Spectral and temporal changes of optical pulses propagating through time-varying linear media

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We present universal formulas for the spectral and temporal output optical fields from a linear traveling-wave medium whose refractive index changes during its propagation within the medium. These formulas agree with known changes in central wavelength and energy that are associated with adiabatic wavelength conversion (AWC). Moreover, they reveal new changes to the optical pulses that have not been noticed, such as pulse compression and spectral broadening. Most significantly, we find that AWC alters the pulse power, pulse chirp, and pulse delay. All of these effects depend on whether the central wavelength is blueshifted or redshifted, the first sign of asymmetry to be reported for AWC. These findings impact the applications of AWC to optical signal processing in microphotonic and nanophotonic structures as well as in lightwave systems.

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It is well known that neither the shape nor the spectrum of an optical pulse change during its propagation inside a nondispersive lossless linear medium of temporally constant refractive index. However, it was observed recently that the spectrum of an optical pulse can change inside a linear medium whose refractive index changes with time [1–9]. The term *adiabatic wavelength conversion* (AWC) is sometimes used to describe such frequency changes. AWC is a linear process that does not depend on optical power or phase matching and was first observed in dynamically tuned resonators [3–6]. Numerical simulations based on a finite-difference time-domain (FDTD) method [3] and experiments carried out to understand the AWC phenomenon inside optical cavities [4–6] focused on a shift in central wavelength and pulse energy. Recently, a theory based on Bloch modes was developed for the photonic crystal optical waveguide [8], which shows that AWC does not require a resonator and can occur in traveling waveguide structures, as first simulated using a FDTD model [9].

In this Letter, we present universal formulas for the temporal and spectral changes of a pulse inside a time-varying medium. These formulas are universal in that they are applicable to all shapes of the input optical field as well as any dynamic index profile of the medium. The only requirement is that the index change occurs for the whole pulse. We show that rapid temporal changes in the refractive index of a linear medium not only shift the pulse spectrum but also lead to pulse compression and spectral broadening as well as variation of the pulse power, pulse chirp, and pulse delay.

Pulse propagation through a time-varying, lossless linear system is governed by a generalized convolution that relates the output electric field $E_{\text{out}}(t)$ to the input electric field $E_{\text{in}}(t)$ as

$$E_{\text{out}}(t) = \int_{-\infty}^{\infty} h(t,t')E_{\text{in}}(t')\,\mathrm{d}t', \tag{1}$$

where $h(t,t')$ is the impulse response function of the system. If the system is time invariant, a case depicted in Fig. 1(a), the impulse response function is $h(t,t') = \delta(t - t' - T_r)$, because all parts of the pulse are delayed by a constant transit time $T_r = n_1L/c$, where n_1 is the refractive index, L is the medium length, and c is the speed of light in a vacuum. In this case, a filterlike equation relating the input and output spectra, $\tilde{E}_{\text{out}}(\omega) = H(\omega)\tilde{E}_{\text{in}}(\omega)$, makes it clear that no new frequencies can be generated.

For the time-varying case, the refractive index is assumed to change by the same amount for the whole medium, and different parts of the pulse are delayed by different times as they complete their passage through the medium, as depicted in Fig. 1(b). The transit time $T_r(t')$ is now a function of the input time t, and the impulse response function for a linear time-variant medium in terms of $T_r(t')$ is

$$h(t,t') = \delta[t - t' - T_r(t')]. \tag{2}$$

Notably, $h(t,t')$ is written as a function of a single variable: $t - t' - T_r(t')$.

For an optical medium of length L, the time-dependent transit time $T_r(t')$ is determined using

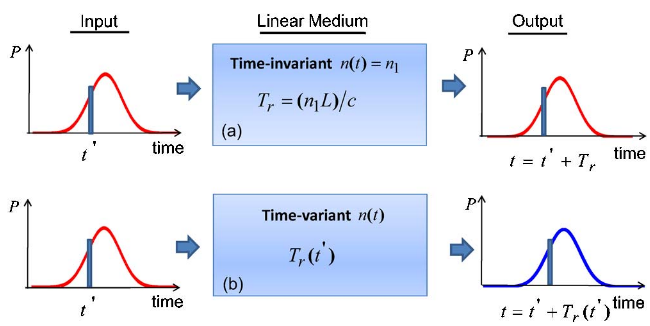

Fig. 1. (Color online) Input and output pulses of linear systems: (a) time-invariant and (b) time-variant medium. Each temporal slice of input pulse is delayed by a transit time T_r, which is constant for the time-invariant case, but time dependent for the time-variant case.

