delta\mathbf{B}_0 + \mathbf{Q}_k)l_d]. \tag{7}$$

3. Averaged nonlinear equations

The set of coupled NLS equations in Eq. (5) is stochastic and any meaningful result requires solving the entire set numerically for different realizations of the random matrix $\mathbf{T}$. However, by averaging each term in Eq. (5) with respect to random realizations of $\mathbf{T}$, we can obtain a set of averaged equations for $\bar{\mathbf{A}}$. This approach is similar to that used for single-mode fibers to include the impact of birefringence fluctuations in an average manner [4]. For the averaging of the nonlinear terms, consider $\hat{\mathcal{N}_k}$ for the k^{th} mode of the fiber. While this quantity contains a large number of terms ($\sim N^7$ for a N-mode fiber), some of them contribute little on average. Using the orthogonality relations between modes and the fact that intermodal four-wave mixing (IM-FWM)-like terms are not phase-matched [11] for strongly-coupled modes, we obtain

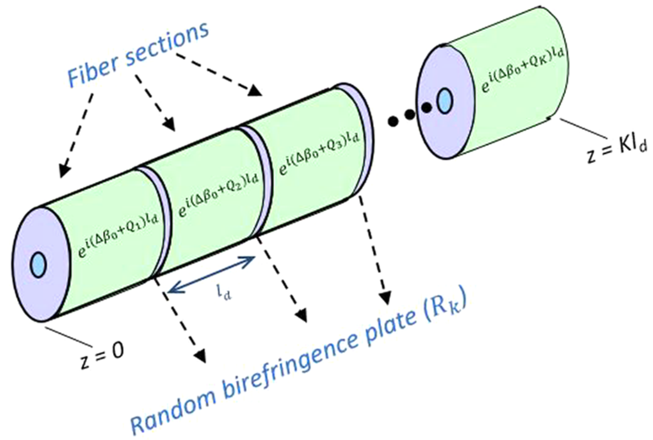

Fig. 1. Model used for calculating the random linear transfer matrix $\mathbf{T}$. The fiber is divided into K sections of length l_d, and each section is followed by a random birefringence plate.

$$\hat{\mathcal{N}_k} = i\gamma\sum_{l=1}^{N}\sum_{m=1}^{N}\sum_{n=1}^{N}\frac{2(|t_{ml}|^2|t_{nk}|^2 + t_{ml}t_{nk}t_{mk}^{*}t_{nl}^{*})}{(1 + d_{kl})(1 + d_{mn})}f_{mmnn}\left|\bar{A}_l\right|^2\bar{A}_k, \tag{8}$$

where t_{ij} are the transfer matrix elements and d_{ij} is equal to 1 if the modes i and j belong to the same mode group but 0 otherwise. The spatial overlap factors that govern nonlinear coupling are given by

$$f_{plmn} = A^{\mathrm{eff}}\iint F_p^{*}F_lF_m^{*}F_n\,dx\,dy. \tag{9}$$

We can use Eq. (8) to write the final averaged NLS equation for each fiber mode. Let S_m be the group of modes that couple strongly with the m^{th} mode (with the convention $m \in S_m$) and let n_m denote the number of modes in S_m. Using these notations, we can write the average propagation equation for the k^{th} mode in the following form

$$\frac{\partial \bar{A}_k}{\partial z} + \left(B_{1k}' - \frac{1}{v_{g1}}\right)\frac{\partial \bar{A}_k}{\partial t} + \frac{i}{2}B_{2k}'\frac{\partial^2 \bar{A}_k}{\partial t^2} = i\sum_{l=1}^{N}\sum_{m\in S_l}\sum_{n\in S_k}C_{klmn}\left|\bar{A}_l\right|^2\bar{A}_k, \tag{10}$$

where B_{1k}' and B_{2k}' are the elements of diagonal matrices $\mathbf{B}_1' = \langle\mathbf{T}^{\mathrm{H}}\mathbf{B}_1\mathbf{T}\rangle$ and $\mathbf{B}_2' = \langle\mathbf{T}^{\mathrm{H}}\mathbf{B}_2\mathbf{T}\rangle$. Physically, we would expect the modes that are strongly coupled to travel at the same group velocity on average. This analytic prediction has been verified numerically in the MGCR and SCR. However, we should note that Eq. (10) misses the randomness of group delay and such averaging is not necessarily the best indicator of DGD [12].

