Reduction of Nonlinear Penalties Due to Linear Coupling in Multicore Optical Fibers

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***Abstract*— We present a general model for studying nonlinear effects in multicore fibers. Our results show that a strong linear coupling among fiber cores can reduce nonlinear impairments and improve system performance when digital signal processing is used to compensate for all linear degradations.**

***Index Terms*— Multicore fibers, multimode fibers.**

I. Introduction

TRANSMISSION rates of lightwave systems, built using single-mode fibers, are now approaching the fiber-capacity limit [1] and may fail to cope with the growing demand in data traffic over the next decade. Designing next-generation systems with multimode (MMFs) or multicore fibers (MCFs) appears to be a possible solution to increase the capacity of future optical networks. This is so because many spatial modes can propagate simultaneously in such fibers. As a result, transmission rates can be considerably increased by multiplexing different data signals over individual spatial modes. Recently, several experiments have demonstrated high bit-rate transmission over MCFs [2], [3] by exploiting space-division multiplexing (SDM).

Before SDM becomes a practical solution, several issues specific to MMFs and MCFs need to be studied because co-propagating modes in such fibers can interact both linearly and nonlinearly. Linear coupling in MCFs is especially of concern since it leads to a periodic transfer of optical power from one core to other neighboring cores. In principle, linear coupling among modes can be efficiently compensated using digital signal processing at the receiver with a multiple-input multiple-output (MIMO) technique. The important question that remains to be answered is how linear coupling in MCFs affects the nonlinear penalty.

In this letter we present the results of a theoretical study on the impact of linear coupling on the nonlinear effects in MCFs. We first present a general model for studying the nonlinear propagation in such fibers and show that a large number of intermodal nonlinear terms can often be neglected in practice. We then discuss the interaction between the intramodal nonlinear effects and linear coupling and show through numerical simulations that linear coupling can mitigate the nonlinear impairments to some extent and improve the system performance when MIMO techniques are used to compensate all the linear effects. Note that birefringence effects are not considered in this letter. Our results indicate that, although a weak linear coupling reduce the receiver complexity [3], it also fails to take advantage of the reduced nonlinear penalty occurring for MCFs with strongly coupled cores.

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II. Nonlinear Propagation in Multicore Fibers

Our derivation of the nonlinear propagation equation follows the procedure in Ref. [4]. However, we assume that the axial component of the optical field is negligible in the weakly guiding approximation. This allows us to employ the following vectorial notation for the electric field:

$$\widetilde{\mathbf{E}}(x,y,z,\omega)=\sum_m \widetilde{\mathbf{A}}_m(z,\omega)\mathrm{F}_m(x,y)e^{\imath\beta_m(\omega)z}/\sqrt{\mathrm{N}_m} \quad (1)$$

where $\widetilde{\mathbf{A}}_m(z,\omega) = [\widetilde{\mathrm{A}}_{m,x}(z,\omega)\ \ \widetilde{\mathrm{A}}_{m,y}(z,\omega)]^{\mathrm{T}}$ includes the slowly varying amplitudes of both polarization components for the mode m with the spatial distribution $\mathrm{F}_m(x,y)$ and the propagation constant $\beta_m(\omega)$. Note that $\mathrm{F}_m(x,y)$ is real in the weakly guiding approximation, and the phase of the optical wave is contained in the complex amplitude $\widetilde{\mathbf{A}}_m$. The power carried by the m^{th} mode can be expressed as $\mathrm{N}_m = \frac{1}{2}\epsilon_0 n_{\mathrm{eff},m} c\, \mathrm{I}_m^2$, where $\mathrm{I}_m^2 = \iint \mathrm{F}_m(x,y)^2 dx\,dy$, where ϵ_0 is the vacuum permittivity and $n_{\mathrm{eff},m}$ the fiber effective refractive index of mode m.

