Phase-Switched All-Optical Flip-Flops Using Two-Input Bistable Resonators

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***Abstract*—We show that switching between the two stable states of an optical flip-flop, made using a two-input Kerr resonator, can be realized through pure phase modulation. The proposed switching mechanism is compatible with cross-phase modulation induced by set and reset pulses for realization of an ultrafast, passive, all-optical memory element.**

***Index Terms*—All-optical flip-flops, cross-phase modulation, optical bistability.**

I. Introduction

Optical flip-flops made with integrated optical resonators could conceivably be used for memory and buffering applications [1], [2]. The Kerr effect is ideal for realizing such devices since it allows for much faster switching speeds than those achieved by using free-carrier or thermal nonlinearities [3]. In practice, however, thermal nonlinearities inevitably arise as a result of optical absorption which heats the device and changes the refractive index via the thermo-optic effect. The resulting thermally-induced index changes dominate the nonlinear response and limit the switching time between the 'on' state (for which the resonator is full of light) and the 'off' state (for which it is empty) to microsecond time scales [3], [4]. Two-input bistability, for which the resonator is equally full of light in both stable states, has the potential to circumvent this limitation. In such a device the thermal effect would be equally strong in both 'on' and 'off' states, producing a background refractive index change that does not respond on a time scale comparable to that of the much faster Kerr nonlinearity. Two-input bistable resonators were proposed in the 1980s [5], [6]. Haeltermann later showed that such a device could be used as an optical flip-flop by modulating the amplitudes of the two input beams [7]. In this Letter we show that two-input bistable resonators can be switched in a robust way by pure phase modulation of the input beams, which can be realized through cross-phase modulation by set and reset pulses for all-optical flip-flop operation.

II. Two-Input Optical Bistability

Figure 1 shows schematically the basic idea behind the proposed two-input optical flip-flop using a micro-ring resonator. In this implementation, two resonator modes, excited by two input beams near a specific resonance frequency ω_r, propagate in the clockwise and counter-clockwise directions. We note, however, that the results obtained here are applicable to any kind of dielectric resonator (such as photonic-crystal microcavities or whispering-gallery-mode resonators) so long as the two excited modes are distinguishable (by having different resonance frequencies for instance). The physical origin of two-input bistability is the asymmetry between Kerr-induced self- and cross-phase modulations (SPM and XPM). Consider the case when mode a_1 in Fig. 1(a) is much more intense than mode a_2. In this situation, mode a_1 experiences an SPM-induced change Δn_{NL} in the medium's refractive index, causing the resonance frequency to shift by an amount $-\omega_r \Delta n_{NL}/n_0$, where n_0 is the linear refractive index of the medium inside the resonator. The clockwise wave a_2, however, experiences an XPM-induced change of $2\Delta n_{NL}$, and its resonance frequency shifts by twice that amount.

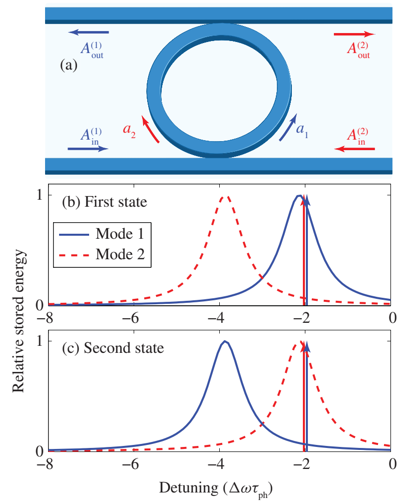

Fig. 1. (a) Schematic of the proposed two-input optical flip-flop. (b) and (c) Description of its two stable states under appropriate biasing conditions. The red and blue arrows indicate the detunings of the two CW input beams, dashed and solid lines show the Lorentzian responses of the two cavity modes in each stable state.