Following the transformation $\mathbf{A} = \mathbf{T}\bar{\mathbf{A}}$, all the random fluctuations are explicitly included through $\mathbf{T}$ and $\bar{\mathbf{A}}$ is deterministic. The nonlinear coefficients in Eq. (10) are given by

$$C_{klmn} = 2\gamma f_{mmnn}\frac{\langle|t_{ml}|^2|t_{nk}|^2\rangle + \langle t_{ml}t_{nk}t_{mk}^{*}t_{nl}^{*}\rangle}{(1 + d_{kl})(1 + d_{mn})}. \tag{11}$$

Eqs. (10) and (11) can be used to model the average behavior of an SDM system in any regime of RLMC, including the ICR. We next discuss how to evaluate the averaged nonlinear coefficients.

Eq. (11) shows that the coefficients of the averaged nonlinear term depend on the average values of the fourth-order moments of the elements of the transfer matrix **T**. More specifically, we need the following four types of fourth-order moments:

$$m_{ij}^{(1)}(\mathbf{X}) = \langle |x_{ij}|^4 \rangle, \tag{12a}$$

$$m_{ikl}^{(2)}(\mathbf{X}) = \langle |x_{ik}|^2 |x_{il}|^2 \rangle \qquad (k \neq l), \tag{12b}$$

$$m_{ijkl}^{(3)}(\mathbf{X}) = \langle |x_{ik}|^2 |x_{jl}|^2 \rangle \qquad (i \neq j \text{and} k \neq l), \tag{12c}$$

$$m_{ijkl}^{(4)}(\mathbf{X}) = \langle x_{ik} x_{jl} x_{il}^* x_{jk}^* \rangle \qquad (i \neq j \text{and} k \neq l), \tag{12d}$$

where $\mathbf{X}$ is any matrix and x_{ij} are its elements. Although our analysis is general and valid for any SDM fiber, to show how to calculate these average nonlinear coefficients, we consider here a six-mode step-index fiber with a core diameter of 11μm and $n_{clad} = 1.444$ such that $V = 3.8$ at $\lambda = 1.55\mu$m. Such a fiber supports three spatial modes (denoted by LP_{01}, LP_{11a} and LP_{11b} in the basis of linearly polarized (LP) modes), resulting in $N = 6$.

For our example fiber, **T** is a 6×6 random matrix. However, LP_{11a} and LP_{11b} modes can be strongly coupled because of their degenerate nature. Also, orthogonally polarized components of any spatial mode are assumed to be strongly coupled owing to birefringence fluctuations. It is thus useful to express **T** in the form

$$\mathbf{T} = \begin{bmatrix} \mathbf{T}_1 & \mathbf{T}_{12} \\ \mathbf{T}_{12}^{\mathrm{H}} & \mathbf{T}_2 \end{bmatrix} \tag{13}$$

where $\mathbf{T}_1$ is a 2×2 matrix for the LP_{01} mode group g_1, $\mathbf{T}_2$ is a 4×4 matrix for the LP_{11} mode group g_2, and $\mathbf{T}_{12}$ is a 2×4 matrix representing coupling between these two mode groups. This coupling is induced by random index perturbations and its magnitude may vary considerably for different SDM fibers.

We calculate the average values for the individual blocks of **T** numerically. For this purpose, we introduce a dimensionless coupling parameter

$$\varkappa = \langle |q_{g1,g2}| \rangle / |\Delta\beta_{0,g1,g2}|, \tag{14}$$

where $\Delta\beta_{0,g1,g2}$ is the difference in propagation constants between the LP_{01} and LP_{11} mode groups and $q_{g1,g2}$ governs the linear coupling between them. Physically, $\varkappa$ represents the normalized coupling strength between the two mode groups. As an example, we choose a fiber with $L = 1$ km and $l_d = 50$ m, and calculate the values of the fourth-order moments as a function of this ratio. Fig. 2 shows such a plot for $\mathbf{T}_1$. It should be noted that the moments shown in that figure are averaged over all possible elements (i, j) of $\mathbf{T}_1$ in Eq. (12). A striking feature in this figure is that all moments change rapidly in the range $0.01 < \varkappa < 0.1$, but remain virtually constant outside this range. Since small and large values of $\varkappa$ correspond to weak and strong coupling between mode-groups, respectively, it is possible to predict the limiting values of $m^{(p)}$ analytically in these two regimes. For this purpose, we use the concept of a Haar matrix, which is a unitary random matrix whose elements are uniformly distributed over the entire range. If the coupling among mode groups is totally random in the sense that all unitary transformations are equally probable, then we expect the corresponding transfer matrix block to be a Haar matrix. Interestingly, the fourth-order moments of a Haar matrix depend only on the dimension n of the matrix and are given by [13]:

