

OPTICAL PHYSICS

Temporal waveguides for optical pulses

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Temporal total internal reflection (TIR), in analogy to the conventional TIR of an optical beam at a dielectric interface, is the total reflection of an optical pulse inside a dispersive medium at a temporal boundary across which the refractive index changes. A pair of such boundaries separated in time acts as the temporal analog of planar dielectric waveguides. We study the propagation of optical pulses inside such temporal waveguides, both analytically and numerically, and show that the waveguide supports a finite number of temporal modes. We also discuss how a single-mode temporal waveguide can be created in practice. In contrast with the spatial case, the confinement can occur even when the central region has a lower refractive index. © 2016 Optical Society of America

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1. INTRODUCTION

The fundamental concept of total internal reflection (TIR) at a dielectric interface has been known since 1840 and is discussed thoroughly in optics textbooks [1,2]. It has been used to make optical waveguides that confine an optical beam to the vicinity of a central core region whose refractive index is chosen to be higher than the surrounding cladding regions [3,4]. A multitude of applications have been found for waveguides, notably in the optical fibers used extensively for designing modern telecommunication systems [5].

The temporal analog of time reflection and refraction was first explored in the context of photon acceleration in plasmas 8-6] and occurs when an optical pulse approaches a moving temporal interface separating two regions with different refractive indices. This process has since been studied in several different contexts [9–16]. In the context of nonlinear optics, rapid rise (or fall) in the intensity of a pump pulse creates a moving temporal boundary, and the spectrum of a probe pulse shifts in such a way that the probe appears to being reflected from the temporal boundary. This nonlinear phenomenon has recently been explored in the contexts of analog gravity [17–19] and optical solitons [20–22].

Although optical nonlinearities are often used to create temporal boundaries, temporal TIR is a general concept, and any technique that shifts the refractive index in time can be used for realizing it [16,17,22]. Here we show that two temporal boundaries that satisfy the temporal TIR condition can be used to make a temporal analog of an optical waveguide, which confines the pulse to a central time window inside which the refractive index is different from the outer regions. A similar arrangement was first examined as a temporal support structure for reducing soliton jitter [23]. More recently, this idea has been explored for bouncing an optical pulse between the two boundaries [19,24–26]. In this work we explore, both analytically and numerically, the propagation of optical pulses inside a temporal waveguide and address the existence of temporal modes supported by such waveguides.

The paper is organized as follows. In Section 2, we discuss the conditions under which the temporal analog of TIR can occur. We use these results in Section 3 to form a temporal waveguide in which a short pulse bounces back and forth between the two temporal boundaries. Analogous to conventional spatial waveguides, a temporal waveguide supports a finite number of modes. This topic is discussed in Section 4. As shown in Section 5, single-mode temporal waveguides can be designed such that a pulse trapped inside it maintains its width even in the presence of group-velocity dispersion (GVD). The main results are summarized in Section 6.

2. TEMPORAL TIR

To simplify the following discussion, we consider an optical pulse in the form of a plane wave propagating inside a medium with the dispersion relation $\beta(\omega)$, where $\beta = n(\omega)\omega/c$ is the propagation constant at a specific frequency ω and $n(\omega)$ is the refractive index at that frequency. When the pulse contains multiple optical cycles (>10), its spectral width is short enough that $\beta(\omega)$ can be Taylor-expanded around its central frequency ω_0 as

$$\beta(\omega) = \beta_0 + \beta_1(\omega - \omega_0) + \frac{\beta_2}{2}(\omega - \omega_0)^2,$$
 (1)

where β_1 is the inverse of the group velocity, and β_2 is the GVD parameter. The higher-order terms in this Taylor expansion can be neglected for sufficiently wide pulses.