When both input fields ($A^{(1)}_{in}$ and $A^{(2)}_{in}$) are equally intense and equally detuned from resonance, the asymmetry between SPM and XPM effects can lead to the existence of two stable states, as illustrated in Figs. 1(b) and 1(c) for the case when both input fields are detuned from the low-intensity resonance frequency by an amount $\Delta\omega = -2/\tau_{ph}$, where τ_{ph} is the photon lifetime of the ring cavity. In Fig. 1(b), the input $A^{(1)}_{in}$ (blue arrow) is on resonance with the cavity mode, while $A^{(2)}_{in}$ is off resonance. The opposite happens in Fig. 1(c) where $A^{(2)}_{in}$ is on resonance, and $A^{(1)}_{in}$ is off resonance.

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To quantify the performance of our two-input optical flip-flop, we use the theoretical methodology developed in Ref. [8]. Its use leads to the following two coupled nonlinear equations for the mode amplitudes a_1 and a_2:

$$\frac{da_1}{dt} = i\Delta\omega_1 a_1 - \frac{a_1}{2\tau_{ph}} + \kappa A_{in}^{(1)}(t) + i(\gamma_{11}|a_1|^2 + 2\gamma_{12}|a_2|^2)a_1, \quad (1)$$

$$\frac{da_2}{dt} = i\Delta\omega_2 a_2 - \frac{a_2}{2\tau_{ph}} + \kappa A_{in}^{(2)}(t) + i(\gamma_{22}|a_2|^2 + 2\gamma_{21}|a_1|^2)a_2, \quad (2)$$

where $|a_k(t)|^2$ represents the optical energy stored in mode k at time t, $\Delta\omega_k = \omega_k - \omega_r$ is the detuning from the cavity resonance ω_r, and $A_{in}^{(k)}$ is the input field normalized so that $|A_{in}^k|^2$ is the optical power. Coupling of the input field into the resonator is governed by κ; it is given by $\kappa = (2\tau_{ph})^{-1/2}$ when cavity losses from absorption and scattering are relatively small [9]. The SPM and XPM effects are included through the nonlinear parameters γ_{kl} given by

$$\gamma_{kl} = \frac{\omega_k n_2 c \eta_{kl}}{n_0^2 (V_k V_l)^{1/2}}, \quad (3)$$

where n_2 is the Kerr coefficient, V_k is the effective mode volume, and η_{kl} is a parameter which measures how well the resonator mode overlaps with the nonlinear medium. Typically $\eta_{kl} \approx 1$ and $V_k \approx V_l$. In what follows we make the approximation $\gamma_{12} \approx \gamma_{22} \approx \gamma_{11} = \gamma$ and solve Eqs. (1) and (2) numerically. The power transmissivity of the two output ports is calculated using [9]

$$T_k = \left|A_{out}^{(k)}/A_{in}^{(k)}\right|^2 = \left|\kappa a_k/A_{in}^{(k)}\right|^2. \quad (4)$$

Before considering the phase-switching dynamics, we identify the stable steady states of the optical flip-flop by considering the continuous-wave (CW) case for which $A_{in}^{(k)} = \sqrt{P_k}$ is a constant. The steady-state solutions are found by setting the time derivatives in Eqs. (1) and (2) equal to zero. Introducing the mode energy $E_k = |a_k|^2$, we find the following set of two algebraic equations ($k = 1$ or 2):

$$E_k\left[\left(1/2\tau_{ph}\right)^2 + (\Delta\omega_k + \gamma E_k + 2\gamma E_{3-k})^2\right] = \kappa^2 P_k. \quad (5)$$

We consider a flip-flop which is biased such that the two input beams have the same power ($P_1 = P_2 = P_0$) and the same detuning ($\Delta\omega_1 = \Delta\omega_2 = \Delta\omega_0$). In this case, the solutions of Eqs. (5) can be divided into two categories, one includes the symmetric ($E_1 = E_2$) and another the asymmetric ($E_1 \neq E_2$) kinds of solutions [6]. The asymmetric solutions come in pairs since the roles of E_1 and E_2 can always be reversed. Following [6], Fig. 2(a) shows important boundaries for each of these two groups of solutions in the $(\Delta\omega_0, P_0)$ plane. The solid curve bounds the region in which asymmetric solutions exist and the dashed curve bounds a region for which there is more than one symmetric solution (at least one symmetric solution is found to exist everywhere in the plane). In the presence of an asymmetric solution, one of the symmetric solutions always becomes unstable. For flip-flop operation, an ideal bias point would have only two stable asymmetric states, similar to the ones shown in Fig. 1. The set of such bias points is shaded in blue in Fig. 2(a). In the following analysis, we focus on a device biased such that $\Delta\omega_0 = -2/\tau_{ph}$ and $P_0 = 1/\gamma\tau_{ph}^2$; as indicated by a star in Fig. 2(a).

