Optics Letters

Controlling the degree of polarization of partially coherent electromagnetic beams with lenses

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Received 20 February 2018; accepted 8 April 2018; posted 13 April 2018 (Doc. ID 324583); published 9 May 2018

We show theoretically that the degree of polarization of a partially coherent electromagnetic beam changes dramatically as the beam is being focused. A low numerical aperture lens can considerably enhance the degree of polarization at its geometrical focus. When two identical lenses are employed in a 4*f* configuration, the degree of polarization of a beam can be tailored by using amplitude masks in the Fourier plane located in the middle of the two lenses. Our findings open up the possibility to control this fundamental property of random beams in a simple manner.

OCIS codes: (070.0070) Fourier optics and signal processing; (070.6110) Spatial filtering; (030.1640) Coherence; (260.5430) Polarization.

https://doi.org/10.1364/OL.43.002344

The study of polarization of electromagnetic fields was pioneered, among others, by Stokes [1] and Poincaré [2]. In much more recent times, the work of Wolf [3] has shown that polarization is intimately related to the concept of coherence. The state of polarization (SOP) of a coherent electromagnetic beam can change during its propagation inside a birefringent medium, even though the beam remains fully polarized. In the case of a partially coherent electromagnetic beam, the notion of the degree of polarization (DOP) becomes relevant. It is defined such that $0 \leq P \leq 1$, with the extreme values corresponding to completely unpolarized beams and fully polarized beams, respectively [1]. The DOP is a fundamental property of light that plays a crucial role in any scattering process [4,5]. It is used as a diagnostic tool in examining material strain [6], in optical coherence tomography [7], and in remote sensing [8]. Since the DOP is directly related to the Stokes parameters, its value can be determined by using standard polarimetry [9]. It is well established that both the SOP and the DOP of a partially coherent beam may change as it propagates, even if the propagation is through free space [10–12].

In this Letter, we study how the DOP is affected when a partially coherent beam is passed through a lens. Lenses are used routinely for making a broad variety of optical devices. Their Fourier transforming properties are widely used in optical signal processing to manipulate the spectral components of fully coherent fields [13–15]. Such Fourier-based filtering techniques can also be applied to partially coherent scalar beams [16–20]. In particular, it was found in Ref. [20] that, in a focusing process, the cross-spectral density [21], the correlation function that characterizes partially coherent scalar fields in the space-frequency domain, satisfies certain Fourier transform relations. This means that the coherence properties of a random beam can be controlled by spatial filtering in a $4f$ setup. This way, for example, an incident, δ-correlated beam can be converted into a partially coherent beam with an adjustable transverse coherence length.

Here we generalize this approach from scalar beams to partially correlated, partially polarized electromagnetic beams. We derive Fourier transform relations for the cross-spectral density matrix that describes the coherence and polarization properties of such beams. These relations are then applied to study a $4f$ configuration. We find that a strong and tunable increase of the DOP can be achieved.

Consider the situation in which a partially coherent beam is incident on a thin lens of focal length f. We begin by recalling a well-known result from the scalar theory of light focusing, namely that the optical field in the back focal plane of a lens is proportional to the spatial Fourier transform of the field in the front focal plane (Sec. 5.2, [13]):

$$U^{(f)}(\boldsymbol{\rho}, \omega) = \frac{1}{j\lambda f} \int_{-\infty}^{\infty} U^{(\mathrm{in})}(\boldsymbol{\rho}', \omega) A(\boldsymbol{\rho}') \times \exp(-jk\boldsymbol{\rho} \cdot \boldsymbol{\rho}'/f) \mathrm{d}^2\rho'. \quad (1)$$

Here $U^{(f)}(\boldsymbol{\rho}, \omega)$ denotes the scalar field in the back focal plane at frequency ω at a transverse position $\boldsymbol{\rho} = (x, y)$, $U^{(\mathrm{in})}(\boldsymbol{\rho}', \omega)$ is the field in the front focal plane, and the wavenumber is $k = 2\pi/\lambda = \omega/c$, with λ denoting the wavelength and c denoting the speed of light. The lens is assumed to have its central axis along the z direction. In Eq. (1) $A(\boldsymbol{\rho}')$ is a pupil function, i.e., $A(\boldsymbol{\rho}') = 1$ for points within the lens aperture, and 0 elsewhere.

