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Effect of Raman scattering on soliton interactions in optical fibers

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We investigate numerically the effect of Raman scattering on the interaction of two temporally separated pulses with identical spectra that propagate inside a single-mode fiber as fundamental solitons. We take into account all interpulse Raman scattering terms in the generalized nonlinear Schrödinger equation and study the interplay between the Kerr, intrapulse Raman, and interpulse Raman effects. We observe considerable differences from the well-known two-soliton interaction behavior caused by the Kerr nonlinearity. We study in detail the mechanism for a net energy transfer from the leading pulse to the trailing pulse caused by the delayed nature of the Raman response in the case of two identical in-phase solitons. Long-range interactions, where the pulses do not temporally overlap, are found to not cause any energy transfer between the pulses, but solitons are still shown to affect each others' phase owing to the long tail of the Raman response function. We also study and compare how a phase difference between two otherwise identical solitons affects the interaction scenario and changes the collision dynamics, both in the presence and in the absence of the Raman effect. Finally, we look at the effect of changing the relative amplitude of the two interacting solitons by a small amount.

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1. INTRODUCTION

Formation and propagation of temporal solitons have been studied extensively in the context of optical fibers [1]. Solitons propagate stably only when the peak power of an optical pulse is large enough for the Kerr nonlinearity to balance the effects of group-velocity dispersion. But several higher-order dispersive and nonlinear effects can perturb a soliton, leading to the Raman-induced spectral shifts and creation of dispersive waves at new frequencies [1].

When two or more closely spaced solitons propagate together inside an optical fiber, they may attract or repel each other, depending on their relative phases. This interesting phenomenon of soliton interaction was studied during the 1980s in single-mode fibers [2–12]. Since then, it has been studied in several different contexts [13–29], and many of these studies include the effect of Raman scattering [13–26].

While the nonlinear response of a Kerr medium is instantaneous, the Raman response is known to be retarded. Moreover, stimulated Raman scattering can manifest in two different ways. In the case of relatively wide pump pulse, it leads to the generation of a new pulse at a frequency that is redshifted from the pump by about 13 THz in the case of silica fibers. However, for pulses shorter than a few picoseconds, the pulse spectrum itself shifts toward the red side in a continuous fashion through intrapulse Raman scattering. Such a Raman-induced frequency shift (RIFS) is also called the self-frequency shift since no other pulse is involved. However, when two or more closely spaced pulses are propagating through the fiber, because of the delayed nature of the Raman response, the leading pulse can affect the trailing pulses through the so-called interpulse Raman scattering. Indeed, such a Raman-induced interaction between two pulses was studied in 2015 inside a gas-filled fiber [18]. It is important to stress that interpulse Raman interaction can occur even when pulses are separated far enough that little temporal overlap occurs between them, as long as their temporal separation is smaller than the Raman response time. However, as we show in this paper, such long-range interactions cannot lead to any energy transfer between the two pulses in silica fibers.

The effects of Raman scattering in optical fibers have been studied in several different contexts. Experimental and numerical studies have shown a clear occurrence of energy transfer between two solitons propagating in a fiber, owing to interpulse Raman effects [24,25]. This has also been shown to significantly affect the collision dynamics. Raman-induced energy transfer between two solitons of different wavelengths plays a key role in supercontinuum generation in optical fibers [19,20]. Interpulse Raman scattering has also been studied in the context of wavelength-division multiplexed systems

[14,15]. More recently, this effect has been behind the generation of the so-called "rogue" solitons with extreme redshifts [21–23]. Most of these studies involve two or more solitons at different wavelengths traveling at different speeds within the fiber, which induces a collision between the pulses.

In this paper, we focus on the case of two temporally separated solitons *of the same wavelength* propagating at the same speed inside a single-mode fiber. This case was studied during the 1980s without including the Raman effect [2–12], and two solitons were found to interact substantially only when they were close enough that their tails overlapped significantly. The question we ask is how the Raman effect influences the Kerr-induced nonlinear interaction between the two solitons. Our numerical results show that, apart from the RIFS experienced by each soliton, interpulse Raman scattering modifies considerably the Kerr-induced interaction between the two solitons. We also consider the case of two pulses with little temporal overlap so that the nonlinear interaction depends solely on interpulse Raman scattering. For closely spaced pulses, the interplay among four-wave mixing (FWM), RIFS, and interpulse Raman scattering modifies the interaction behavior considerably from what has been known previously. Apart from the mutual attraction and repulsion of the two solitons, we also observe energy transfer between them [24,25] and we study how this affects collision dynamics in the case of two identical solitons.

The paper is arranged as follows. We discuss in Section 2 the underlying physical model and describe the numerical method employed. In Section 3 we study how the inclusion of Raman scattering modifies soliton interaction and analytically interpret these results. We study the impact of an initial phase difference between the pulses in Section 4 and of different initial amplitudes in Section 5. Finally, the main conclusions are summarized in Section 6.