Each frequency component of the field satisfies

$$\nabla^2\widetilde{\mathbf{E}}(\omega)+n_{\mathrm{f}}^2\frac{\omega^2}{c^2}\widetilde{\mathbf{E}}(\omega)=-\omega^2\mu_0\widetilde{\mathbf{P}}^{\mathrm{NL}}(\omega) \quad (2)$$

where $n_{\mathrm{f}}(x,y)$ is the spatial distribution of the refractive index and $\widetilde{\mathbf{P}}^{\mathrm{NL}}(\omega)$ is the Fourier transform of the third-order nonlinear response

$$\mathbf{P}^{\mathrm{NL}}(t)=\frac{\epsilon_0}{4}\chi^{(3)}[(\mathbf{E}(\mathrm{t})^{\mathrm{T}}\mathbf{E}(\mathrm{t}))\mathbf{E}^*(\mathrm{t})+2(\mathbf{E}(\mathrm{t})^{\mathrm{H}}\mathbf{E}(\mathrm{t}))\mathbf{E}(\mathrm{t})] \quad (3)$$

where the superscripts $^{\mathrm{T}}$ and $^{\mathrm{H}}$ denote the transpose and Hermitian conjugates, respectively.

To isolate the evolution of a specific mode, say the pth spatial mode, we use the orthogonal nature of spatial modes. The propagation equation is then converted to the time domain by following a standard procedure and expanding $\beta_p(\omega)$ in a Taylor series around the carrier frequency ω_0. The final result can be written in the form

$$\frac{\partial \mathbf{A}_p}{\partial z} = \left(-\frac{\alpha}{2} + \imath\beta_{p0}\right)\mathbf{A}_p - \beta_{p1}\frac{\partial \mathbf{A}_p}{\partial \mathrm{t}} - \frac{\imath\beta_{p2}}{2}\frac{\partial^2 \mathbf{A}_p}{\partial \mathrm{t}^2} + \imath\sum_{lmn} f_{plmn}\frac{\gamma}{3}\left[(\mathbf{A}_l^{\mathrm{T}}\mathbf{A}_m)\mathbf{A}_n^* + 2(\mathbf{A}_n^H\mathbf{A}_m)\mathbf{A}_l\right] + \imath\sum_m q_{mp}\mathbf{A}_m \quad (4)$$

where $\mathbf{A}_p(z, \mathrm{t})$ is the Fourier transform of $\widetilde{\mathbf{A}}_p(z, \omega)e^{\imath\beta_p(\omega)z}$, $\beta_{p,n} = \partial^n\beta_p/\partial\omega^n|_{\omega=\omega_0}$, β_{p1} is the inverse of the group velocity, β_{p2} is the group-velocity dispersion of the pth spatial mode, and α is the fiber loss coefficient. The nonlinear parameter is defined as $\gamma = 3k_0\chi^{(3)}/(4\epsilon_0 c n_{\mathrm{eff}}^2 A_{\mathrm{eff}})$, A_{eff} being the effective mode area. The linear and nonlinear couplings among spatial modes are governed respectively by

$$q_{mp} = \frac{k_0^2}{2\beta_{p0}\mathrm{I}_m\mathrm{I}_p}\iint (n_{\mathrm{f}}^2 - n_{\mathrm{c,m}}^2)\mathrm{F}_m\mathrm{F}_p\, dx\, dy \quad (5)$$

$$f_{plmn} = \frac{A_{\mathrm{eff}}}{\mathrm{I}_l\mathrm{I}_m\mathrm{I}_n\mathrm{I}_p}\iint \mathrm{F}_l\mathrm{F}_m\mathrm{F}_n\mathrm{F}_p\, dx\, dy \quad (6)$$

where $n_{\mathrm{c,m}}(x, y)$ is the spatial index distribution of the isolated core in which the mode m propagates.

Nonlinear propagation over MMFs with uncoupled spatial modes has been previously studied in [5]. In the following discussion, we consider MCFs with single-mode cores and we choose to use the coupled-core approach [6], where modes refer to the fields propagating in individual cores (rather than supermodes). The spatial distribution $\mathrm{F}_p(x, y)$ of each mode p, is calculated first assuming an infinite cladding and using a standard procedure [7].

We consider MCFs with identical cores of radius a, separated by a distance d (see the inset of Fig. 1). The strength of linear and nonlinear coupling among the spatial modes (q_{mp} and f_{plmn}, respectively) depends not only on the distance d but also on the extent of mode confinement governed by the parameter $V = k_0 a(n_{\mathrm{core}}^2 - n_{\mathrm{clad}}^2)^{1/2}$. In the absence of fiber nonlinearity, the linear coupling leads to a periodic exchange of energy among fiber cores. The power launched into one core is totally or partially transferred to the neighbor cores and finally comes back to its original core after a distance $2L_c$, where L_c is the coupling length.