In the following we assume that we are dealing with a narrow electromagnetic beam whose transverse extent is smaller than the lens aperture. This allows us to replace $A(\boldsymbol{\rho}')$ by unity in Eq. (1). Furthermore, as long as we are dealing with a paraxial system, Eq. (1) will hold for both Cartesian transverse components, E_x and E_y, of the electric field vector. The axial component of such a beam is negligible and will be ignored from now on. With these simplifications, the statistical properties of a partially coherent electromagnetic beam in a cross-sectional plane z are fully described by its cross-spectral density matrix, which is defined as [21]

$$\mathbf{W}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,z,\omega) = \begin{pmatrix} W_{xx} & W_{xy} \\ W_{yx} & W_{yy} \end{pmatrix}, \tag{2}$$

where all the matrix elements are functions of the same four variables and given by the expression

$$W_{ij}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,z,\omega)=\langle E_i^*(\boldsymbol{\rho}_1,z,\omega)E_j(\boldsymbol{\rho}_2,z,\omega)\rangle, \quad (i,j=x,y). \tag{3}$$

Here the angular brackets indicate an average taken over an ensemble of beam realizations. Using Eq. (1) for the two field components and interchanging the order of ensemble averaging and integration, yields the interesting result of

$$W_{ij}^{(f)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\omega) = \frac{1}{\lambda^2 f^2}\tilde{W}_{ij}^{(\mathrm{in})}(-k\boldsymbol{\rho}_1/f, k\boldsymbol{\rho}_2/f,\omega), \tag{4}$$

where the spatial, four-dimensional, Fourier transform of the cross-spectral density elements is defined as

$$\tilde{W}_{ij}^{(\mathrm{in})}(\mathbf{f},\mathbf{g},\omega) = \iint_{-\infty}^{\infty} W_{ij}^{(\mathrm{in})}(\boldsymbol{\rho}',\boldsymbol{\rho}'',\omega) \times e^{-j(\mathbf{f}\cdot\boldsymbol{\rho}'+\mathbf{g}\cdot\boldsymbol{\rho}'')}\mathrm{d}^2\rho'\mathrm{d}^2\rho''. \tag{5}$$

All preceding equations have an explicit frequency dependence, indicating that they are defined for a specific frequency component of the optical field. For brevity, we will no longer display the ω dependence.

The usefulness of the Fourier relation (4) can be illustrated by considering the spectral DOP, which is defined as the ratio of the intensity of the polarized part of the beam and its total intensity. It is given by the formula (Sec. 6.3.3, [21])

$$P(\boldsymbol{\rho},z) = \sqrt{1-\frac{4\,\mathrm{Det}\,W(\boldsymbol{\rho},\boldsymbol{\rho},z)}{[\mathrm{Tr}\,W(\boldsymbol{\rho},\boldsymbol{\rho},z)]^2}}, \tag{6}$$

where Det denotes the determinant and Tr denotes the trace of the W matrix. The DOP can be obtained experimentally by measuring the four spectral Stokes parameters, S_i, with $i=0,1,2,3$. More specifically (Sec. 10.9.3, [1]),

$$P(\boldsymbol{\rho},z) = \frac{\sqrt{S_1^2+S_2^2+S_3^2}}{S_0}. \tag{7}$$

To illustrate how a lens affects the DOP, we choose a specific type of random electromagnetic beam, known as the Gaussian Schell–model electromagnetic beam (Ch. 9, [3]). If we assume the spectral amplitudes and coherence radii of E_x and E_y to be equal, the elements of the cross-spectral density matrix are

$$W_{xx}^{(\mathrm{in})}(\boldsymbol{\rho}',\boldsymbol{\rho}'') = A^2 e^{-(\rho'^2+\rho''^2)/4\sigma^2} e^{-(\rho''-\rho')^2/2\delta^2}, \tag{8}$$

$$W_{xy}^{(\mathrm{in})}(\boldsymbol{\rho}',\boldsymbol{\rho}'') = A^2 B_{xy} e^{-(\rho'^2+\rho''^2)/4\sigma^2} e^{-(\rho''-\rho')^2/2\delta_{xy}^2}, \tag{9}$$

