Soliton stability and trapping in multimode fibers

Shaival Buch* and Govind P. Agrawal

The Institute of Optics, University of Rochester, Rochester, New York 14627, USA

**Corresponding author: shaival.buch@rochester.edu*

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We investigate the stability of optical solitons in few-mode fibers by solving numerically coupled nonlinear Schrödinger equations that include intermodal nonlinear coupling. While a single fundamental soliton propagating in any fiber mode is stable, simultaneous propagation of two or more solitons in different fiber modes is not always stable and leads to interesting effects resulting from intermodal nonlinear coupling. Under some conditions, soliton trapping is observed such that two solitons in different modes shift their spectra and travel at the same speed in spite of considerable intermodal differential group delay between them.

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Traditionally, single-mode optical fibers have been used for observing nonlinear phenomena [1] as well as for designing optical communication systems [2]. However, multimode and multicore fibers are attracting attention recently in the context of space-division multiplexing [3]. Such fibers provide an extra dimension over which communication channels can be multiplexed to meet the demand for ever-increasing capacity.

Although optical solitons have been studied extensively in the context of single-mode fibers [1], it is only recently that nonlinear propagation in multimode fibers has become of interest [4–6]. A new feature of such fibers is the presence of intermodal nonlinear coupling, even when linear modal coupling is negligible. This nonlinear coupling results from two nonlinear phenomena known as intermodal cross-phase modulation (XPM) and degenerate four-wave mixing (FWM). In this Letter, we present our numerical results exploring the stability of optical solitons in a six-mode fiber under different input conditions. We begin by introducing the set of coupled nonlinear Schrödinger (NLS) equations that we solve numerically with the split-step Fourier method. We briefly discuss the case of a single isolated fundamental soliton and then focus on the case where two or more solitons are launched simultaneously into different fiber modes. To maximize the intermodal nonlinear coupling, we assume that all solitons travel with the same speed and analyze our results. Finally, we introduce finite differential group delay (DGD) between two spatial modes and show that soliton trapping can occur for relatively small values of DGD.

Two different notations exist to denote various modes of an optical fiber [1], called the HE or the LP modes (LP stands for linear polarization). We have chosen to employ the LP terminology, valid in the weakly guiding approximation for fibers with a relatively small refractive-index difference between the core and the cladding materials. We consider a graded-index fiber designed to support three modes, referred to as LP_{01}, LP_{11a} and LP_{11b}, the last two being twofold degenerate. In the following, we number them from 1 to 3, successively. Since each mode can have two orthogonally polarized components (denoted by the subscripts x and y), our fiber actually supports 6 distinct modes.

To describe propagation of optical pulses inside a multimode fiber, we employ the mathematical model of [7]. However, we normalize all equations with the so-called soliton units and solve numerically the following set of six coupled NLS equations:

$$\frac{\partial \mathbf{u}_p}{\partial \xi} + d_{1p}\frac{\partial \mathbf{u}_p}{\partial \tau} + i\frac{d_{2p}}{2}\frac{\partial^2 \mathbf{u}_p}{\partial \tau^2} = iN_1^2 \times \sum_l \sum_m \sum_n f_{lmnp}\left[\frac{2}{3}(\mathbf{u}_l^H \mathbf{u}_m)\mathbf{u}_n + \frac{1}{3}(\mathbf{u}_n^T \mathbf{u}_m)\mathbf{u}_l^*\right] e^{i\Delta\beta_{lmnp}L_D\xi}, \quad (1)$$

where $p, l, m, n = 1, 2, 3$, superscripts T and H stand for the transpose and Hermitian conjugate of a matrix, $\mathbf{u}_p = \mathbf{A}_p/\sqrt{P_1}$, $\mathbf{A}_p$ is the Jones vector, $\mathbf{A}_p = [A_{px}, A_{py}]^T$, and A_{px} is the slowly varying envelope of the x-polarized pulse propagating in pth spatial mode of the fiber. The power P_1 used for normalization is the peak power needed to form a fundamental soliton in the LP_{01} mode. Dispersion length of this mode, defined as $L_D = T_0^2/|\beta_{21}|$, is used to normalize the propagation distance, i.e., $\xi = z/L_D$. Similarly, $\tau = t/T_0$ is the normalized time, where T_0 is a reference time related to the width of input pulses.