If the refractive index changes across a temporal boundary located at $T = T_{\rm B}$, $\beta(\omega)$ will be different on each side of the boundary. In general, the refractive index boundary may be moving, and we consider such a situation in this paper. For a temporal boundary that is moving with the speed $v_{\rm B}$, we work in a reference frame in which the boundary is stationary. Using the coordinate transformation $t = T - z/v_{\rm B}$, where T is the time in the laboratory frame, the dispersion relation in the moving frame becomes

$$\beta'(\omega) = \beta_0 + \Delta\beta_1(\omega - \omega_0) + \frac{\beta_2}{2}(\omega - \omega_0)^2 + \beta_B(t), \quad (2)$$

where $\Delta\beta_1 = \beta_1 - 1/\nu_B$ is a measure of the optical pulse's speed relative to the boundary. The quantity $\beta_B(t) = k_0 \Delta n(t)$ ($k_0 = \omega_0/c$) represents the change in the propagation constant caused by the time-dependent index change $\Delta n(t)$. For a temporal boundary located at $t = T_B$, β_B takes on different values for $t > T_B$ and $t < T_B$. For simplicity, we assume that $\beta_B = 0$ for $t < T_B$. In this case, the dispersion curve shifts upward if $\Delta n > 0$ for $t > T_B$, causing different propagation constants in the two temporal regions.

Using Maxwell's equations together with the dispersion relation in Eq. (2) and making the slowly varying envelope approximation, we obtain [27]

$$\frac{\partial A}{\partial z} + \Delta \beta_1 \frac{\partial A}{\partial t} + \frac{i\beta_2}{2} \frac{\partial^2 A}{\partial t^2} = i\beta_B(t)A,$$
(3)

where A(z, t) is the pulse envelope at a distance z. To simplify our discussion, we have taken $\beta_{\rm B}(t)$ to be a step function located at $t = T_{\rm B}$. In practice, a temporal boundary will have a finite rise time that may also change during propagation owing to dispersion. The exact form of $\beta_{\rm B}(t)$ depends on the physical mechanism used to produce the temporal boundary. This issue is discussed further in Section 6.

We solve Eq. (3) numerically with the standard split-step Fourier method used commonly for nonlinear problems [27]. Figures 1(a) and 1(b) show the temporal and spectral evolutions, respectively, of a Gaussian input pulse, $A(0, t) = A_0 \exp[-t^2/(2T_0^2)]$, using parameter values $T_0 = 1.5$ ps, $T_B = 5$ ps, $\Delta\beta_1 = 0.667$ ps/m, $\beta_2 = 0.05$ ps²/m, and $\beta_B = 5.6$ m⁻¹. The value of β_B used here requires an index change of $\Delta n < 10^{-6}$ for the temporal boundary at a wavelength of 1 µm. The dispersion length, defined as $L_D = T_0^2/|\beta_2|$, is 45 m for these parameter values, resulting in a pulse broadening of only 5.4% over a distance of 15 m [27]. We stress that our choice of parameters is arbitrary. The numerical results and the conclusion of this paper apply for distances ranging from <1 cm to >1 km with a proper choice of the pulse width and other parameters.

The temporal evolution in Fig. 1(a) shows that the pulse behavior across a temporal boundary is strikingly similar to an optical beam undergoing TIR at a spatial boundary. We emphasize that the pulse keeps moving in the forward direction, but its speed changes in such a way that it appears to recede from the temporal boundary. The spectral evolution in Fig. 1(b) shows that TIR is accompanied by a large spectral



Fig. 1. Evolution of pulse (a) shape and (b) spectrum in the presence of a temporal boundary (dashed white line) for $\beta_{\rm B} = 5.6$ m⁻¹. (c) Dispersion curves for $t < T_{\rm B}$ (dashed blue) and $t > T_{\rm B}$ (solid orange). The red-shaded region shows the spectral extent of the input pulse and the corresponding range of the propagation constants.

shift that is responsible for this speed change. The spectral changes can occur because a temporal boundary breaks symmetry in time. As a result, photon momentum in the moving frame (or β') must be conserved while photon energy (or ω) may change. In contrast, energy is conserved in the spatial case while the momentum undergoes a change.