$$W_{yy}^{(\mathrm{in})}(\boldsymbol{\rho}',\boldsymbol{\rho}'') = W_{xx}^{(\mathrm{in})}(\boldsymbol{\rho}',\boldsymbol{\rho}''), \tag{10}$$

$$W_{yx}^{(\mathrm{in})}(\boldsymbol{\rho}',\boldsymbol{\rho}'') = W_{xy}^{*(\mathrm{in})}(\boldsymbol{\rho}',\boldsymbol{\rho}''), \tag{11}$$

where A denotes an amplitude, σ is the effective width of the spectral densities of the x component and the y component of the electric field, δ and δ_{xy} are the coherence radii, and B_{xy} is the (complex) correlation coefficient of E_x and E_y. On using these expressions for the matrix elements in Eq. (6), we find that

$$P^{(\mathrm{in})}(\boldsymbol{\rho}) = |B_{xy}|. \tag{12}$$

It is thus seen that the DOP in the front focal plane of this specific partially coherent beam is independent of position, and it equals the modulus of the correlation coefficient.

To investigate how the DOP changes as the beam passes through a lens, we substitute the matrix elements given by Eqs. (8)–(11) and perform the four-fold integration indicated in Eq. (5) by introducing sum and difference variables as

$$\mathbf{R}_+ = \frac{\boldsymbol{\rho}'+\boldsymbol{\rho}''}{2}, \qquad \mathbf{R}_- = \boldsymbol{\rho}''-\boldsymbol{\rho}'. \tag{13}$$

We then obtain, for the elements of the cross-spectral density matrix in the back focal plane, the expressions

$$W_{xx}^{(f)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2) = \left(\frac{2\pi A\sigma\Omega}{\lambda f}\right)^2 e^{-k^2\Omega^2(\boldsymbol{\rho}_1+\boldsymbol{\rho}_2)^2/8f^2} \times e^{-k^2\sigma^2(\boldsymbol{\rho}_2-\boldsymbol{\rho}_1)^2/2f^2}, \tag{14}$$

$$W_{xy}^{(f)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2) = B_{xy}\left(\frac{2\pi A\sigma\Omega_{xy}}{\lambda f}\right)^2 e^{-k^2\Omega_{xy}^2(\boldsymbol{\rho}_1+\boldsymbol{\rho}_2)^2/8f^2} \times e^{-k^2\sigma^2(\boldsymbol{\rho}_2-\boldsymbol{\rho}_1)^2/2f^2}, \tag{15}$$

$$W_{yy}^{(f)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2) = W_{xx}^{(f)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2), \tag{16}$$

$$W_{yx}^{(f)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2) = W_{xy}^{*(f)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2), \tag{17}$$

where we introduced the two new quantities as

$$\frac{1}{\Omega^2} = \frac{1}{4\sigma^2}+\frac{1}{\delta^2}, \qquad \frac{1}{\Omega_{xy}^2} = \frac{1}{4\sigma^2}+\frac{1}{\delta_{xy}^2}. \tag{18}$$

Using the preceding expressions for the elements of the cross-spectral density matrix in Eq. (6), while setting $\boldsymbol{\rho}_1=\boldsymbol{\rho}_2=\boldsymbol{\rho}$, we obtain the following simple result for the DOP in the back focal plane of the lens:

$$P^{(f)}(\boldsymbol{\rho}) = \frac{\Omega_{xy}^2}{\Omega^2}|B_{xy}|e^{-k^2\rho^2(\Omega_{xy}^2-\Omega^2)/2f^2}. \tag{19}$$