The nonlinear coupling can be separated into an *intermodal* part for which at least two subscripts in f_{plmn} are distinct and into an *intramodal* part $f_{pppp} = 1$. An important question is how large intermodal coupling is compared to the intramodal one. Dominant contributions to the intermodal terms come from f_{pppm} and other 3 possible index permutations. Figure 1 shows f_{pppm} as a function of V for a 3-core MCF when the core separation d varies in the range of $4a$ to $10a$. One can see that the intermodal nonlinear terms are several orders of magnitude smaller than the intramodal terms until d is reduced to below $4a$. In the following discussion, we neglect the intermodal coupling but include all intramodal nonlinear effects fully. The MCFs used in [2] and [8] for data transmission operated in this regime.

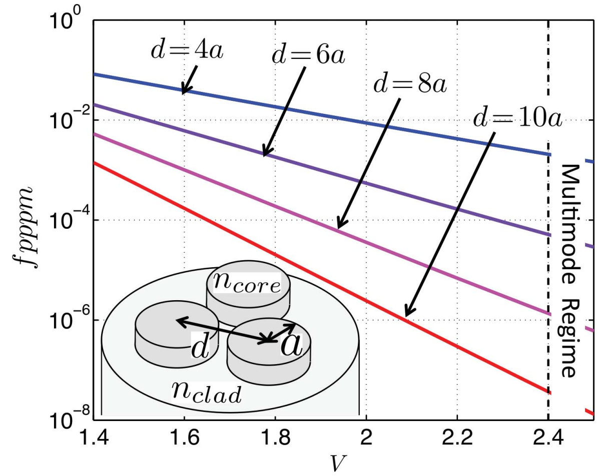

Fig. 1. Largest intermodal nonlinear coupling coefficient f_{pppm} as a function of V, for different distances between neighboring cores.

An important question is how the intramodal nonlinear effects are affected by the linear coupling that occurs in MCFs. This question is even more relevant for modern lightwave systems that employ the PDM technique with phase-modulated symbols. In the following, we numerically solve Eq. (4) with the intramodal nonlinear terms to evaluate the impact of linear coupling on nonlinear transmission in 3-core MCF fibers.

III. 80-Gb/s SDM-PDM-QPSK System

In this section, we focus on transmission of 80-Gb/s PDM-QPSK signals over a multicore fiber. Each PDM-QPSK signal is constructed from an independent random bit stream that is Gray-mapped onto each QPSK symbol. The transmission rate is $R_s = 20$ Gbaud for each polarization, with a pulse shape corresponding to a raised-cosine spectrum with a roll-off factor of 0.2 [1]. Each fiber core carries two QPSK signals, multiplexed onto two orthogonal linear polarizations. Each PDM-QPSK signal is made of 2^{15} modulated symbols with 4 numerical samples per symbol. The transmission line consists of 100-km sections, each followed by a fiber amplifier that compensates for all span losses. The fiber parameters are chosen such that each core of the MCF has the same characteristics as a standard single-mode fiber: Core diameter is 8 μm ($\Delta = 0.36\%$) with $V = 2$, $A_{\mathrm{eff}} = 75\mu\mathrm{m}^2$, $D = 17$ ps/(nm-km) and $\gamma = 1.5\ \mathrm{W}^{-1}\mathrm{km}^{-1}$. The distance d between the cores is adjusted to obtain the desired coupling length; in the case of a 2-core fiber, $L_c = 100$ m corresponds to $d = 10.8\,a$. In our simulations, L_c is varied to study the role of coupling among fiber cores. We ignore fiber birefringence effects in this letter. We solve (4) numerically with the split-step Fourier method. At the receiver, we artificially degrade the bit-error rate (BER) by adding noise to account for amplified spontaneous emission (ASE). This is justified since nonlinear interaction between the ASE and the signal has been verified to be negligible.