$$\bar{m}^{(1)}(n) = \frac{2}{n(n+1)}, \ \bar{m}^{(3)}(n) = \frac{1}{(n-1)(n+1)}, \tag{15}$$

$$\bar{m}^{(2)}(n) = \frac{1}{n(n+1)}, \ \bar{m}^{(4)}(n) = \frac{-1}{(n-1)n(n+1)}, \tag{16}$$

where a bar is used to indicate the Haar limit. These expressions agree with the numerical results in Fig. 2 in the two extreme limits of weak and strong intergroup coupling. In the case of weak coupling, $n = 2$ as $\mathbf{T}_1$ is an isolated 2×2 matrix. In contrast, in the case of strong intergroup coupling, $n = 6$ since all modes are equally coupled. These analytical predictions are shown by the solid and dashed horizontal lines in Fig. 2.

Fig. 2 provides the averaged values of the moments of matrix elements for the LP_{01} mode group. A similar figure can be generated for the LP_{11} mode group (moments of $\mathbf{T}_2$), whose values in the weak and strong coupling limits can be predicted analytically from Eqs. (15) and (16). The moments of the off-diagonal matrix blocks $\mathbf{T}_{12}$ and $\mathbf{T}_{21}$, which will have significant contributions in the ICR, can also be computed numerically using Eqs. (12). Once values of all these averaged fourth-order moments are known, they can be used in the propagation Eqs. (10) and (11) to model the average behavior of an SDM system in any regime of linear coupling. It should also be noted here that the coupling strength parameter and the associated averaged moments are not an indicator of the amount of energy coupled between mode-groups [14 Eq. (8)]. Indeed, we would expect that even in the MGCR, there would be exchange of power between mode-groups after propagating very long distances within the fiber.

To validate further our general expression in Eq. (11), we apply it to the two extreme coupling regimes that have been studied earlier. In the MGCR, only some modes (with a larger value of $\varkappa$) are strongly coupled. In this limit,

$$C_{klmn}(\text{MGCR}) = \frac{2\gamma}{n_k} \frac{1}{n_l + d_{lk}} f_{mmnn}, \tag{17}$$

where, as before, d_{mk} is 1 when m and k belong to a coupled mode-group but 0 otherwise. This is a general expression which allows for strong coupling between degenerate modes. In the SCR, all modes are assumed to be equally coupled, which means that S_k and S_m contain all N modes ($n_k = n_m = N$). Under this condition, we get

$$C_{klmn}(\text{SCR}) = \frac{2\gamma}{N(N+1)} f_{mmnn}. \tag{18}$$

Both of these cases have also been studied in the past [5–7], and our formulas agree with earlier results. As a final check, in the limit $N = 2$ and using the known values of $f_{1111} = f_{2222} = 1$ and $f_{1122} = f_{2211} = 1/3$, our results reduce to the well known standard Manakov equation for a single-mode fiber [4]. In general, the nonlinear terms are smaller when all modes of a multimode fiber are strongly coupled because of the presence of the factor $N(N + 1)$ in the denominator of Eq. (18). Indeed, linear coupling has been shown to reduce nonlinear penalties in multicore fibers with coupled cores [15,16].

In the ICR, the nonlinear terms in Eq. (11) depend on the coupling parameter $\varkappa$, as shown in the Fig. 2. In this case, the transfer matrix does not reduce to a Haar matrix as its elements are not uniformly distributed. However, we can compute the nonlinear coefficients in Eq. (11) numerically and use them in Eq. (10) to model the system performance. We have observed that the transition region of intermediate coupling seen in Fig. 2 becomes slightly narrower as the system length increases. Thus, our results allows us to predict where the transition to the ICR would occur for any SDM fiber.