Momentum conservation is imposed at the central frequency ω_0 by setting $\beta'(\omega) = \beta_0$ in the dispersion relation given in Eq. (2) to find the reflected and transmitted frequencies respectively as

$$\omega_r = \omega_0 - 2\frac{\Delta\beta_1}{\beta_2},\tag{4}$$

$$\omega_t = \omega_i + \frac{\Delta\beta_1}{\beta_2} \left[-1 \pm \sqrt{1 - \frac{2\beta_B \beta_2}{(\Delta\beta_1)^2}} \right].$$
 (5)

From Eq. (5), we see that the transmitted frequency becomes complex and loses its physical meaning when the following TIR condition is satisfied:

$$\sqrt{2\beta_{\rm B}\beta_2} > \Delta\beta_1. \tag{6}$$

Figure 1(c) shows dispersion curves for $t < T_{\rm B}$ (dashed blue) and $t > T_{\rm B}$ (solid orange). In the case of TIR, the solid curve in Fig. 1(c) is shifted enough that momentum conservation cannot be achieved, so the pulse must be completely reflected. This is the temporal analog of the well-known phenomenon of total internal reflection. Since the GVD parameter β_2 can take either positive or negative values, temporal TIR exhibits much richer behavior than its spatial counterpart. In particular, it can occur even when the refractive index is increased across the boundary ($\beta_{\rm B} > 0$), provided β_2 is also positive. The well-known spacetime analogy shows that the case of anomalous dispersion ($\beta_2 < 0$) corresponds to the usual diffraction in space, so the refractive index must decrease across the boundary for TIR to occur [28].

3. EXAMPLE OF A TEMPORAL WAVEGUIDE

We now ask what happens when an optical pulse is located between two temporal boundaries that both move at the same speed and satisfy the TIR condition given in Eq. (6). If the group velocity of the pulse differs from that of the temporal boundaries, we expect that the pulse will travel toward one of the temporal boundaries and be reflected completely. Since the spectrum of the reflected pulse will be shifted as indicated in Eq. (4), it will move away from that first boundary with a new group velocity. However, unlike the single boundary case, the pulse will now arrive at the second temporal boundary, where it will once again experience TIR, but this time its center frequency will shift back to the initial value. This process should repeat itself, trapping the pulse between the two temporal boundaries. This behavior is analogous to that of an optical beam inside two spatial boundaries that form the core of a conventional waveguide such that the beam undergoes TIR multiple times as it travels through the waveguide. Because of this analogy, we refer to the configuration with two temporal boundaries as a temporal waveguide. The trapping effect has been previously examined in the context of soliton fission [24,29], and we stress that the temporal waveguide does not require optical nonlinearities.

To explore the behavior of an optical pulse inside such a temporal waveguide, we launch a Gaussian pulse ($T_0 = 1.5$ ps) with its peak located in the middle of a 10 ps wide temporal waveguide and study how the pulse shape and spectrum evolve along a 100 m long optical fiber by solving Eq. (3) numerically. Figure 2 shows the temporal and spectral evolutions using the same parameters as Fig. 1, i.e., $\Delta\beta_1 = 0.667$ ps/m, $\beta_2 = 0.05$ ps²/m, and $\beta_B = 5.6$ m⁻¹ for |t| > 5 ps but 0 for |t| < 5 ps. Notice that with this choice of β_B , the refractive index is smaller inside the core of our temporal waveguide.

Figure 2 shows how the optical pulse bounces back and forth between the two temporal boundaries, where it is completely reflected as predicted by the TIR condition in Eq. (6). The reflection at the top boundary is accompanied by the frequency shift of $\Delta \nu_r = -4.24$ THz (see Fig. 1), which changes the relative group velocity of the pulse so that it now moves toward the lower temporal boundary located at t = -5 ps. When the pulse is totally reflected at this boundary, its center frequency is shifted back to its original value, and it begins to travel at its original group velocity toward the top temporal



Fig. 2. Evolution of the (a) shape and (b) spectrum of a Gaussian pulse ($T_0 = 1.5$ ps) inside a 10 ps wide temporal waveguide. Dashed horizontal lines show the temporal boundaries that form the waveguide core.

boundary. This process repeats multiple times with further propagation. Temporal fringes near the two boundaries are a consequence of the spectral shifts required for TIR to occur. Indeed, the 0.23 ps fringe spacing correlates perfectly with the 4.24 THz spectral shift.