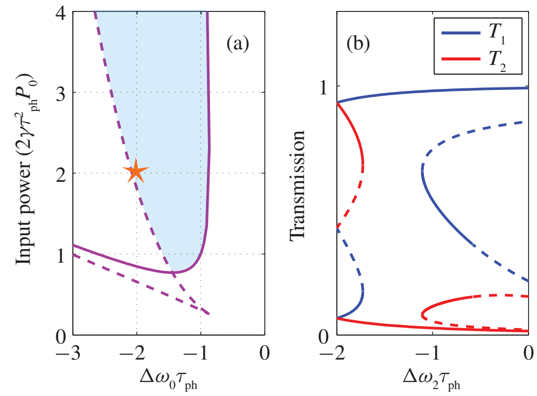

Fig. 2. (a) Boundaries for the symmetric (dashed curve) and asymmetric (solid curves) solutions in the $(\Delta\omega_0, P_0)$ plane. The shaded region shows the ideal biasing region in which only two asymmetric stable states exist. The star indicates the bias point used in this letter. (b) Available transmission states when detuning of beam 2 deviates from the chosen bias point. Solid and dashed lines show stable and unstable solutions, respectively.

III. Phase-Switched Optical Flip-Flops

The possibility of switching a single-input bistable resonator by modulating the input phase was considered in 1979 [10]. This work showed that phase-based switching is a non-trivial problem even when only one optical input is employed. Here, we first develop a conceptual understanding of the influence of phase modulation on two-input bistable resonators and then corroborate that understanding with numerical simulations.

Phase switching dictates that input phases of the two beams may change with time such that $A_{in}^{(k)}(t) = \sqrt{P_0}\exp[i\phi_k(t)]$. A time-dependent phase is equivalent to imposing a frequency chirp which, in turn, modifies the biasing conditions in a transient fashion. Mathematically, an instantaneous change in the detuning of an input beam from resonance is given by

$$\Delta\omega_k(t) = \Delta\omega_0 - \frac{d\phi_k}{dt}. \quad (6)$$

This change in detuning modifies the available steady states towards which the system will evolve. Figure 2(b) shows the behavior of the solutions of Eqs. (5) when $\Delta\omega_2\tau_{ph}$ deviates from its bias value of -2. Stability of each possible solution was examined through a standard linear stability analysis of Eqs. (1) and (2) (see, for example, section 7.2 of Ref. [11]). The solid and dashed lines show stable and unstable solutions, respectively. Figure 2(b) shows that the system will evolve toward a stable state in which T_1 is high and T_2 is low during the time for which $\Delta\omega_2$ is increased from its bias value. If the system is initially in a state for which T_1 is low and T_2 is high, a phase modulation of beam 2 can flip the device.

The phase of an input optical beam can be changed through a variety of techniques. A phase modulator can be used for this purpose to switch a flip-flop electro-optically. An all-optical flip-flop is designed to be switched using set and reset

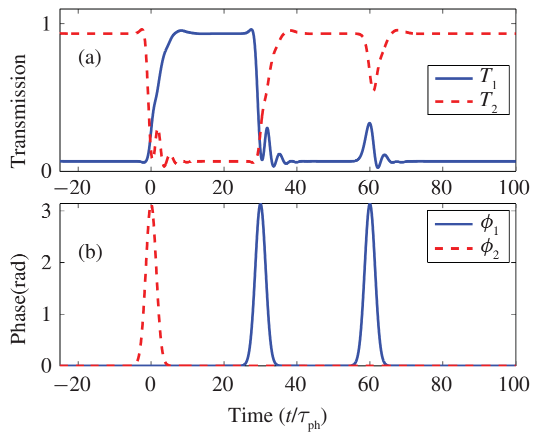

Fig. 3. Numerical simulations of phase-induced switching of the proposed optical flip-flop. T_1 and T_2 at the two output ports (a) when the input beams are kept at constant power but undergo phase changes as shown in (b).