We note that if the beam in the front focal plane is completely unpolarized ($B_{xy}=0$), then, according to Eq. (19), the DOP will be zero in the back focal plane as well. However, when the beam is partially polarized, i.e., $|B_{xy}|>0$, several interesting effects occur. First of all, whereas the DOP in the front focal plane, as given by Eq. (12), is uniform (i.e., independent of $\boldsymbol{\rho}$), the DOP in the back focal plane is not. It reaches its highest value at the geometrical focus ($\boldsymbol{\rho}=\mathbf{0}$), and then decreases as a Gaussian function. In the second place, the DOP at the focus is enhanced by the action of the lens. The enhancement factor equals

$$\frac{P^{(f)}(\mathbf{0})}{P^{(\mathrm{in})}} = \frac{4\sigma^2+\delta^2}{4\sigma^2+\delta_{xy}^2}\left(\frac{\delta_{xy}}{\delta}\right)^2 \geq 1. \tag{20}$$

The inequality follows from using Eq. (12) together with the so-called realizability condition of {see Eq. (25) of Ref. [22]}

$$\delta \le \delta_{xy} \le \delta/|B_{xy}|^{1/2}. \tag{21}$$

An example is shown in Fig. 1. It is seen that the DOP at the geometrical focus is more than twice as large as the DOP in the front focal plane (indicated by the horizontal dashed line). It follows from Eq. (19) that the width of the DOP curve can be tuned by changing the focal length f of the lens. This is illustrated by the two curves that correspond to the case $f = 1$ m (blue) and 2 m (red), respectively. At a lateral distance of ~0.6 mm and ~1.2 mm the degree of polarization drops below its value in the front focal plane. At larger distances, the field becomes essentially unpolarized.

Under certain circumstances the field at the geometrical focus can become practically fully polarized. This occurs when the effective intensity width σ is substantially larger than the two coherence radii δ and δ_{xy}. In that case the source is quasi-homogeneous, and

$$\frac{\Omega_{xy}^2}{\Omega^2} \approx \frac{\delta_{xy}^2}{\delta^2}. \tag{22}$$

If, additionally, δ_{xy} takes on its maximum value allowed by the inequalities (21), i.e., $\delta_{xy} = \delta/|B_{xy}|^{1/2}$, then Eq. (19) reduces to

$$P^{(f)}(\boldsymbol{\rho} = \mathbf{0}) \approx 1. \tag{23}$$

This effect is illustrated in Fig. 2. Here a weakly polarized input beam (in this case with $P^{(\text{in})} = 0.25$) becomes locally fully polarized by the focusing process. For the lens with $f = 3$ m (green curve), the field is highly polarized (DOP > 0.6) in a circle with a radius of 1 mm around the geometrical focus. It is worth pointing out that one cannot increase the focal length indefinitely. At a certain point, the Fresnel number of the lens becomes so small that the so-called focal shift phenomenon comes into play [23,24].

The combination of two lenses is often used for making optical instruments such as microscopes and telescopes. Another application of two consecutive lenses is the Fourier processing of signals and images, a well-known technique in the context of coherent light [13–15]. Of particular interest is a $4f$ Fourier processor, as shown in Fig. 3, made with two identical lenses.

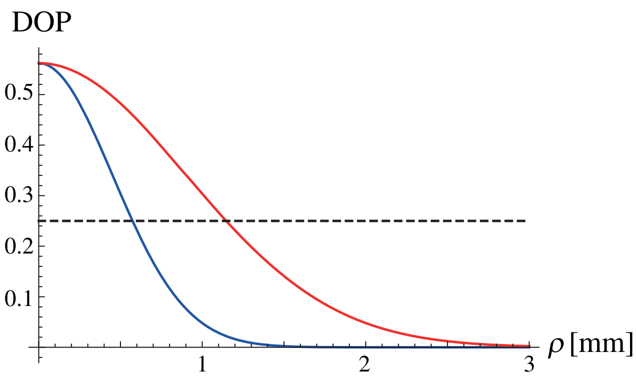

Fig. 1. Degree of polarization (DOP) in the back focal plane of a lens as a function of the radial distance ρ. The dashed horizontal line indicates $|B_{xy}|$, the modulus of the DOP in the front focal plane. In this example, $\lambda = 632.8$ nm, $f = 1$ m (blue curve) or $f = 2$ m (red curve), $B_{xy} = 0.25$, $\sigma = 5$ mm, $\delta = 0.2$ mm, and $\delta_{xy} = 0.3$ mm.