4. Nonlinear transmission simulation

To show how useful the results of this paper are for modeling the realistic SDM systems, we perform full numerical simulation for a specific SDM system and predict the optical signal-to-noise ratio (OSNR) in different coupling regions. Using the same six-mode fiber (core diameter $= 1.1\ \mu$m, $n_{\text{clad}} = 1.444$ and $V = 3.8$ at $1.55\ \mu$m), we transmit six QPSK-format data streams, each at 28.5 Gbaud (bit rate 57 Gb/s). The 1000-km-long transmission line consists of 10 spans of 100-km fiber, each followed by a fiber amplifier that compensates for all span losses. No dispersion management is employed along the fiber link, but all linear impairments are assumed to be perfectly

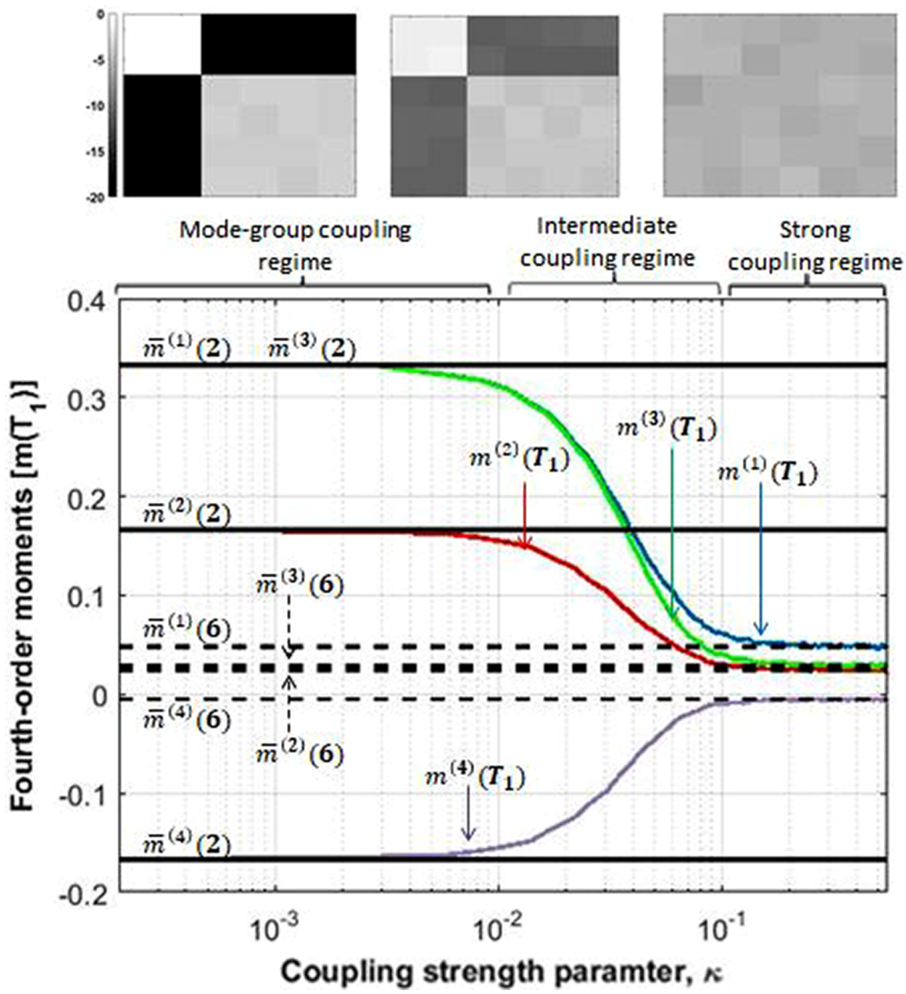

Fig. 2. Averaged fourth-order moments of $\mathbf{T}_1$ in Eq. (13) as a function of κ when L = 1 km and l_d = 50 m. Haar-limit predictions are shown by solid and dashed horizontal lines for n = 2 and 6, respectively. Magnitude of transfer matrix elements [Eq. (13)] in the three coupling regions, computed numerically using Eq. (7), is shown on top using a gray-shaded logarithmic scale.

compensated at the digital receiver.