2. NUMERICAL MODEL

We consider an ideal single-mode fiber and use the well-known generalized nonlinear Schrödinger equation in the form [1]

$$\frac{\partial A}{\partial z} - i\sum_{n=1}^{\infty} i^n \frac{\beta_n}{n!}\frac{\partial^n A}{\partial t^n} = i\left(\gamma(\omega_0) + i\gamma_1 \frac{\partial}{\partial t}\right) \times \left(A(z,t)\int_0^{\infty} R(t')|A(z,t-t')|^2 \mathrm{d}t'\right), \tag{1}$$

where $A(z,t)$ is the slowly varying pulse envelope, β_n are the dispersion coefficients appearing in the Taylor expansion of the propagation constant, $\gamma(\omega_0)$ is the nonlinear coefficient at the pulses' central frequency ω_0, and γ_1 is the self-steepening parameter. The Raman effects are included through the nonlinear response function $R(t)$ defined as

$$R(t) = (1 - f_R)\delta(t) + f_R h_R(t), \tag{2}$$

where f_R represents the fractional contribution of the delayed Raman response to the nonlinear polarization and h_R is the Raman response function related to vibrations of silica molecules. We use its form given in [30]:

$$h_R(t) = (1 - f_b)H_R(t) + f_b[(2\tau_b - t)/\tau_b^2]\exp(-t/\tau_b), \tag{3}$$

where $f_b = 0.21$, $\tau_b \approx 96$ fs, and $H_R(t)$ has the form

$$H_R(t) = \frac{\tau_1^2 + \tau_2^2}{\tau_1\tau_2^2}\exp\left(-\frac{t}{\tau_2}\right)\sin\left(\frac{t}{\tau_1}\right), \tag{4}$$

with $\tau_1 = 12.2$ fs, $\tau_2 = 32$ fs, and $f_R = 0.245$.

We retain only the first two terms in the sum appearing in Eq. (1) and normalize this equation using the so-called soliton units:

$$\tau = (t - \beta_1 z)/T_0, \qquad \xi = z/L_D,$$
$$u(\xi,\tau) = A(z,t)/\sqrt{P_0}, \tag{5}$$

where T_0 and P_0 are the width and the peak power of input pulses and $L_D = T_0^2/|\beta_2|$ is the dispersion length of the fiber. Using $\beta_2 < 0$ in the case of anomalous dispersion required for soliton formation, Eq. (1) takes the form

$$i\frac{\partial u}{\partial \xi} + \frac{1}{2}\frac{\partial^2 u}{\partial \tau^2} = -N^2\left(1 + is\frac{\partial}{\partial \tau}\right) \times \left(u(\xi,\tau)\int_0^{\infty} R(\tau')|u(\xi,\tau-\tau')|^2 \mathrm{d}\tau'\right). \tag{6}$$

The parameter N, defined as $N^2 = \gamma P_0 T_0^2/|\beta_2|$, represents the soliton order and $s = \gamma_1/(T_0\gamma)$ governs self-steepening. We solve the preceding equation in the frequency domain by taking the Fourier transform as

$$\tilde{u}(\xi,\omega) = \int_{-\infty}^{\infty} u(\xi,\tau)e^{i\omega\tau}\mathrm{d}\tau. \tag{7}$$

The resulting set of first-order differential equations is solved with the fourth-order Runge–Kutta method.

3. INTERACTION OF TWO IN-PHASE SOLITONS

To study soliton interaction, the input is in the form of two temporally separated pulses that can differ in phase and amplitude but have the same wavelength. More specifically, we use

$$u(0,\tau) = \mathrm{sech}[\tau + q_0] + r\,\mathrm{sech}[r(\tau - q_0)]e^{i\theta}. \tag{8}$$

$q_0 = \Delta t_0/2T_0$ depends on the initial temporal separation Δt_0 between the two pulses, θ is their relative phase difference, and r is the ratio of their amplitudes at $\xi = 0$. In this section, we set $\theta = 0$ and $r = 1$ and vary only q_0. In all simulations we choose $N = 1$ so that each pulse would propagate as a fundamental soliton in the absence of the other. We use $T_0 = 100$ fs, a value that corresponds to a full width at half-maximum of 176 fs for sech-shape pulses. We also set $s = 0$ since self-steepening effects are negligible for such wide pulses.

We begin by studying how the Raman effect qualitatively changes the nonlinear interaction of two identical in-phase solitons ($\theta = 0$ and $r = 1$) by turning the Raman term on and off through the parameter f_R. We choose $q_0 = 3.5$ or $\Delta t_0 = 7T_0$ so that pulse tails overlap to some extent. The top panel of Fig. 1 shows the evolution of two solitons over $100\,L_D$ in the absence of the Raman effect. As seen there, solitons attract each other through the Kerr nonlinearity and collide periodically, the first collision occurring at a distance of about $26\,L_D$, where the spectrum also broadens considerably. After each collision, the solitons cross over and start to move away from each other until the separation between them equals their initial