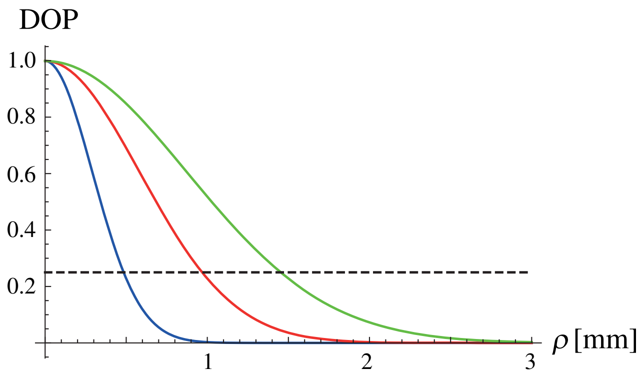

Fig. 2. Degree of polarization (DOP) in the back focal plane of a lens as a function of the radial distance ρ. The dashed horizontal line indicates $|B_{xy}|$, the modulus of the DOP in the front focal plane. In this example, $\lambda = 632.8$ nm, $f = 1$ m (blue curve), $f = 2$ m (red curve), or $f = 3$ m (green curve), $B_{xy} = 0.25$, $\sigma = 5$ mm, $\delta = 0.2$ mm, and $\delta_{xy} = 0.4$ mm.

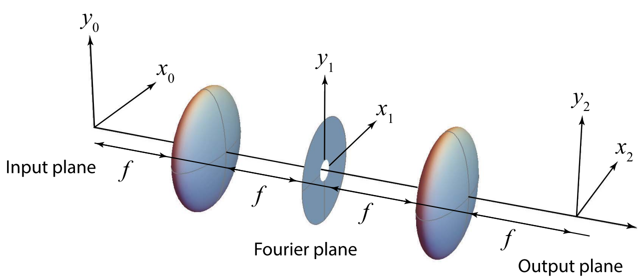

Fig. 3. $4f$ setup for Fourier processing consisting of two identical lenses with focal length f. An iris with radius a is situated in the Fourier plane.

As described in Refs. [17–20], the concept of spatial filtering can be extended to partially coherent scalar fields. In that case not the field itself, but rather its correlation function undergoes a Fourier transform. We now show how the degree of polarization of a partially coherent electromagnetic beam can be controlled using a $4f$ Fourier processor. We start with Eq. (4), which shows how the first lens generates in its back focal plane, (f_1), the four-dimensional Fourier transform of each cross-spectral density matrix element, as defined in Eq. (5). Let $F(\boldsymbol{\rho})$ denote the modifications induced by the phase or amplitude mask, i.e.,

$$E_i^{(m)}(\boldsymbol{\rho}) = F(\boldsymbol{\rho}) E_i^{(f_1)}(\boldsymbol{\rho}), \tag{24}$$

with the modified field indicated by the superscript m. The second lens will produce in the output plane the four-dimensional Fourier transform of the modified cross-spectral density matrix, given by the formula

$$W_{ij}^{(m)}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) = F^*(\boldsymbol{\rho}_1) F(\boldsymbol{\rho}_2) W_{ij}^{(f_1)}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2). \tag{25}$$

We can now proceed as before and derive that the cross-spectral density matrix elements in the output plane, denoted as f_2, the back focal plane of the second lens, satisfy the formula

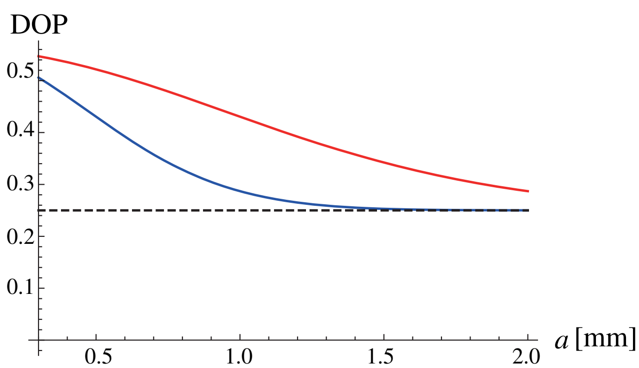

Fig. 4. Degree of polarization in the output plane as a function of the aperture radius a for $f = 1$ m (blue) and $f = 2$ m (red). The DOP in the input plane is indicated by the dashed horizontal line. All other parameters are defined in Fig. 1.