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$$\int_{t'}^{t'+T_r(t')} [c/n(\tau)] \mathrm{d}\tau = L, \tag{3}$$

where we focus on a temporal slice of the input pulse at time t', as shown in Fig. 1. It is evident from this equation that $T_r(t')$ will depend on the refractive index change's temporal profile.

To solve for $T_r(t')$, we assume that the refractive index begins to change from its initial value of n_1 at time T_i, follows some function $n(t)$ until the final time T_f such that $n(T_f) = n_2$, and stays at n_2 after that. The integral in Eq. (3) then has the form

$$\int_{t'}^{T_i} (c/n_1) \mathrm{d}\tau + \int_{T_i}^{T_f} [c/n(\tau)] \mathrm{d}\tau + \int_{T_f}^{t'+T_r(t')} (c/n_2) \mathrm{d}\tau = L. \tag{4}$$

This formula requires that the pulse is entirely within the medium as the index changes. The time-dependent transit time $T_r(t')$ in this general dynamic case can be calculated from Eq. (4) as

$$T_r(t') = (1 - s)t'/s + T_{r0}, \tag{5}$$

where we introduce a stretching factor $s = n_1/n_2$ and an effective transit time delay T_{r0}, expressed fully as

$$T_{r0} = n_2 L/c + T_f - T_i/s - n_2 \int_{T_i}^{T_f} [1/n(\tau)] \mathrm{d}\tau. \tag{6}$$

If the refractive index does not change during the time interval $T_f - T_i$, then $n_2 = n_1$, $s = 1$, and $T_r(t')$ is reduced to the constant value $n_1 L/c$ as seen above for the time-invariant case.

Combining Eq. (1), (2), and (5) and performing the integral over a scaled time variable t'/s result in the universal formula in time domain [Eq. (7)]. The frequency domain formula [Eq. (8)] is found by its Fourier transform:

$$E_{\text{out}}(t) = sE_{\text{in}}(st - sT_{\text{ro}}), \tag{7}$$

$$\tilde{E}_{\text{out}}(\omega) = \tilde{E}_{\text{in}}(\omega/s) \exp(-i\omega T_{\text{ro}}). \tag{8}$$

These formulas show that the major effect of temporally changing the medium's refractive index manifests on the optical pulse through a stretching parameter $s = n_1/n_2$ (which depends only on the ratio of the initial and final values of the refractive index) and on an effective transit-time delay T_{ro}.

As an example of how the stretching factor s affects optical pulses, we look at a Gaussian pulse:

$$E_{\text{in}}(t) = E_0 e^{-t^2/2T_0^2 - i\omega_1 t}, \tag{9}$$

$$\tilde{E}_{\text{in}}(\omega) = E_0 T_0 \sqrt{2\pi}\, e^{-T_0^2(\omega-\omega_1)^2/2}, \tag{10}$$

where T_0 is the pulse width and ω_1 is the carrier frequency. The corresponding output field is obtained by applying the universal formulas [Eqs. (7) and (8)] to the input field [Eqs. (9) and (10), respectively]:

$$E_{\text{out}}(t) = sE_0 e^{-s^2(t-T_{r0})^2/2T_0^2 - is\omega_1(t-T_{r0})}, \tag{11}$$

$$\tilde{E}_{\text{out}}(\omega) = E_0 T_0 \sqrt{2\pi}\, e^{-T_0^2(\omega - s\omega_1)^2/(2s^2) - i\omega_1 T_{r0}}. \tag{12}$$

For the special case $s = 1$ where the medium is time invariant, neither the temporal shape $P(t)$ nor the spectrum $P(\omega)$ of the pulse changes, a case shown in Fig. 2 by solid curves, where the optical field is expressed as $E = \sqrt{P} e^{i\phi}$. The only effect of the time-invariant medium is to induce a phase offset and linear slope in the temporal and spectral domains, respectively, as depicted by the solid curves in Fig. 2.