In deriving Eq. (1), the propagation constant β_p of each mode was expanded in a Taylor series as

$$\beta_p(\omega) = \beta_{0p} + \beta_{1p}(\omega - \omega_0) + \beta_{2p}(\omega - \omega_0)^2/2 + \ldots, \quad (2)$$

where $\beta_{kp} = (d^k\beta_p/d\omega^k)|_{\omega=\omega_0}$. The parameters d_{1p}, and d_{2p} are defined as

$$d_{1p} = (\beta_{1p} - \beta_{11})L_D/T_0, \qquad d_{2p} = \beta_{2p}/|\beta_{21}|. \quad (3)$$

Because of the degenerate nature of LP_{11a} and LP_{11b} modes, $d_{k2} = d_{k3}$. Since $d_{11} = 0$ and $d_{21} = -1$ (anomalous dispersion), we need to specify only two dimensionless parameters to account for the dispersive effects, namely d_{12} and d_{22}. Third and higher order dispersion terms can be included but are ignored in this work to focus on the intermodal nonlinear coupling. The phase mismatch on the right side of Eq. (1) is given by $\Delta\beta_{lmnp} = \beta_{0m} + \beta_{0n} - \beta_{0l} - \beta_{0p}$. It governs the strength of FWM among the four fields involved. In practice, only terms that are nearly phase matched contribute to the triple sum in Eq. (1).

In Eq. (1), N_1 corresponds to the soliton order of the fundamental mode and is defined as $N_1^2 = \gamma P_1 T_0^2/|\beta_{21}|$, where γ is the nonlinear parameter. The intermodal nonlinear coupling is governed by overlapping spatial profiles of the modes. It is included in Eq. (1) through the dimensionless f_{lmnp} factor given by

$$f_{lmnp} = A_1^{\mathrm{eff}} \iint F_l^* F_m F_n F_p^* \mathrm{d}x\mathrm{d}y, \quad (4)$$

where $F_p(x, y)$ is the spatial distribution of the pth mode, assumed to be normalized such that $\iint |F_m|^2(x, y)\mathrm{d}x\mathrm{d}y = 1$. The effective area A_1^{eff} of the LP_{01} mode appears when we use the normalization $f_{1111} = 1$. Noting that l, m, n, p can vary from 1 to 3, one needs to specify 81 parameters to fully account for the intermodal nonlinear coupling. However, a large number of them vanish because of mode orthogonality and FWM efficiency, and many coincide in values because of the degeneracy of LP_{11a} and LP_{11b} modes. We are left with 17 nonzero terms. For numerical simulations, we use the values of fiber parameters given in [8]. Using the effective areas given there, we find $\gamma = 1.77\ \mathrm{W}^{-1}/\mathrm{km}$ and $f_{2222} = f_{3333} = 0.747$. Eight nonlinear terms responsible for XPM between the LP_{01} and LP_{11a} (or LP_{11b}) modes have values $f_{lmnp} = 1/2$. Six phase-matched terms coupling the degenerate LP_{11a} and LP_{11b} modes have value $f_{lmnp} = 1/4$. For our choice of $T_0 = 1$ ps, $L_D = 41.15$ m, and $d_{22} = 1.06$. The DGD parameter d_{12} is varied from 0 to 4.

Equation (1) is solved with the standard split-step Fourier method for all modes [1]. The input in each mode is in the form $u(0, \tau) = u_0 \mathrm{sech}(\tau)$, with u_0 set to zero for modes in which no soliton is launched. We propagate one or more solitons along the fiber and study their stability in the presence of intermodal nonlinear coupling. We assume an ideal fiber and neglect any linear coupling among its modes (as well as fiber losses).

