

Effect of free carriers on pump-to-signal noise transfer in silicon Raman amplifiers

Ivan D. Rukhlenko,^{1,*} Indika Udagedara,¹ Malin Premaratne,¹ and Govind P. Agrawal²

¹Advanced Computing and Simulation Laboratory, Monash University, Clayton, Victoria 3800, Australia

²The Institute of Optics, University of Rochester, Rochester, New York 14627, USA

*Corresponding author: ivan.rukhlenko@monash.edu

Received May 4, 2010; revised June 11, 2010; accepted June 11, 2010;
posted June 15, 2010 (Doc. ID 128014); published July 5, 2010

We study noise transfer from pump to signal in silicon Raman amplifiers, with particular emphasis on the regimes of strong cumulative free-carrier absorption and heavy pump depletion. We calculate the relative intensity noise (RIN) transfers in copumped and counterpumped amplifiers and provide intuitive explanations for RIN peculiarities. We show that noise transfer at low frequencies may be suppressed by carefully choosing the pump intensity, effective free-carrier lifetime, or amplifier length, but only at the expense of a rise in noise at high frequencies. © 2010 Optical Society of America

OCIS codes: 190.4360, 230.0230, 230.4480, 250.4390.

Despite several experimental demonstrations of net Raman gain in silicon-on-insulator (SOI) waveguides [1–3], the problem of efficient Raman amplification in silicon is far from being fully solved. The main reason for this is the need to minimize nonlinear optical losses, caused by two-photon absorption (TPA) and free-carrier absorption (FCA), while avoiding excessive signal noise. The noise performance is especially important for silicon Raman amplifiers (SRAs), because they operate at high pump powers that result in enhanced transfer of the relative intensity noise (RIN) from pump to the signal.

Unlike the challenge of nonlinear losses, which has been much investigated in recent years [4–6], the physics of RIN transfer in SRAs is poorly understood. The results of a few recent theoretical studies on this phenomenon [7,8] are not widely applicable, for they are based on strong simplifying assumptions, which may not be met in practice. Specifically, in order to estimate the impact of RIN transfer on the noise figure in SRAs, Sang *et al.* [7] assumed instantaneous FCA and employed the undepleted-pump approximation. While the first assumption is valid for low-frequency noise components, the second holds only when the signal power remains much smaller than the pump power all along the amplifier. In our recent work [8], we abandoned the undepleted-pump approximation and analytically calculated the low-frequency RIN in copumped SRAs. Although the results allowed us to qualitatively predict the possibility of zero noise transfer under certain conditions, a precise quantitative assessment of this phenomenon is still required. In this Letter, we present a general study of the RIN-transfer problem in SRAs. In our work, we consider both the depletion of the pump and the cumulative nature of FCA, draw an intuitive picture of the physics behind the noise transfer, and present guidelines for RIN minimization.

The starting point for our study is the set of partial differential equations that describe the interaction of two cw fields (pump and signal) propagating through an SOI waveguide of constant cross section. These equations relate the pump intensity $I_p(z, t)$, the signal intensity $I_s(z, t)$, and the density $N(z, t)$ of free carriers as [8,9]

$$\pm \frac{1}{I_p} \left(\frac{\partial I_p}{\partial z} \pm \frac{1}{v_p} \frac{\partial I_p}{\partial t} \right) = -\alpha_p - \beta_p I_p - \sigma_p N - \gamma_p I_s, \quad (1a)$$

$$\frac{1}{I_s} \left(\frac{\partial I_s}{\partial z} + \frac{1}{v_s} \frac{\partial I_s}{\partial t} \right) = -\alpha_s - \beta_s I_s - \sigma_s N + \gamma_s I_p, \quad (1b)$$

$$\frac{\partial N}{\partial t} = -\frac{N}{\tau_c} + \rho_p I_p^2 + \rho_s I_s^2 + \rho_{ps} I_p I_s, \quad (1c)$$

where the + (or –) sign is chosen depending on whether the SRA is copumped or counterpumped. The parameters v_j , α_j , and β_j are, respectively, the group velocity, linear-loss coefficient, and TPA coefficient at the wavelength λ_j of the j th beam; τ_c is the effective free-carrier lifetime. The other parameters are defined as follows:

$$\begin{aligned} \sigma_j &= \sigma_0 (\lambda_j / \lambda_0)^2, \quad \sigma_0 = 1.45 \times 10^{-21} \text{ m}^2, \quad \lambda_0 = 1.55 \text{ } \mu\text{m}, \\ \gamma_p &= g_R + 2\beta_{ps}, \quad \gamma_s = (\lambda_p / \lambda_s)(g_R - 2\beta_{ps}), \\ \rho_j &= \beta_j \lambda_j / (4\pi \hbar c), \quad \rho_{ps} = \beta_{ps} \lambda_p / (\pi \hbar c), \end{aligned}$$

where g_R is the Raman gain coefficient, β_{ps} is the cross-TPA coefficient, c is the speed of light in vacuum, and the signal is assumed to be separated from the pump by the Raman shift of 15.6 THz.

The RIN spectrum of a pump laser consists of noise components at frequencies ranging from zero to a few gigahertz. We can study the RIN-transfer problem by focusing on a specific noise component at frequency ω and writing the solution of Eqs. (1) in the form [7]:

$$\begin{aligned} I_j(z, t) &= \mathcal{I}_j(z) \{1 + a_j(z) \exp[i\omega(t \mp z/v_p)]\}, \\ N(z, t) &= \mathcal{N}(z) [1 + n(z) \exp(i\omega t)], \end{aligned}$$

where $a_j(z)$ and $n(z)$ are the complex-valued quantities satisfying the conditions $|a_j(z)|, |n(z)| \ll 1$. The time-independent quantities $\mathcal{I}_j(z)$ and $\mathcal{N}(z)$ represent the average intensities and carrier density, respectively.