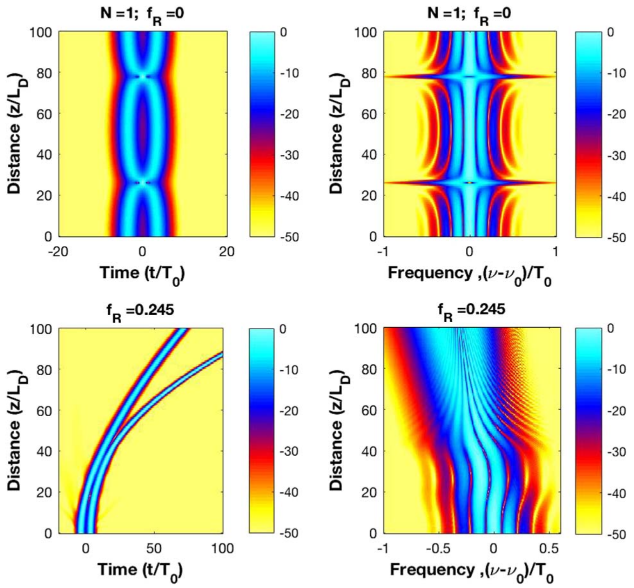

Fig. 1. Temporal and spectral evolutions over $100\,L_D$ of two temporally separated pulses when the Raman term is excluded (top row) or included (bottom row). The parameters used are $q_0 = 3.5$, $\theta = 0$, and $r = 1$. The color bar shows power on a decibel scale.

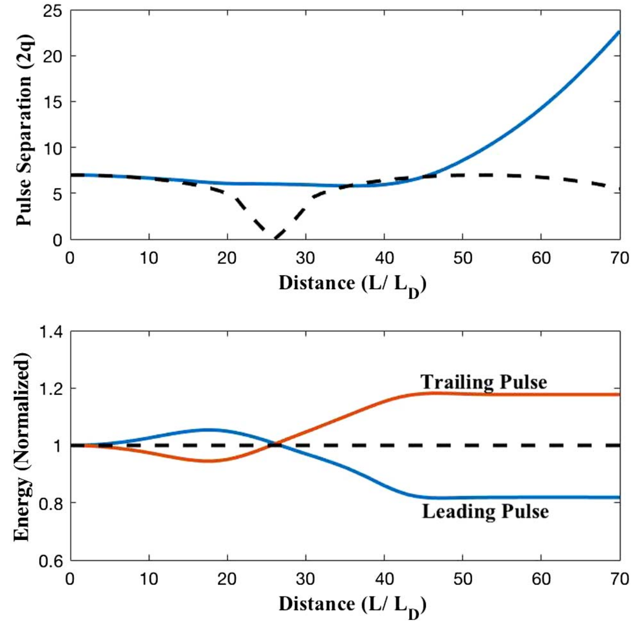

Fig. 2. Separation ($2q$) between two pulses (top) and energy in each pulse (bottom) as a function of distance ξ for the case shown in the bottom row of Fig. 1. The dotted line shows the corresponding plots when the Raman effect is not included ($f_R = 0$). No net energy transfer exists in this case.

separation. This process keeps repeating. Indeed, we can see the second collision occurring at a distance of $78\,L_D$. This behavior is well known and has been studied in detail [2–12].

The bottom panel of Fig. 1 shows how this classical interaction behavior changes when the Raman effects are included. As seen there, both solitons slow down as their spectra redshift because of the RIFS through intrapulse Raman scattering. Temporal separation between the pulses decreases initially owing to the Kerr effect. The solitons come very close to each other near $\xi = 40$, after which they begin to separate. This behavior is indicative of an out-of-phase collision [22] and occurs because the two solitons acquire different phase shifts as they propagate owing to the asymmetric nature of the interpulse Raman interaction. The solitons never fully overlap at the point of collision because of the destructive interference caused by the two out-of-phase pulses. Moreover, the trailing pulse becomes narrower and its RIFS increases considerably since it scales with the local pulse width T_s as T_s^{-4} [1]. The origin of temporal narrowing is related to the Raman-induced transfer of energy from the leading pulse to the trailing pulse, which must reduce its width to maintain $N = 1$, as required for fundamental solitons. The net result of energy transfer is that the two pulses begin to separate from each other owing to the different RIFS experienced by each pulse. A large RIFS is also apparent from tilting of the spectrum in the last panel in Fig. 1.

To quantify the extent of interplay between the Raman and Kerr effects, Fig. 2 shows the temporal separation of two pulses (top) and energy of each pulse (bottom) as a function of distance for the results shown in Fig. 1. In this figure, we clearly observe a net transfer of energy to the trailing pulse after $\xi = 25$, but before that distance it is the leading pulse that has more energy than the trailing pulse. To understand this strange behavior, we have to consider how the temporal separation of two pulses changes from its initial value of $2q_0 = 7$. As seen in Fig. 2, initially the two pulses move closer due to a Kerr-induced attractive force. At the same time, the spectra of both pulses redshift owing to the RIFS. Initially the Kerr effect dominates, but the interpulse Raman effects take over beyond $\xi = 20$. In the region between $\xi = 25$ to $\xi = 40$, there is a net energy transfer from the leading pulse to the trailing one, which becomes narrower to remain a soliton and undergoes even more RIFS, resulting in an increasing temporal separation of the two pulses. In short, there is an initial phase of attraction between the pulses, followed by an out-of-phase collision, after which the two pulses move away from each other owing to the energy transfer initiated by interpulse Raman scattering. Once they start moving apart, the Kerr effect weakens even more. The pulses eventually drift apart and stop interacting, with the trailing pulse becoming narrower and more intense than the leading pulse.