For the initial analysis, we neglect the DGD and set $d_{12} = 0$ to ensure nonlinear coupling along the entire length of the fiber. We first launch a single fundamental soliton into a specific mode of the fiber. Our results show that the soliton remains unchanged and retains all of its energy even after hundreds of dispersion lengths, irrespective of the mode into which it was launched. To judge whether the soliton will remain stable against perturbations, we added white noise with spectral density of 10^{-17} W/Hz to all channels to seed the nonlinear effects. Remarkably, the soliton propagated in a stable fashion even in the presence of noise. As an example, Fig. 1 shows the case when a fundamental soliton is propagated in the LP_{01x} mode over 100 dispersion lengths. A small amount of energy is transferred to the orthogonally polarized mode seeded with noise, but its magnitude remains well below 0.1% of the input energy. These results indicate that a single isolated soliton launched into any mode propagates in a stable fashion inside a multimode fiber.

Much more interesting behavior is observed when two fundamental solitons are launched simultaneously into different fiber modes. Figure 2 shows their evolution over 100 dispersion lengths in two cases. In part (a), two copolarized solitons propagate in the LP_{01x} and LP_{11ax} modes of the fiber. Both solitons lose energy initially in the form of dispersive waves and become narrower as they interact through intermodal XPM and reshape themselves. To understand this behavior, we note that spatial profiles of two solitons overlap considerably, even though they are traveling in distinct spatial modes. As a result, each soliton modulates substantially the phase of the other soliton through XPM. In effect, the total nonlinear phase shift seen by each soliton is larger than that of a single isolated soliton. As this situation is similar to that of a soliton launched with $N > 1$, each soliton undergoes an initial narrowing phase. The same

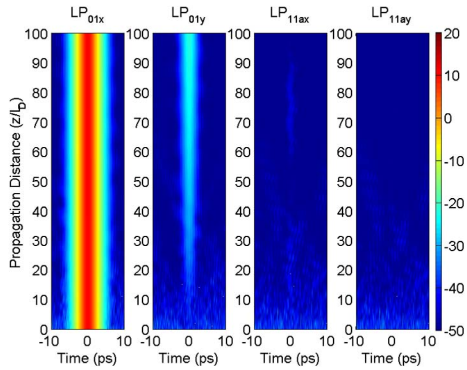

Fig. 1. Single fundamental soliton in LP_{01x} mode together with noise in all modes (The LP_{11b} modes behave similar to the LP_{11a} modes and are not shown here). The color bar shows optical power on a dB scale.

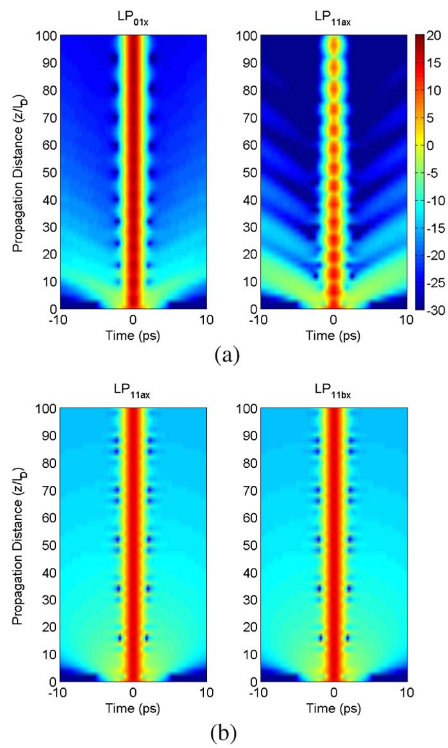

Fig. 2. Simultaneous propagation of fundamental solitons in two fiber modes: (a) LP_{01x} and LP_{11ax}, (b) LP_{11ax} and LP_{11bx}. The color bar shows optical power on a dB scale.

XPM-induced perturbation is responsible for the radiation shed by each soliton. However, as seen in Fig. 2(a) most of radiation is shed over the first few dispersion lengths, and energy lost through radiation remains well below 1% of the input energy of each soliton.