$$W_{ij}^{(f_2)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2)=\frac{1}{\lambda^2 f^2}\iint_{-\infty}^{\infty} W_{ij}^{(m)}(\boldsymbol{\rho}',\boldsymbol{\rho}'')\times\exp[-jk(\boldsymbol{\rho}_2\cdot\boldsymbol{\rho}''-\boldsymbol{\rho}_1\cdot\boldsymbol{\rho}')/f]\mathrm{d}^2\rho'\mathrm{d}^2\rho''. \tag{26}$$

We can use this equation to calculate the degree of polarization by using Eq. (6) after setting $\boldsymbol{\rho}_1 = \boldsymbol{\rho}_2 = \boldsymbol{\rho}$.

As an example of spatial filtering, let us consider a circular aperture with radius a, i.e., a filter function F such that

$$F(\rho) = \begin{cases} 1 & \text{if } \rho \le a, \\ 0 & \text{if } \rho > a. \end{cases} \tag{27}$$

For $W_{ij}^{(f_1)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2)$, the elements of the cross-spectral density matrix in the Fourier plane, we choose the expressions given by Eqs. (14)–(17). After some algebra, we obtain that

$$W_{xx}^{(f_2)}(\boldsymbol{\rho},\boldsymbol{\rho}) = \left(\frac{kA\sigma}{f}\right)^2 M(\rho)(1 - e^{-k^2\Omega^2 a^2/2f^2}), \tag{28}$$

$$W_{xy}^{(f_2)}(\boldsymbol{\rho},\boldsymbol{\rho}) = B_{xy}\left(\frac{kA\sigma}{f}\right)^2 M(\rho)(1 - e^{-k^2\Omega_{xy}^2 a^2/2f^2}), \tag{29}$$

$$W_{yy}^{(f_2)}(\boldsymbol{\rho},\boldsymbol{\rho}) = W_{xx}^{(f_2)}(\boldsymbol{\rho},\boldsymbol{\rho}), \tag{30}$$

$$W_{yx}^{(f_2)}(\boldsymbol{\rho},\boldsymbol{\rho}) = W_{xy}^{*(f_2)}(\boldsymbol{\rho},\boldsymbol{\rho}), \tag{31}$$

where the function $M(\rho)$ is defined as

$$M(\rho) = \int_0^{2a} R_{-} e^{-k^2\sigma^2 R_{-}^2/2f^2} J_0\left(\frac{k\rho R_{-}}{f}\right)\mathrm{d}R_{-}. \tag{32}$$

On substituting from Eqs. (28)–(31) into Eq. (6), we find that the DOP in the output plane is uniform (i.e., independent of the lateral distance ρ), and given by the formula

$$P^{(f_2)} = |B_{xy}|\frac{(1 - e^{-k^2\Omega_{xy}^2 a^2/2f^2})}{(1 - e^{-k^2\Omega^2 a^2/2f^2})}. \tag{33}$$

In Fig. 4, the DOP in the output plane is shown as a function of the iris radius a. It is seen that for both choices of the focal length f, a substantially increased DOP can be obtained, the precise value of which can be controlled by selecting the radius a of the iris in the Fourier plane. When the aperture radius increases, the DOP in the output plane gradually tends to its value in the input plane, as is expected. In summary, the incident beam has a low and uniform DOP. The DOP in the Fourier plane is increased near the focus, and has a Gaussian distribution. The amplitude filtering creates a field with a uniform and high DOP in the output plane.

The degree of polarization is a fundamental property that characterizes an electromagnetic beam. It describes the ratio of the intensity of its fully polarized part and its total intensity. We have derived Fourier transform relations that show that the DOP is strongly affected when a beam is focused by a lens. This new formalism was then applied to study a $4f$ Fourier processing setup. With such a device a significantly increased, and tunable DOP can be achieved.

Funding. Air Force Office of Scientific Research (AFOSR) (FA9550-16-1-0119); Directorate for Mathematical and Physical Sciences (MPS) (ECCS-1505636); China Scholarship Council (CSC).

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