One may ask why the power is transferred from the leading pulse to the trailing pulse, and not *vice versa*. The answer becomes apparent after one considers the physics behind Raman scattering that leads to an asymmetry between the pulses. Even though both pulses induce molecular vibrations in the medium, the molecular vibrations induced by the leading pulse affect the trailing pulse much more than the other way around. Thus, because of the delayed nature of the Raman effect, some energy of the leading pulse used to excite molecular vibrations is transferred to the trailing pulse through Raman amplification. In a recent work, the leading pulse affected the trailing pulse through Raman-induced soliton interaction inside a gas-filled fiber [18].

If the same mechanism is at play in silica fibers, we should be able to see some Raman-induced interaction between the two input pulses that are separated far enough that Kerr-induced attraction is negligible between them. Figure 3 shows the temporal and spectral evolutions when the two pulses are initially separated by $16T_0$ (1.6 ps for $T_0 = 0.1$ ps) so that there is negligible overlap between them. The solitons still influence each other, but this is limited to a Raman-induced perturbation that forces pulses to shed some energy early on, after which they reshape to form Raman solitons whose spectra redshift continually. Both the intrapulse and interpulse Raman effects are at play here. The asymmetric nature of the interpulse Raman interaction leads to slightly different redshifts for the two pulses (resulting from the intrapulse Raman effect). Since the radiation is emitted at different frequencies, its interference produces the temporal fringe pattern seen in Fig. 3. The spectral fringes, on the other hand, result from beating of two temporally separated solitons, which maintain their temporal separation as they propagate along the fiber. Such long-range interactions can occur as long as temporal separation of the two pulses is within the Raman response time (~1 ps for silica fibers). Somewhat surprisingly, we do not observe any energy transfer between the pulses for such interactions. As we discuss below, interpulse Raman scattering responsible for energy transfer requires some temporal overlap between the pulses.

Two distinct nonlinear effects can lead to energy transfer among temporally separated optical pulses. One of them is interpulse FWM (related to the Kerr nonlinearity) that transfers energy from one pulse to its two neighboring pulses in a symmetric fashion. The other is interpulse Raman scattering, a nonlinear process that is inherently asymmetric because of its delayed nature. In this case the leading pulse transfers energy to trailing pulses through the onset of molecular vibrations. The combined effect of the two phenomena depends on the initial conditions. To understand how the competing Raman and Kerr nonlinearities affect soliton interaction, we need to isolate their contributions.

If we write the input field in the form $u(0,\tau) = u_1(0,\tau) + u_2(0,\tau)$, where u_1 and u_2 correspond to two temporally separated pulses, and substitute this form in Eq. (6),

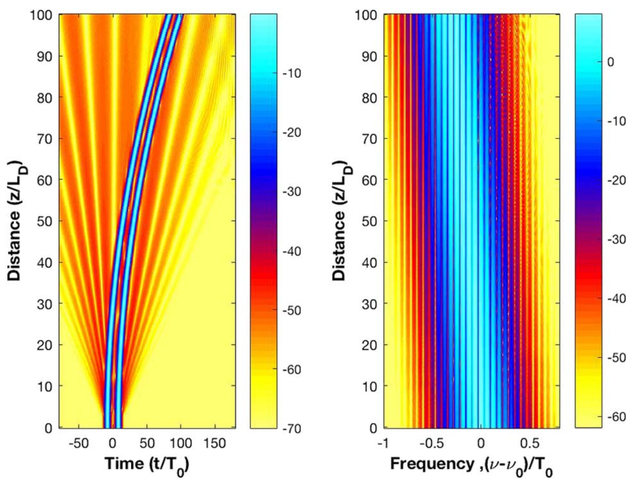

Fig. 3. Temporal (left) and spectral (right) evolution of two widely separated solitons with $q_0 = 8$. All other parameters remain the same. The color bar shows power on a decibel scale.

we get two coupled equations. The equation for u_1 can be written as

$$i\frac{\partial u_1}{\partial \xi} + \frac{1}{2}\frac{\partial^2 u_1}{\partial \tau^2} + (1 - f_R)[|u_1|^2 + 2|u_2|^2]u_1$$
$$= -(1 - f_R)u_1^2 u_2^* - f_R u_1 \int_0^\infty (|u_1|^2 + |u_2|^2)h_R(\tau')\mathrm{d}\tau'$$
$$- f_R u_2 \int_0^\infty (u_1 u_2^* + u_1^* u_2)h_R(\tau')\mathrm{d}\tau'. \tag{9}$$

The three terms on the right side respectively correspond to interpulse FWM, RIFS, and interpulse Raman scattering. There are also two nonlinear terms on the left side of Eq. (9). Some of these five terms cause nonlinear phase shifts while the others initiate an energy transfer. To isolate the energy transfer terms, we look at the evolution of the pulse intensity $|u_1|^2$ and obtain

$$\frac{\partial |u_1|^2}{\partial \xi} = -2\,\mathrm{Im}[(1 - f_R)u_2^2 u_1^{*2}]$$
$$- 2\,\mathrm{Im}[f_R u_1^* u_2 \int_o^\infty (u_1 u_2^* + u_2 u_1^*)h_R(\tau')\mathrm{d}\tau']. \tag{10}$$

The first term on the right side is due to the Kerr effect and represents energy transfer initiated by interpulse FWM. The second term is due to the Raman effect and represents energy transfer initiated by interpulse Raman scattering. The evolution equation for $|u_2|^2$ can be found by interchanging u_1 and u_2 in Eq. (10).