At the receiving end, we demodulate the symbol stream by implementing numerically a coherent receiver, followed by analog to digital converters that sample the signal at the symbol rate. Digital signal processing includes ideal an

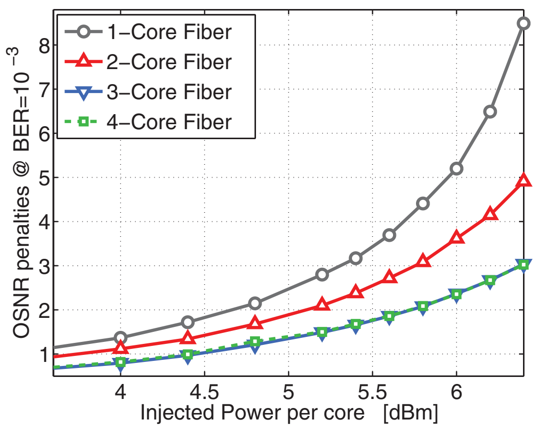

Fig. 2. OSNR penalties at a BER of 10^{-3} after 1000 km, as a function of input power per core P_{in} for MCFs with $L_c = 100$ m.

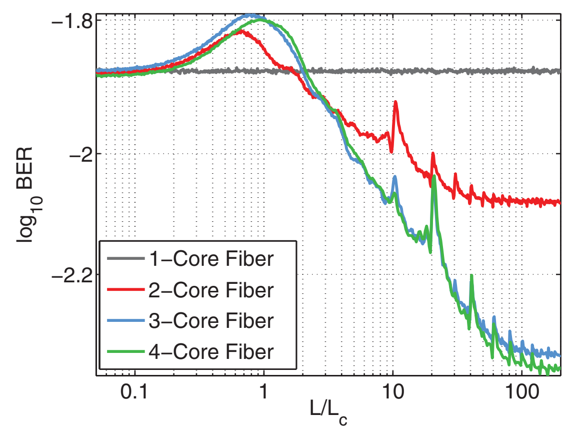

Fig. 3. BER versus L/L_c for 1000 km transmission for three MCFs, when $P_{\text{in}} = 8$ dBm and OSNR = 20 dB.

chromatic dispersion compensator and a least-square equalizer [9] for spatial mode demultiplexing and phase recovery. This linear equalizer is not adaptive and relies in the transmission of training sequences inserted periodically within the data.

Figure 2 shows OSNR penalties at a BER of 10^{-3} after 1000 km as a function of the injected power per core for MCFs with 2, 3, and 4 cores. Penalties are referenced to the back-to-back performance where MCFs perform the same as a single-core fiber. Transmission over single-core fiber is also shown for comparison. Our results clearly show that the penalty is reduced for MCFs, i.e., MCFs are more robust to nonlinear impairments. The results for 3-core and 4-core fibers nearly coincide because we employ for the 4-core fiber, a square configuration that has the same spacing between neighbor cores as the 3-core fiber.

To understand the origin of reduced penalty for MCFs, we show in Fig. 3 the BER as a function of L/L_c using $P_{\text{in}} = 8$ dBm at a fixed OSNR of 20 dB at the receiver. Same behavior is observed for lower values of P_{in} as long the system operates in the nonlinear regime. For a large coupling length ($L/L_c < 0.1$), MCF's performance is comparable to a single-core fiber since little power transfer occurs among the cores over the distance L. For $L \sim L_c$, linear coupling in MCFs actually degrades the performance. However, when L becomes much larger than L_c ($L > 3L_c$), the BER is reduced for MCFs, resulting in improved performance. We think that this improvement is due to an averaging of nonlinear distortions in MCFs with strongly coupled cores.

Nonlinear penalties are caused because nonlinear distortions vary from symbols to symbol. Linear coupling increases power variations of the signal in each core during signal's propagation through the fiber. In the case of strongly coupled cores, power variations become so rapid that they result in an averaging of the symbol nonlinear distortions. Because of this averaging, nonlinear distortions are compensated to a much larger extent during the phase recovery processing, and system performance improves compared to the single-core case. The oscillatory structure in Fig. 3 is a consequence of the periodic transfer of energy between cores and the presence of amplifiers at every span of the system. Larger penalties are observed for the values L_c for which most of the power transfer occurs when power inside the fiber is high.

IV. Conclusion

We have developed a general model for studying nonlinear propagation in MCFs. We used this model to calculate nonlinear penalties in MCFs with up to four cores when 80-Gb/s PDM-QPSK signals are transmitted through each core. Our results reveal that a strong linear coupling can mitigate the nonlinear impairments and improve the system performance. Rapidly varying birefringence has not been considered in this letter but similar behavior with larger performance improvement is expected in that case. These results should prove useful for the development of future SDM lightwave systems as they suggest that the cores of MCFs can be much closer than what is currently thought practical [3].

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