Fig. 3 shows the OSNR penalties (compared to back-to-back performance) at a bit-error rate (BER) of 10^{-3} as a function of the standard deviation, σ_ϵ, of refractive-index fluctuations. The OSNR has been calculated using a noise bandwidth of 0.1 nm (12.5 GHz) and the BER value chosen here is commonly used as the forward-error correction threshold. The plot is obtained by solving Eq. (5) with the split-step method. The dashed horizontal lines correspond to the values predicted by using the averaged nonlinear terms in the MGCR and SCR. The most important feature of this figure is the sharp peak observed in the ICR, indicating that the OSNR penalty degrades considerably in the ICR. We have verified that in the absence of nonlinearity, no degradation is observed. Therefore, the increased penalty in the ICR in Fig. 3 can be attributed to nonlinear effects. Moreover, as seen in Fig. 3, it is preferable to operate in the SCRthan in the MGCR. The transfer matrix approach used in this paper can predict the transition region between the two coupling regimes and can be useful in designing SDM fibers for optimal performance.

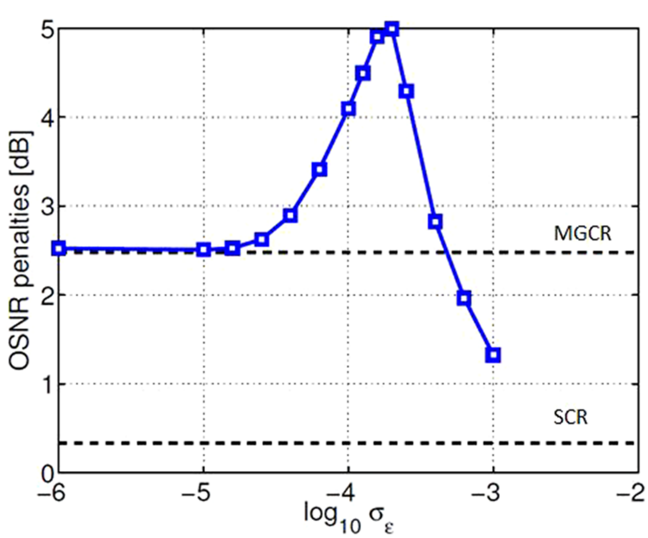

Fig. 3. OSNR penalties for a 6 channel SDM system, transmitted over 1000 km 28.5-Gbaud channels in the QPSK format, as a function of σ_ϵ. The dashed horizontal lines show the penalties obtained by using the averaged equations.

5. Conclusions

In this work we have studied the impact of random linear mode coupling in fibers on the performance of lightwave systems designed for space-division multiplexing. Our physical model of linear mode coupling is based on random perturbations of the refractive index in the transverse plane that also change along the fiber's length with a relatively short decorrelation length. We show that the resulting coupling between any two modes depends on a dimensionless parameter κ, which is a ratio of the coupling coefficient and the difference in their propagation constants $\Delta\beta$. We use this ratio to identify the two extreme regimes, referred to as the mode-group coupling and strong-coupling regimes.

We develop a vectorial nonlinear propagation equation that describes evolution of all modes simultaneously. It includes linear mode coupling through a random coupling matrix as well as variations in the modal propagation constants via birefringence fluctuations through a block diagonal matrix. Since the use of such a stochastic equation is time consuming in practice, we develop an averaging procedure similar to that used to study the random birefringence effects in single-mode fibers. The averaging of the nonlinear effects requires knowledge of the fourth-order moments of the elements of a random transfer matrix. We discuss the dependence of the magnitude of these moments on $\varkappa$ for a specific three-mode fiber. We also find simple analytical expressions for these fourth-order moments in the mode-group and strong coupling limits using Haar matrices. In these two limits, we were able to obtain analytic expressions for various nonlinear terms.

The averaging method discussed here can be applied to all SDM fibers with any number of spatial mode-groups. For fibers with more than two mode groups, each mode-group pair can have different strengths of RLMC (different $\varkappa$). One needs to calculate the nonlinear coefficients in Eq. (11) separately for each mode-group pair and then use those coefficients in Eq. (10) to model the average nonlinear transmission. For the example shown here, the agreement of the full numerical simulations with the predicted averaged values indicates that this approach can be used for reducing the computation time for SDM systems.

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