Equation (10) is quite useful for understanding the results shown in Figs. 1–3. The right side of Eq. (10) is 0 initially since both u_1 and u_2 are real at $\xi = 0$ for two in-phase solitons. For the energy transfer to occur, there has to be a relative phase difference between the two pulses. In the case of two in-phase pulses launched at the same frequency, a change in the relative phase can be initiated by cross-phase modulation, Raman scattering, or both.

Once a finite relative phase is established between the two pulses, the Raman term in Eq. (10) leads to an asymmetric transfer of energy between the two pulses. The asymmetry arises from the Raman response function $h_R(t)$, which has a long oscillatory tail and produces a net transfer of energy from the leading pulse to the trailing pulse as the two pulses propagate along the fiber. This energy transfer produces changes in the soliton widths, which in turn leads to different RIFS for each soliton. As a result, the solitons begin to move away from each other. So, while interpulse FWM and the Raman effect both can cause energy transfer between the solitons, it is the asymmetry of the Raman process that eventually leads to increasing separation of the two pulses. It is important to note that the RIFS term from Eq. (9) does not show up in the equation for energy transfer [Eq. (10)]. The right-hand side of Eq. (10) would remain 0 if there were no temporal overlap between the pulses. Thus, temporal overlap between the pulses is a necessity for energy transfer to occur, which explains the absence of any energy transfer in Fig. 3. On the other hand, the RIFS term from Eq. (9) would still survive, despite a lack of temporal overlap between the pulses, owing to the long tail of the delayed Raman response function ($h_R(t)$). This term affects soliton dynamics for pulses that are widely separated in time.

If we consider the spectral domain, the Raman gain is known to be zero for the zero frequency shift. So when there is interpulse energy transfer because of the Raman effect, one would expect energy to flow from the blue edge of the leading soliton to the red edge of the trailing soliton. We have verified this numerically by plotting the output spectrum of each pulse for the case shown in Fig. 1. The spectrum of each pulse was asymmetric such that the blue and red sides were steeper for the leading and trailing pulses, respectively. Such spectrally asymmetric energy transfer can result in an additional cross-frequency shift for the two solitons [21].

4. SOLITONS WITH A RELATIVE PHASE DIFFERENCE

The results discussed so far consider two in-phase fundamental solitons. Having an initial phase difference between the two pulses can change their nonlinear interaction considerably. Even in the absence of the Raman effect, an initial phase difference between the two solitons leads to out-of-phase collisions that exhibit different behavior compared to Fig. 1. The inclusion of the Raman effect can cause the relative phase to change in such a way that the interaction force on average is canceled out [17]. In this section, we investigate numerically how the interaction of solitons is affected by their initial relative phases.

To put the dependence of interaction on the initial relative phase into context, we begin by briefly reviewing the phase dependence of Kerr-induced interactions. As soon as a finite nonzero value of θ is introduced, the symmetry and periodicity seen in the top row of Fig. 1 are broken. For values of θ between 0 and $\pi/2$, the two pulses undergo an initial phase of attraction before they move away from each other. In the range $\theta = \pi/2$ to π, the pulses experience monotonic repulsion and begin to move away from each other right away. We can quantify this behavior by plotting the distance at which the pulses come closest to each other as a function of θ. This is shown in Fig. 4, where we also show the smallest separation of pulses. When θ is between $\pi/2$ and π (or between $-\pi$ and $-\pi/2$), the smallest separation occurs at $z = 0$.

Inclusion of Raman scattering changes the behavior shown in Fig. 4 drastically. Figure 5 shows the same two quantities as in Fig. 4, but with the Raman term included. A direct comparison of the two figures reveals the drastic changes induced by the Raman effect, the most noteworthy being that the behavior is not symmetric about $\theta = 0$. We still have a region of no attraction where pulses undergo monotonic repulsion, but this region does not begin exactly at $|\theta| = \pi/2$. In the region $\theta < |\pi/2|$, the pulses undergo an initial phase of attraction, before beginning to move away from each other. However, the minimum pulse separation and the distance at which that occurs do not follow any uniform pattern and even exhibit maxima and minima at specific values of θ. In our opinion, this behavior is related to temporal oscillations in the Raman response function $h_R(t)$. Figure 6 shows examples of the temporal evolution of the two pulses for two specific values of $\theta = \pi/4$ and $\theta = 3\pi/4$. Notice the different widths of two solitons in the first case after a distance of 40 L_D because of the transfer of energy from the leading soliton to the trailing one.