For the general time-variant case, the output pulse remains Gaussian in shape but its carrier frequency changes as $\omega_2 = s\omega_1$. This frequency shift matches previous FDTD and experimental studies of AWC [1–9]. The magnitude of the central frequency shift depends on the ratio $s = n_1/n_2$ and is relatively small in practice, typically $<0.01\%$, but recently $\sim 0.2\%$ [8]. For an optical frequency $\nu \sim 200$ THz, a 0.2% change in frequency is about 0.4 THz, which can be easily measured experimentally.

Importantly, this spectral shift is accompanied by temporal reshaping of the pulse. It is evident from Eqs. (9) and (11) that the temporal pulse width changes from T_0 to T_0/s. Moreover, pulse spectral width ($W_0 = 1/T_0$) changes to $W_0' = sW_0$, as evident from Eqs. (10) and (12). A blueshift of the central wavelength ($s > 1$) results in temporal pulse compression, amplitude enhancement, and spectral broadening, as depicted by the dashed (blue) curves in Fig. 2. On the other hand, a redshift ($s < 1$) results in temporal pulse broadening, amplitude

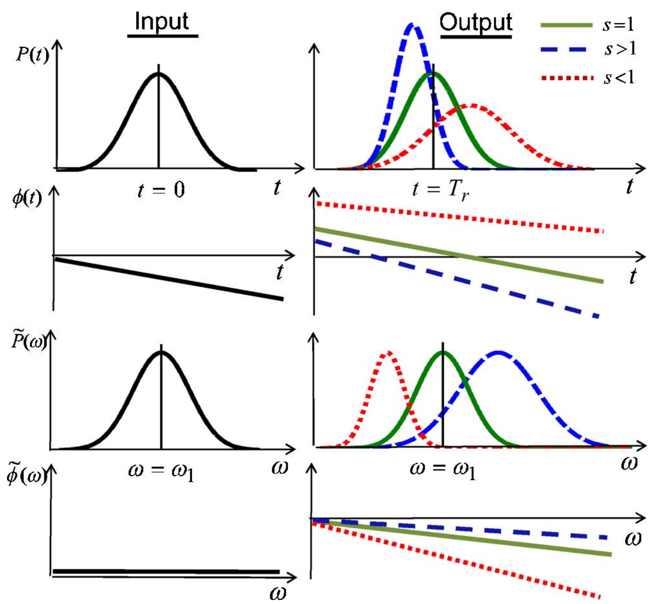

Fig. 2. (Color online) Input (left column) and output (right column) traces showing the power and phase profiles of Gaussian pulses in the temporal and spectral domains. The dashed (blue), solid (green), and dotted (red) curves correspond to stretching factors of $s = 3/2$, 1, and 2/3, respectively. We choose a linear variation of $n(\tau)$ for $s \neq 1$, while preserving the interval $T_f - T_i = 0.5 n_1 L/c$..

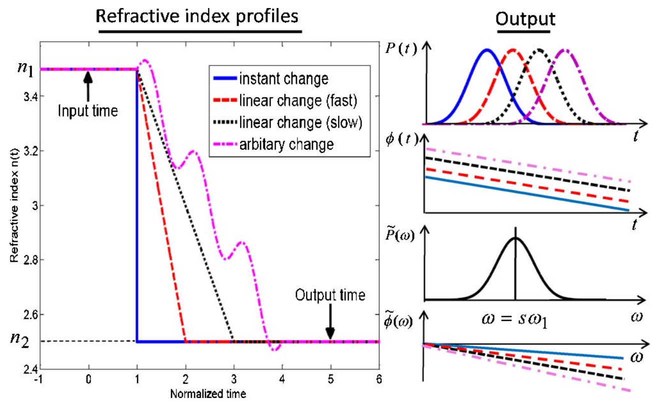

Fig. 3. (Color online) Four dynamic refractive index profiles and the corresponding output pulses. In all cases, the refractive index is initially n_1 and becomes n_2 after the change. The output pulses are identical in temporal shape and spectral power, but have different time delays.