One may ask whether the 1.5 ps pulse can remain confined within the 10 ps time window indefinitely, while maintaining its original shape and size. In Fig. 2(a), we see that the pulse broadens noticeably after 50 m. Broadening occurs because the dispersion length is 45 m for the parameter values used here. We cannot make dispersion negligible because TIR does not occur in its absence. The GVD also causes certain wavelengths to reflect sooner, leading to the sloped appearance of the spectrum during propagation, as seen in Fig. 2(b). The predicted behavior is analogous to that observed in spatial multimode waveguides designed with a core whose size is much larger than the beam width. Since the 10 ps core of our temporal waveguide is much wider than the 1.5 ps pulse, multiple modes are excited, which interfere with each other as the pulse propagates down the medium. This suggests that we should analyze the optical modes supported by temporal waveguides.

4. MODES OF A TEMPORAL WAVEGUIDE

The spatial modes of planar waveguides have been extensively studied [3,4]. By definition, the shape of an individual mode does not change during propagation. To develop an analogous theory for the modes of a temporal waveguide, we seek solutions to Eq. (3) that do not change with propagation except for a phase shift. Therefore, we assume a modal solution of the form

$$A(z,t) = M(t) \exp[i(Kz - \Omega t)],$$
(7)

where M(t) is the temporal shape of the mode, K is the rate at which the mode accumulates phase during propagation, and Ω is a frequency shift such that $\omega_0 + \Omega$ becomes the new central frequency of the mode. There is no spatial analog of this frequency shift. It is needed in the temporal case because the first derivative in Eq. (3) is related to the speed at which the pulse approaches a temporal boundary.

Substituting Eq. (7) into Eq. (3) and equating the real and imaginary parts, we obtain

$$(\Delta\beta_1 + \beta_2 \Omega) \frac{dM}{dt} = 0,$$
(8)

$$\frac{d^2M}{dt^2} + \frac{2}{\beta_2} \left(K - \Omega \Delta \beta_1 - \frac{\beta_2 \Omega^2}{2} - \beta_B \right) M = 0.$$
 (9)

From Eq. (8), we find that the frequency shift Ω must be chosen as $\Omega = -\Delta \beta_1 / \beta_2$. This corresponds to a pulse that will propagate at the same speed as the two temporal boundaries. A spectral shift is necessary if the mode were to remain confined with the temporal waveguide.

Using this value for $\Delta\beta_1$ to replace Ω in Eq. (9), the mode shape is governed by the simple equation

$$\frac{d^2M}{dt^2} + \frac{2}{\beta_2} \left(K + \frac{(\Delta\beta_1)^2}{2\beta_2} - \beta_B \right) M = 0.$$
 (10)

To be as general as possible, we assume that $\beta_{\rm B}$ takes different values for $|t| < T_{\rm B}$, $t > T_{\rm B}$, and $t < -T_{\rm B}$. We label these three values as $\beta_{\rm B0}$, $\beta_{\rm B1}$, and $\beta_{\rm B2}$, respectively. The temporal waveguide becomes symmetric when $\beta_{\rm B1} = \beta_{\rm B2}$.

To solve Eq. (10), we follow a procedure analogous to that used for spatial planar waveguides [3] and write its solution in the form

$$M(t) = \begin{cases} B_1 \exp[-\Omega_1(t - T_B)] & t > T_B, \\ A \cos(\Omega_0 t - \phi) & |t| < T_B, \\ B_2 \exp[\Omega_2(t + T_B)] & t < -T_B, \end{cases}$$
(11)

where the parameters Ω_0 , Ω_1 , and Ω_2 are defined respectively as

$$\Omega_0^2 = \frac{2K}{\beta_2} + \left(\frac{\Delta\beta_1}{\beta_2}\right)^2 - \frac{2\beta_{B0}}{\beta_2},$$
 (12)

$$\Omega_1^2 = \frac{2\beta_{\text{B1}}}{\beta_2} - \frac{2K}{\beta_2} - \left(\frac{\Delta\beta_1}{\beta_2}\right)^2,$$
(13)

$$\Omega_2^2 = \frac{2\beta_{B2}}{\beta_2} - \frac{K}{\beta_2} - \left(\frac{\Delta\beta_1}{\beta_2}\right)^2.$$
 (14)