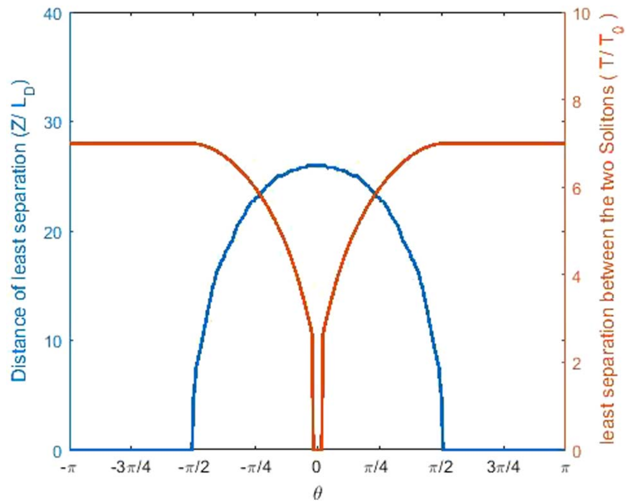

Fig. 4. Distance at which two pulses have smallest separation (blue) and the value of this separation (red) as a function of the relative phase θ. The other parameters are $q_0 = 3.5$ and $r = 1$. The Raman effect is not included here ($f_R = 0$).

Energy transfer is less significant in the second case where the two solitons begin to separate from each other right away.

We can use Eq. (9) to gain some physical insight into the numerical results shown in Figs. 4–6. If we assume that pulses are so wide that the dispersion terms can be neglected (the continuous-wave approximation) and use $u_j = \sqrt{P_j} e^{i\phi_j}$, where P_j is related to the peak power ($j = 1, 2$), we obtain the following set of three coupled equations for P_1, P_2, and $\theta = \phi_2 - \phi_1$:

$$\frac{\partial P_1}{\partial \xi} = -2c_f P_1 P_2 \sin(2\theta) - 2f_R P_1 P_2 \sin(2\theta), \tag{11a}$$

$$\frac{\partial P_2}{\partial \xi} = 2c_f P_1 P_2 \sin(2\theta) + 2f_R P_1 P_2 \sin(2\theta), \tag{11b}$$

$$\frac{\partial \theta}{\partial \xi} = (P_2 - P_1)[c_s - c_x - c_f \cos(2\theta) - 2f_R \cos(\theta)], \tag{11c}$$

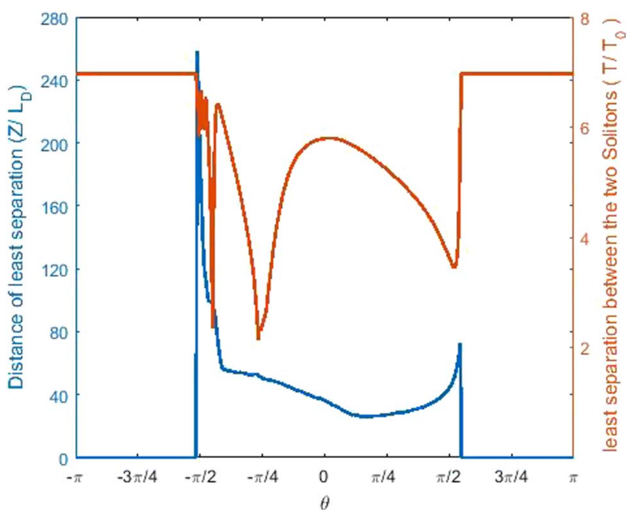

Fig. 5. Same as in Fig. 4 except that the Raman effect is included here.

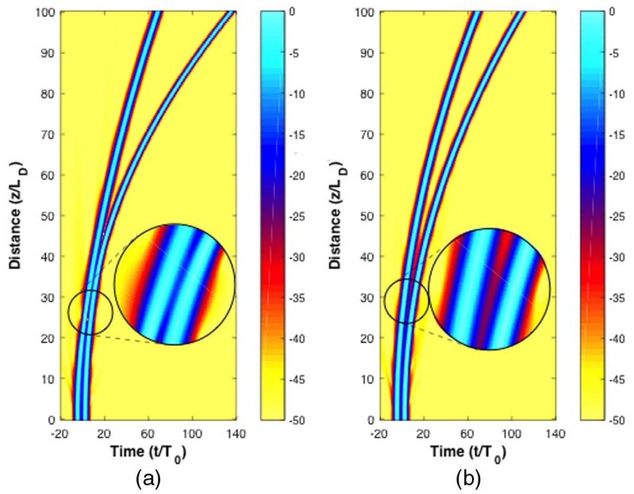

Fig. 6. Temporal evolution over 100 L_D of two solitons for an initial relative phase of (a) $\theta = \pi/4$ and (b) $\theta = 3\pi/4$. In both cases $q_0 = 3.5$ and $r = 1$. The color bar shows power on a decibel scale.

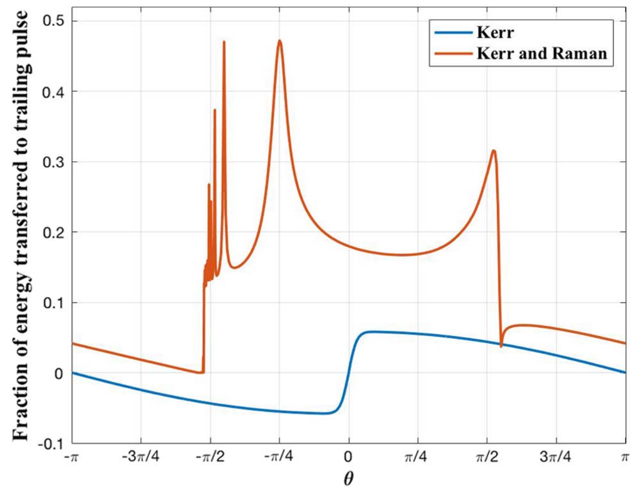

Fig. 7. Fraction of energy transferred to the trailing pulse as a function of the relative phase θ. The red curve is for the case when the Raman term is included. The other parameters are $q_0 = 3.5$ and $r = 1$. A positive value indicates that the trailing pulse gains energy.