The most important feature of Fig. 2 is that the LP_{11a} soliton undergoes an oscillatory phase during which it transfers its energy to the LP_{01} soliton. Our results show that all energy is transferred to the LP_{01} soliton after $200L_D$. Further, at that distance, the width of the LP_{01} soliton is reduced by >50%, while its peak power is enhanced by more than a factor of 4 to ensure that the soliton order N remains close to 1. The physical mechanism behind this power transfer is under investigation. Our numerical investigations indicate that energy is always transferred from a higher order mode to the LP_{01} mode, and it is not necessary that pulses propagate as a soliton. One may interpret this phenomenon as a kind of self-focusing inside a multimode fiber.

Figure 2(b) shows the case of two fundamental solitons launched into the degenerate LP_{11ax} and LP_{11bx} modes. Similar to the case shown in Fig. 1(a), both solitons initially shed energy in the form of dispersive waves and reshape themselves by becoming narrower. Moreover, they exhibit a periodic evolution pattern and do not fully stabilize even after 100 dispersion lengths. However, in contrast to the case seen in Fig. 2(a), no power transfer occurs between the two degenerate modes. Indeed, our results show that each soliton contains more than 99% of its original energy even after 200 dispersion lengths.

One may ask what happens when three copolarized solitons propagate along the three spatial modes supported by the fiber, two of which are degenerate. Figure 3 shows their evolution over 100 dispersion lengths in the situation. The evolution pattern is similar to that seen in Fig. 2(a). All solitons lose some energy initially in the form of dispersive waves, but this energy loss is relatively small. In contrast, both solitons propagating in two higher-order modes transfer their energy to the LP_{01} soliton at a rate much faster than that of Fig. 2(a). The main conclusion is that solitons propagating in higher order modes of the fiber are unstable, while the one traveling in the fundamental LP_{01} mode is not only stable but also becomes shorter as energy is transferred to it. This behavior is reminiscent of self-focusing in the sense that the spatial extent of the beam also narrows down and may be related to the formation of optical bullets discussed in Ref. [6].

Finally, we study the effect of intermodal DGD that invariably occurs in realistic multimode fibers. Soliton robustness has been studied previously in the case of birefringent fibers having DGD between the two orthogonally polarized modes [9,10]. In our case, it leads to a group-velocity mismatch between the LP_{01} and LP_{11} modes. We quantify this mismatch through the parameter $\delta\beta_1 = \beta_{12} - \beta_{11} = d_{12}T_0/L_D$. Pulses in the LP_{11a} and LP_{11b} modes travel at the same speed because of their degenerate nature. Figure 4 shows the numerical results for (a) $\delta\beta_1 = 10$ and (b) 100 ps/km when fundamental solitons are launched simultaneously into the LP_{01} and LP_{11a} modes. For the smaller value, $\delta\beta_1 = 10$ [Fig. 4(a)], the soliton in the fundamental mode slows down, and the soliton in the LP_{11a} mode speeds up, till they are both traveling with the same speed. This is the phenomenon of soliton trapping that has been observed in birefringent fibers [9], and it is accomplished though shifts in the spectrum of each soliton in opposite directions. Using simple arguments, we can estimate the spectral shift as $\Delta\omega = \Delta\beta_1/\beta_2 = L_D\Delta\beta_1/T_0^2$. For the parameter values used in Fig. 4(a), the estimated shift in the frequency is about 40 GHz. A comparison of Fig. 4(a) with Fig. 2(a) shows that, except the trapping and a temporal tilt induced by spectral shifts, the two solitons evolve in a similar fashion. In particular, the LP_{11} soliton remains unstable and transfers its energy to the LP_{01} soliton as it propagates down the fiber.