The four constants B_1 , B_2 , A, and ϕ can be related by imposing the boundary conditions that both M(t) and its derivative dM/dt be continuous across the two temporal interfaces. The boundary conditions lead to the following two relations:

$$\tan(\Omega_0 T_{\rm B} - \phi) = \frac{\Omega_1}{\Omega_0}, \qquad \tan(\Omega_0 T_{\rm B} + \phi) = \frac{\Omega_2}{\Omega_0}.$$
 (15)

These equations can be used to find the eigenvalue equation in the form

$$2\Omega_0 T_{\rm B} = m\pi + \tan^{-1}\left(\frac{\Omega_1}{\Omega_0}\right) + \tan^{-1}\left(\frac{\Omega_2}{\Omega_0}\right), \qquad (16)$$

where the integer m = (0, 1, 2, ...) denotes the mode order. For each value of *m*, the eigenvalue equation can be solved to find the value of *K* for that specific mode at that value of $\Delta\beta_1$. We stress that the value of *K* changes with $\Delta\beta_1$ such that $K + (\Delta\beta_1/2\beta_2)^2$ is the same for a given mode of the waveguide. In analogy with spatial waveguides, we call the m = 0 mode the fundamental temporal mode of the waveguide.

For simplicity, we focus on symmetric waveguides for which the index jump is identical at both temporal boundaries so that $\Omega_1 = \Omega_2$. In this case, the eigenvalue equation takes a much simpler form:

$$\Omega_1 = \Omega_0 \tan(\Omega_0 T_{\rm B} + m\pi/2). \tag{17}$$

We also introduce a dimensionless parameter as [3]

$$V = T_{\rm B} \sqrt{\Omega_0^2 + \Omega_1^2} = \sqrt{\frac{2(\beta_{\rm B1} - \beta_{\rm B0})T_{\rm B}^2}{\beta_2}}.$$
 (18)

It plays an important role in determining the number of modes supported by the temporal waveguide. In complete analogy with the spatial case, the waveguide supports m modes when $V < (m + 1)\pi/2$. In particular, a temporal waveguide will support only the fundamental m = 0 mode if it is designed such that $V < \pi/2$.

Consider the temporal waveguide used for Fig. 2. Using the known parameter values in Eq. (18), we find that this waveguide has V = 74.8 and supports 48 modes. The temporal and spectral evolutions of several modes (m = 0, 2, 10) of this waveguide are shown in Fig. 3. As expected, the modes propagate without changing their shape or spectrum. In analogy with a spatial waveguide, the *m*th-order mode has m + 1 distinct peaks inside the temporal window of the waveguide.

The new feature in the temporal case is different spectral shifts associated with different modes. The fundamental mode propagates with a central frequency shifted by $\Omega/(2\pi) =$ 2.12 THz, which matches the frequency shift predicted by $\Omega = -\Delta \beta_1 / \beta_2$. The spectra of all of the higher-order modes (m > 0) exhibit two intense peaks located at approximately $(\Omega \pm \Omega_0)/(2\pi)$. The two spectral peaks beat together to form the temporal oscillations of the cosine term in Eq. (11), resulting in the multiple-peaked structure of the higher-order modes (m > 0). For the highest-order mode supported by the waveguide, the value of Ω_0 is the highest frequency offset that still satisfies the TIR condition at both of the temporal boundaries. In addition to the two main spectral peaks, all modes have several lower-intensity spectral peaks that are separated by $\Delta \nu = 1/(2T_{\rm B})$. These spectral peaks are caused by an interference between the evanescent tails of the mode, which are separated in time by $2T_{\rm B}$. For less confined modes, these oscillations become more intense as more of the pulse energy is contained in the evanescent tails.



Fig. 3. Evolution of shapes (top row) and spectra (bottom row) for modes with [(a), (d)] m = 0; [(b), (e)] m = 2; and [(c), (f)] m = 10 for a 10 ps wide temporal waveguide with V = 74.8.