where $c_s = 1 - f_R$, $c_x = 2(1 - f_R)$, and $c_f = 1 - f_R$ represent the relative contributions of self-phase modulation, cross-phase modulation, and FWM terms, respectively. These equations show how the peak powers and the relative phase evolve with ξ. If these quantities stop changing in the limit $\xi \to \infty$, a kind of steady state can be realized. It is easy to see that this can occur only if $P_1 = P_2 = P_0$ and $\theta = \theta_m = m\pi/2$ at $\xi = 0$, where m is an integer. The case of two equal-amplitude, in-phase solitons discussed in Section 3 corresponds to the choice $m = 0$.

We use a standard linear stability analysis to check the stability of the steady-state solutions. Assuming that small perturbations in phase grow with ξ as $e^{g\xi}$, the growth rate is found to be

$$g = 2P_0[(2 - f_R)(-1)^m(c_s - c_x - 2f_R \cos\theta_m) - c_f]^{1/2}. \quad \mathbf{(12)}$$

It turns out that for phase values that are multiples of π (m is an even integer), g is purely imaginary. In other words, the relative phase between the two pulses remains constant on propagation, if the initial value of θ is chosen to be $-\pi$, 0, or π. For other values of θ, the relative phase oscillates about the closest multiple of π. The case of $\theta = \pm\pi/2$ is a special case, since g is 0 at that point. So, if the initial value of θ is chosen to be $\pi/2$, it will stay the same during propagation of pulses along the fiber. However, even a slight perturbation would lead to θ oscillating about the next closest multiple of π. This is the reason that we observe a sudden change in behavior near $\theta = \pm\pi/2$ in Figs. 4 and 5. The term containing f_R in Eq. (12) is responsible for qualitative differences that appear when the Raman term is included.

One may ask how the energy transfer discussed in Section 3 for $\theta = 0$ changes for nonzero values of θ. Figure 7 shows the fraction of energy transferred to the trailing pulse as a function of θ. An interesting feature seen here is that even when the Raman term is not included, we observe some energy transfer between the pulses, but which pulse transfers the energy depends on whether θ is positive or negative. To the best of our knowledge, this Kerr-induced energy transfer has not been noticed in earlier studies. The energy transfer indicates that two closely spaced solitons with a nonzero value of θ do not represent an exact two-soliton solution of the nonlinear Schrödinger equation. Indeed, the well-known two-soliton solution cannot be reduced to the form of our initial conditions at any distance over one period. The physical reason of energy transfer is related to interpulse FWM. Equation (10) in the limit of $f_R \to 0$ (no Raman) shows that the term on the right side is not zero when the two solitons have a relative phase difference. It takes a positive value for one soliton, and a negative value for the other, indicating a net energy transfer.

The inclusion of the Raman term adds asymmetry to the energy transfer mechanism in the sense that the trailing pulse ends up getting more energy for almost any value of θ that we choose. The amount of energy transferred is also much larger compared to the Kerr case. The oscillatory nature of energy transfer is again related to the form of the Raman response function for silica glass.

5. TWO SOLITONS WITH DIFFERENT AMPLITUDES

We briefly consider the case of two in-phase solitons with different amplitudes by choosing $\theta = 0$ and $r \neq 1$ in Eq. (6). If the Raman term is not included in Eq. (2) by setting $f_R = 0$, it is known that solitons become resistant to both attractive and repulsive forces when $r \neq 1$. In other words, spacing between the two solitons oscillates as they propagate along the fiber such that they maintain the initial temporal separation between them on average.

The inclusion of the Raman term once again introduces asymmetry to this scenario in the sense that dramatically different behavior occurs depending on whether $r > 1$ or <1. Figure 8 shows the two cases by choosing $r = 0.9$ and $r = 1.1$ along with the spectrograms at a distance of 60 L_D (bottom row). A relative difference in the peak powers of two solitons

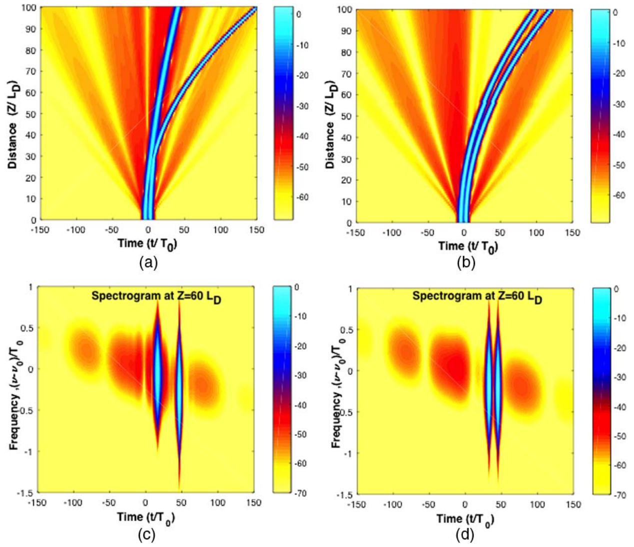

Fig. 8. (top row) Temporal and spectral evolutions over $100\,L_D$ of two in-phase solitons with different amplitudes: (left) $r = 0.9$ and (right) $r = 1.1$. In both cases $q_0 = 3.5$ and $\theta = 0$. (bottom row) Spectrograms at a distance 60 L_D in the two cases showing different redshifts for the two solitons. Temporal fringes are caused by a small frequency difference between the two dispersive waves shed by solitons.