reduction, and spectral compression, as depicted by the dotted (red) curves in Fig. 2. Although the pulse is rescaled, the number of electric-field cycles within the pulse is conserved. Although 0.2% changes (or less) in pulse temporal and spectral width may not be easy to observe, the temporal power scales as s^2, making the asymmetry between red and blue AWC shifts measurable. Furthermore, the nonlinear effects scale as s^4 or higher order and should be more noticeable.

We now consider the impact of different $n(\tau)$ while maintaining the same n_1 and n_2. From Eq. (4) it follows that all four cases of $n(\tau)$ shown in Fig. 3 have $T_r(t') = (1-s)t'/s + T_{r0k}$, where $s = n_1/n_2$ and T_{r0k} is obtained from Eq. (6) using the appropriate form of $n(t)$ for the k_{th} scenario ($k = 1$ to 4). Since the same stretching factor s appears, the temporal and spectral power shapes of the output pulse are identical in all four cases. This conclusion agrees with the numerical results obtained from a Maxwell-equation-based FDTD method [3].

However, the output pulse *does* experience a temporal delay that depends on $n(\tau)$, even for the same s, as depicted in Fig. 3. This dependence on the dynamic index profile may be pursued as a means of controlled optical buffering. Also, pulse delay depends on the "direction" of frequency shift; for the blueshifted case ($s > 1$) in Fig. 2, the pulse is advanced, while for the redshifted case ($s < 1$), the pulse is delayed. The optical phases shown in Fig. 3 also depend on $n(\tau)$. The phase constant of the temporal phase $\phi(t)$ is changed, but this has no bearing on the central frequency $s\omega$. The slope of the spectral phase $\tilde{\phi}(\omega)$ changes with $n(\tau)$, corresponding to the change in the time delay of the temporal pulse.

AWC does cause a *nonlinear* change in the temporal phase for chirped input pulses, a previously unnoticed effect. Specifically, for the first-order chirp e^{-ict^2}, the output chirp is $e^{-is^2c(t-T_{r0})^2}$ from Eq. (7). AWC causes the output chirp to have a factor of s^2. For the blueshifted case ($s > 1$), the chirp is enhanced, whereas for the redshifted case ($s < 1$), the chirp is suppressed. These changes in temporal phase lead to change in pulse width upon further propagation down the fiber-optic transmission system. Moreover, AWC has an even more severe effect on higher-order chirp, converting chirp of the form t^3 and t^4 (for example) to s^3t^3 and s^4t^4, respectively.

The conserved physical quantity for AWC is the so-called *action*, defined as $J = U/\omega$ [10], where U is the pulse energy. Physically, J is proportional to the number of photons contained in the pulse, a conserved quantity. This invariance in action (not energy) was first brought to light in the FDTD simulation in [3] and is the reason why this wavelength-conversion phenomenon is called AWC. Using our universal formulas [Eq. (7) and (8)], this conservation is readily shown:

$$J_{\text{out}} = \frac{\int_{-\infty}^{\infty} |sE_{\text{in}}(st - sT_{r0})|^2 \mathrm{d}t}{s\omega} = \frac{\int_{-\infty}^{\infty} |E_{\text{in}}(t)|^2 \mathrm{d}t}{\omega} = J_{\text{in}}. \tag{13}$$

In conclusion, we presented universal formulas for the spectral and temporal output optical fields from a dynamic linear optical medium. We derive these formulas by extending the delta-function concept of impulse response to time-variant linear systems and incorporating a physical picture of the time delay by a linear medium. This varies greatly from the modal and FDTD approaches used in earlier works. The pulse changes we predict arise from scaling of time by the stretching factor s. We show that frequency shift due to AWC is accompanied by changes in the pulse width, peak power, delay, and chirp of the input pulses. We are currently extending this analysis technique to the case of optical resonators.

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