5. SINGLE-MODE TEMPORAL WAVEGUIDE

Spatial waveguides supporting a single mode have found a variety of applications. It is therefore useful to consider single-mode temporal waveguides. As discussed earlier, only a single mode will propagate if the temporal waveguide is designed with $V < \pi/2$. From Eq. (18), we see that this condition can be satisfied by either reducing the width of the waveguide (parameter $T_{\rm B}$) or by decreasing the magnitude of the index change at the temporal boundaries. Such a waveguide will support only the m = 0 mode with a temporal shape similar to that shown in Fig. 3(a).

An important question is how an optical pulse with a shape different from that of the fundamental mode behaves when launched into such a single-mode waveguide. Figure 4 shows the simulated behavior for a Gaussian input pulse with $T_0 =$ 3.5 ps inside a 10 ps wide symmetric temporal waveguide designed with V = 1.414. The pulse is launched with an initial differential group delay of $\Delta\beta_1 = 10$ ps/km. From Fig. 4(a), we see that the pulse initially bounces off a few times at the two temporal boundaries through TIR, losing a considerable portion of its energy into the "cladding" region (outside of dashed boundaries) in the form of dispersive waves [27]. However, it eventually stops oscillating and acquires the shape of the fundamental mode supported by the waveguide. This behavior is analogous to that which occurs when an optical beam traveling at an angle is launched into a spatial waveguide.

The spectral evolution seen in Fig. 4(b) appears strange and has a "chevron" shape that does not match the spectrum of the fundamental mode. This apparent discrepancy occurs because the total electric field at any location contains both the guided and unguided light at different frequencies. As a result of their interference, the simulated spectrum acquires a fringe-like structure. Numerically, it is easy to filter out the unguided components. Figure 4(c) shows the resulting spectrum, which exhibits the expected behavior. More specifically, the spectrum oscillates initially in a manner discussed in Section 3 but eventually settles down to take the shape associated with the guided mode, with its center frequency shifted by just the right amount (about 32 GHz).

We stress that the shift of the central frequency is not caused by the entire pulse spectrum shifting to this value of Ω . Figure 5 shows the early evolution of the spectrum in Fig. 4 in greater detail. As this figure shows, a portion of the input pulse spectrum overlaps with the spectrum of the single-mode waveguide and is guided. From this perspective, the efficiency with which the pulse couples into the fundamental mode of the temporal waveguide can be improved by launching a pulse at a frequency shifted by Ω . This is analogous to aligning the propagation axis of an optical beam with the axis of an optical fiber to improve the coupling efficiency.

The reshaping of the optical pulse occurs for any pulse that is launched into a single-mode temporal waveguide, regardless of its temporal duration or shape. Figures 6(a)-6(c) show the evolution of three different Gaussian pulses with $T_0 = 2.5$ ps, $T_0 = 5$ ps, and $T_0 = 10$ ps, respectively. Each pulse was launched into the same single-mode temporal waveguide used for Fig. 4, but its spectrum was centered at the shifted frequency Ω to improve coupling into the fundamental temporal mode. The 2.5 ps pulse quickly broadens because of GVD, filling the waveguide in less than 400 m. The pulse then loses considerable energy into the $|t| > T_{\rm B}$ regions as it reshapes itself into the fundamental mode of the waveguide. In contrast, the 10 ps pulse in Fig. 6(c) narrows down as it is initially much wider than the waveguide. Narrowing occurs because the portion of the pulse outside of the guiding region is mostly shed off through dispersion. The portion lying inside the central core region reshapes itself until it once again matches the shape of the fundamental mode. The 5 ps pulse in Fig. 6(b) is just wide enough that most of its energy lies inside the temporal waveguide. As a result, much less energy is shed into the dispersive



Fig. 4. Evolution of the (a) shape and (b) spectrum of a Gaussian pulse with $T_0 = 3.5$ ps inside a 10 ps wide single-mode temporal waveguide designed with $\beta_{\rm B} = 2 \text{ km}^{-1}$. The spectral evolution of only the guided pulse is shown in (c). Dashed horizontal lines show the temporal boundaries that form the waveguide core.

wave as it acquires the shape of the fundamental mode. Clearly, this is the optimum situation if the objective is to couple most of the pulse energy inside a single-mode temporal waveguide. This is analogous to improving the coupling efficiency into



Fig. 5. Early evolution of the spectrum for the guided optical pulse from Fig. 4.