causes them to have different widths, which causes them to redshift through RIFS by different amounts, and thus travel at different speeds along the fiber. If the trailing pulse has a higher peak power ($r > 1$), it experiences a stronger redshift, and the temporal separation increases monotonically between the two pulses. In contrast, if the leading pulse has a higher peak power ($r < 1$), it catches up with the trailing pulse after propagating a certain distance. At this point the pulses appear to cross each other before separating. Closer inspection reveals that, in fact, the two pulses never completely overlap, indicating an out-of-phase collision. As soon as the leading pulse approaches the trailing pulse, they begin to exchange energy through interpulse Raman scattering. As a result, the trailing pulse becomes more energetic and begins to move away from the other one.

Red bands in Fig. 8 represent dispersive waves created when solitons shed some energy as they are perturbed by the Raman effect. This radiation appears to form a fringe pattern whose origin can be understood as follows. As each pulse is perturbed by the other pulse, it sheds some energy in the form of a dispersive wave. But since the pulses have redshifted by different amounts (owing to different initial peak powers), the excess energy shed from each pulse is at slightly different frequencies, leading to interference between the radiation emitted by each pulse and creating a fringe pattern seen in Fig. 8. This behavior has been observed in an experiment where soliton collisions were found to create dispersive waves [31].

6. CONCLUDING REMARKS

We have studied in considerable detail the effect of Raman scattering on the nonlinear interaction of two temporally separated pulses with identical spectra that propagate inside a single-mode fiber as fundamental solitons. We distinguish carefully between the intrapulse and interpulse Raman effects. The former produces spectral redshifts of each pulse, while the latter can lead to energy transfer between the two pulses in addition to spectral redshifts.

We first considered the special case of two identical in-phase solitons. In the absence of the Raman effects, it is known that the two solitons collide periodically along the fiber owing to a Kerr-induced attraction between them. The interplay between the Kerr effect and the delayed Raman response changes considerably this known behavior caused by the Kerr nonlinearity. Our results show that, even though the two solitons still experience some attraction initially, they undergo an out-of-phase collision before beginning to separate from each other because of the Raman-induced spectral redshifts that change the relative speed of two pulses inside the dispersive nonlinear medium. Moreover, the delayed nature of the Raman response leads to considerable transfer of energy from the leading pulse to the trailing pulse through the interpulse Raman interaction between the two solitons. As a result of this energy transfer, the trailing soliton becomes narrower compared to the leading one and its spectrum is redshifted further because of intrapulse Raman scattering. The net result is that the two solitons that were identical in all respects initially develop different spectra, widths, and peak powers as they propagate inside an optical fiber.

We also investigated how different amplitudes or phases of the two solitons affect the interaction scenario in the presence of the Raman effect. In the case of different input phases, the soliton interaction depends considerably on the precise value of the initial phase difference between the two input pulses. We studied the minimum spacing between the solitons and the distance at which that occurs as a function of the relative phase shift and compared the interaction behavior with and without including the Raman effects. The fraction of energy transferred from the leading pulse to the trailing pulse also depends considerably on the precise value of the relative phase shift and this fraction can exceed 40% for some specific values of this parameter. A new feature that remained unnoticed in the past studies is that energy transfer between the two solitons can occur even when solitons interact only through the Kerr nonlinearity if their relative phase is finite initially. However, the fraction of energy transferred is typically below 10%. Moreover, the energy is transferred from the trailing pulse to the leading pulse for negative values of this relative phase, a situation that never occurs when the Raman effects are included.

In the case of different-amplitude solitons, we found considerably different behavior depending on whether the leading pulse or the trailing pulse has a higher amplitude initially owing to the asymmetry caused by the Raman effect. When optical pulses propagate as solitons, the soliton with a higher amplitude is also narrower. As a result, its redshift through intrapulse Raman scattering is enhanced and it moves slower compared to the soliton with a smaller amplitude. The extent of energy transfer also depends on whether the leading soliton has a higher or smaller amplitude compared to the trailing one.

It should be evident from our results that both the intrapulse and interpulse Raman effects play a critical role in the nonlinear interaction of two temporally separated solitons. For pulses shorter than a few picoseconds, these effects cannot be ignored and should be considered in any study where two or more temporally separated short optical pulses are transmitted through a nonlinear dispersive medium. An experimental verification of our predictions would be of considerable interest and can be carried out by launching two short pulses (width <1 ps) into a highly nonlinear fiber (such as a photonic crystal fiber) from a mode-locked laser operating at a wavelength that lies in the region of anomalous dispersion of the fiber.

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