As the group-velocity mismatch between the two solitons increases, one expects the soliton trapping to

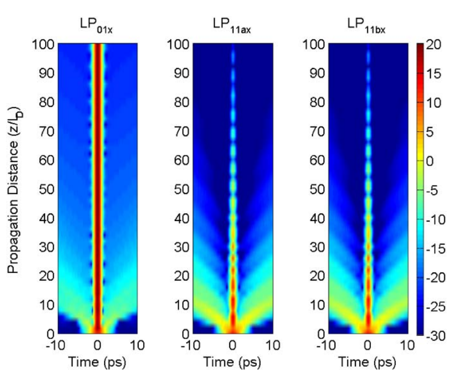

Fig. 3. Simultaneous propagation of x-polarized solitons in three fiber modes. The color bar shows optical power on a dB scale.

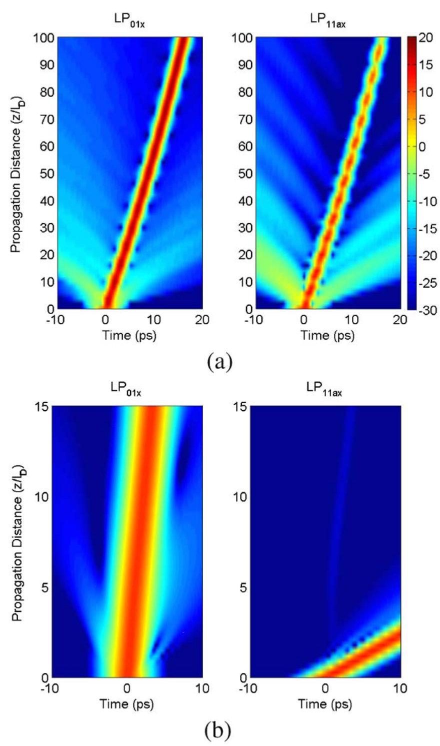

Fig. 4. Simultaneous propagation of fundamental solitons in LP_{01x} and LP_{11ax} modes with (a) $\delta\beta_1 = 10$ ps/km and (b) $\delta\beta_1 = 100$ ps/km. (Colorbar represents power in dB.)

become less effective. For a sufficiently large value of $\delta\beta_1$, trapping should cease to occur completely. The reason is that the two solitons overlap temporally over shorter and shorter distances before they separate from each other, making intermodal XPM to become less and less effective with increasing value of $\delta\beta_1$. Numerical results confirm this behavior. Figure 4(b) where $\delta\beta_1 = 100\ \mathrm{ps/km}$ shows that soliton trapping nearly ceases to occur for such large values of intermodal DGD. In this particular case, the pulses separate in time before they have propagated one half of a dispersion length. Remarkably, even within such a short distance, the spectrum of the soliton in the LP_{01} mode has shifted slightly. This shift results in a slight decrease in its group velocity and is the reason for the tilt seen in Fig. 4(b). Also note that the LP_{11a} soliton no longer transfers its energy to the LP_{01} soliton because XPM-induced copuling ceases to occur as soon as the two solitons separate from each other temporally, even though they overlap spatially inside the fiber.

In summary, we addressed the issue of the stability of optical solitons in a multimode fiber in this Letter. We focused on a fiber supporting three spatial modes, two of which were degenerate. After including two polarization states, we solved a set of six coupled NLS equations that include intermodal nonlinear coupling. The following conclusions can be drawn from our numerical results. When a single fundamental soliton is launched into a specific fiber mode, it can propagate stably over long fiber lengths, even when intramodal nonlinear effects are seeded through noise in other modes. When two orthogonally polarized solitons are launched into a specific fiber mode, the nonlinear birefringence induced through intramodal XPM makes evolution of each soliton periodic such that its width oscillates in a periodic fashion. Finally, when two copolarized solitons are launched into different fiber modes, their evolution depends on the amount of intermodal DGD. For small values of DGD, solitons trapping is observed such that two solitons in different fiber modes shift their spectra and travel at the same speed in spite of considerable intermodal DGD between them. For the specific fiber considered in our numerical simulations, complete trapping was found to occur for DGD values as large as 20 ps/km. A surprising feature revealed by our numerical simulations is that, under certain conditions, pulses propagating in higher-order modes may transfer their energy to the LP_{01} soliton as they propagate inside the fiber. The physical mechanism behind this power transfer is currently under investigation.

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