optical pulses at wavelengths that are different from those of the CW inputs used to bias the device [1]. In our proposed configuration, the XPM phenomenon is used to modulate the phases of the two CW inputs. As an example, if we assume a Gaussian shape for set and reset pulses of width T_0, the XPM-induced phase shift can be written in the form [11]

$$\phi_k(t) = \phi_0 \exp[-(t - t_p)^2/T_0^2], \tag{7}$$

where ϕ_0 is the maximum phase shift occurring at the location $t = t_p$. The phase modulation induces a negative detuning of the CW bias beams for $t < t_p$ and a positive detuning for $t > t_p$. The largest value of the detuning is approximately given by

$$\Delta\omega_k^{max} \approx \Delta\omega_0 + 0.86\phi_0/T_0. \tag{8}$$

If the ratio ϕ_0/T_0 is sufficiently large ($\sim 1/\tau_{ph}$), and if this detuning is maintained for a duration long enough ($\sim \tau_{ph}$) that the resonator can respond, then the phase modulation will flip the device.

To verify that the proposed mechanism will flip/flop the device in a robust fashion, we solve Eqs. (1) and (2) for phase-modulated input beams. The set and reset operations are performed by applying Gaussian phase modulations of the form in Eq. (7) to input beams 2 and 1, respectively. We assume a pulse width $T_0 = 2\tau_{ph}$, for which our simulations show that switching occurs over the range of maximum phase shifts $2.3 < \phi_0 < 9$, leaving a broad window for phase error.

Figure 3(a) shows the power transmissivity for each of the two output ports when the set and reset operations are applied to the two inputs as shown in Fig. 3(b) when $\phi_0 = \pi$. Initially, the cavity is full of clockwise propagating light, causing the XPM-induced shift in the counter-clockwise mode resonance frequency so that $A_{in}^{(1)}$ is off resonance and T_1 is in the low state. When a set pulse modulates the phase of $A_{in}^{(2)}$, the flip-flop enters into a regime in which the only stable state is the one for which T_1 is high and T_2 is low. As a result, the system begins evolving towards this stable state. Physically, what happens during this process is that the cavity empties of clockwise-propagating light, reducing the XPM effect on the counter-clockwise mode so that $A_{in}^{(1)}$ can resonantly excite it. After the phase modulation is complete the cavity has filled up with counter-clockwise propagating light such that T_1 is in the high state, and the device has flipped. A subsequent phase modulation of $A_{in}^{(1)}$ by a reset pulse then causes the device to flop in an analogous fashion.

Figure 3 also shows that, if two consecutive reset signals are applied, the second will not cause the device to flip anomalously. This is not surprising given the conceptual understanding of the effect of the reset signal discussed earlier. The leading edge of this second signal causes some modulation in the power at the output ports, however its trailing edge reliably walks T_1 back into its low state in accordance with the reset operation.

In Fig. 3, the switching time is $\sim 5\tau_{ph}$ for both the on and off states. Typical integrated resonators have photon lifetimes ~10 ps [3], [4] that can be further reduced through a proper resonator design. Thus, it is reasonable to conclude that switching times of under 50 ps are realistic for the proposed device. For the set/reset pulses, peak power levels required to obtain a π phase shift by XPM in a silicon-based device of 2 W have been demonstrated, and lower values may be achievable [12].

In summary, we have proposed pure phase-based switching of two-input optical flip-flops in a manner that is compatible with XPM-induced phase shifts of the holding beams by set and reset pulses. We have identified the conditions under which phase switching can occur. A conceptual understanding of the switching mechanism was developed and confirmed through numerical simulations, showing that both the set and reset operations take place in a robust fashion. Switching on picosecond time scales can in principle be achieved with the proposed device even in the presence of thermal nonlinearity.

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