Fig. 6. Evolution of the shape of Gaussian pulses with (a) $T_0 = 2.5$ ps, (b) $T_0 = 5$ ps, and (c) $T_0 = 10$ ps into the same waveguide used in Fig. 4. Dashed horizontal lines show the temporal boundaries that form the waveguide core.

optical fibers by matching the optical beam to the mode diameter and numerical aperture of an optical fiber.

One may ask how the optical phase varies across the pulse. It follows from Eq. (10) that M(t) is a real quantity, indicating a constant phase. However, we should not forget the phase factor in Eq. (7). If the pulse is launched with its center frequency at ω_0 , the phase across the guided pulse would vary linearly in time. The slope of this temporal phase corresponds to the frequency shift Ω . However, if we shift the center frequency of the incident pulse by Ω , the phase becomes flat in time. This is what occurred in the case of Fig. 6. Since the phase of the waveguide mode is uniform in time, we may say that the pulse is phase locked. We should stress that the phase is not constant since it increases with z at a rate $K + \beta_2 \Omega^2/2$; however, it increases uniformly for the entire pulse duration.

6. CONCLUSIONS

We have shown that the analog of TIR at a temporal boundary can be used to make temporal waveguides. Using numerical simulations, we have shown that a temporal waveguide can be produced by two copropagating temporal boundaries that satisfy the condition for temporal total internal reflection. We were able to solve the underlying equations analytically to obtain the modes of a temporal waveguide. These modes are analogous to those of a spatial waveguide except for a crucial frequency shift. In particular, we introduced a dimensionless parameter V whose value determines the number of modes supported by that waveguide. The single-mode condition $V < \pi/2$ is then identical to that found for spatial waveguides.

We used numerical solutions to show that the modes propagate stably over long distances. Coupling into a single-mode waveguide was studied by launching Gaussian pulses of different widths. We discussed in detail the dynamics of how the launched pulse reshapes its shape and spectrum to evolve into the fundamental mode, shedding energy as dispersive waves in the process. We also showed that pulses that more closely match the shape and spectrum of the fundamental mode couple more efficiently into the temporal waveguide.

Although this paper focused on the simple case of a stepindex boundary (zero rise time), most practical temporal boundaries will experience dispersion during propagation, leading to changes in the boundary rise time. We have verified numerically that temporal boundaries with a finite rise time also exhibit temporal modes with shapes that depend on the magnitude of rise time. When we launched a mode of a step-index boundary into a waveguide with a nonzero rise time (taken to be 5% of the waveguide width for each boundary), the pulses adjusted their shape and evolved toward the mode of the new waveguide. Furthermore, the behavior of a single-mode waveguide remained largely unchanged for rise times as large as 30% of the waveguide width, although the shape of the fundamental mode changed continuously with increasing rise time of the boundary. The precise evolution of a temporal boundary will be determined by the exact physical mechanism used to generate the boundary. For example, when an intense pulse propagating as an optical soliton is used to create a temporal boundary through the nonlinear phenomenon of cross-phase modulation, the temporal boundary created by it will have a finite rise time that is unaffected by dispersion. This leads us to conclude that the results obtained in this paper would apply qualitatively to temporal boundaries with a finite rise time as long as its magnitude is a small fraction (below 10%) of the waveguide's temporal window.

Experimental confirmation of temporal TIR and the temporal waveguide will be of great interest. Our estimates show that the change in refractive index across the temporal boundary can be lower than $\Delta n = 10^{-6}$ for producing temporal TIR. The main issue is controlling the relative speed of the pulse with respect to the temporal boundary. A traveling-wave electro-optic phase modulator driven by copropagating microwave pulses could be used to produce the two moving temporal boundaries. Alternatively, a pump-probe configuration using a rectangular pump pulse could be used to produce the temporal boundaries through cross-phase modulation but will require pump pulses of high energies. In this case, the probe pulse would be launched in the middle of the pump with the two edges of the pump forming the waveguide boundaries. **Funding.** National Nuclear Security Administration (NNSA) (DE-NA0001944); University of Rochester (UR); New York State Energy Research and Development Authority (NYSERDA); National Science Foundation (NSF) (ECCS-1505636).

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