It is easy to show using Eq. (1) that $\mathcal{I}_j(z)$ and $a_j(z)$ with $j = s, p$ satisfy the equations

$$\pm \frac{1}{\mathcal{I}_p} \frac{d\mathcal{I}_p}{dz} = -\alpha_p - \beta_p \mathcal{I}_p - \sigma_p \mathcal{N} - \gamma_p \mathcal{I}_s, \quad (2a)$$

$$\frac{1}{\mathcal{I}_s} \frac{d\mathcal{I}_s}{dz} = -\alpha_s - \beta_s \mathcal{I}_s - \sigma_s \mathcal{N} + \gamma_s \mathcal{I}_p, \quad (2b)$$

$$\pm \frac{da_p}{dz} = -\beta_p \mathcal{I}_p a_p - \sigma_p \mathcal{N} n - \gamma_p \mathcal{I}_s a_s, \quad (2c)$$

$$\frac{da_s}{dz} + \frac{i\omega}{v_{\pm}} a_s = -\beta_s \mathcal{I}_s a_s - \sigma_s \mathcal{N} n + \gamma_s \mathcal{I}_p a_p, \quad (2d)$$

where $v_{\pm} = v_p v_s / (v_p \mp v_s)$,

$$\begin{aligned} \mathcal{N}(z) &= \tau_c (\rho_p \mathcal{I}_p^2 + \rho_s \mathcal{I}_s^2 + \rho_{ps} \mathcal{I}_p \mathcal{I}_s), \\ n(z) &= \frac{2(\rho_p \mathcal{I}_p^2 a_p + \rho_s \mathcal{I}_s^2 a_s) + \rho_{ps} \mathcal{I}_p \mathcal{I}_s (a_p + a_s)}{(1 + i\omega\tau_c)(\rho_p \mathcal{I}_p^2 + \rho_s \mathcal{I}_s^2 + \rho_{ps} \mathcal{I}_p \mathcal{I}_s)}. \end{aligned}$$

The set of four equations (2) needs to be solved numerically, together with the four boundary conditions. When the signal is launched at $z = 0$, the boundary conditions are

$$\mathcal{I}_p(\delta_{\pm}) = \mathcal{I}_{p0}, \quad \mathcal{I}_s(0) = \mathcal{I}_{s0}, \quad a_p(\delta_{\pm}) = a_0, \quad a_s(0) = 0,$$

where $\delta_{\pm} = (L/2)(1 \mp 1)$, L is the amplifier length, and a_0 is a sufficiently small real number ($a_0 \ll 1$). Once the solution of Eq. (2) is obtained, the RIN transferred from the pump to signal is found from the relation $R_{\pm} = |a_s(L)/a_0|^2$ [7,8].

Before showing the numerical results, it is instructive to highlight several features of noise transfer governed by Eqs. (2a) and (2d). First of all, the RIN transfer in SRAs stems from three distinct sources: cross-TPA (included through parameters γ_j), FCA, and stimulated Raman scattering (SRS). Second, free carriers influence signal noise only at pump intensities high enough that the FCA term in Eq. (2d) is comparable to the Raman term. Third, the effect of free carriers on RIN depends drastically on noise frequency. If $\omega \ll |v_{\pm}|/L$ and $\omega \ll \tau_c^{-1}$, the last two terms in Eq. (2d) have opposite signs and FCA reduces the RIN transferred to the signal through SRS. In addition, FCA increases low-frequency noise attenuation resulting from TPA. As the noise frequency ω increases, the effect of FCA weakens—due to insufficient accumulation of free carriers—and becomes negligible for $\omega \gg \tau_c^{-1}$; this enhances the noise. For frequencies exceeding $|v_{\pm}|/L$, the averaging of noise along the amplifier length develops [10], and RIN is reduced considerably.

As a numerical example, we consider a 1.55- μm signal amplified using a pump at 1.434 μm , both propagating in the form of TE modes along an air-clad SOI waveguide. We estimate velocities v_{\pm} using an effective refractive index $n_{\text{TE}} = 2.76$ and a dispersion parameter $D = 1 \text{ ps}/(\text{m} \times \text{nm})$ [9]; $v_{+} \approx [D(\lambda_s - \lambda_p)]^{-1}$, $v_{-} \approx c/(2n_{\text{TE}})$. We also use $\alpha_p = \alpha_s = 1 \text{ dB/cm}$, $\beta_p = \beta_s = \beta_{ps} = 0.5 \text{ cm/GW}$, $g_R = 76 \text{ cm/GW}$ [8], $\mathcal{I}_{s0} = 0.01\mathcal{I}_{p0}$, and $a_0 = 0.001$ in numerical calculations.

Figure 1 shows the RIN-transfer spectra corresponding to different free-carrier lifetimes for copumped and coun-

terpumped SRAs. At high pump intensities, the spectrum exhibits two pronounced plateaus in the range $0 < \omega \ll \tau_c^{-1}$ and $\tau_c^{-1} \ll \omega \ll |v_{\pm}|/L$. As discussed earlier, the lower plateau owes its existence to the cumulative nature of FCA. The interpretation of RIN changes between the two plateaus is nontrivial, as such changes may occur for $\omega \ll \tau_c^{-1}$ (e.g., red spectra). In this case, the growth of RIN results from self-phase modulation (SPM) of the signal, originating as follows. Noise on the signal creates small temporal variations of TPA and cross-TPA. These variations modulate FCA, which not only attenuates signal oscillations but also shifts their phase with respect to the oscillations of free-carrier density. As a result, the efficiency of FCA-induced RIN reduction decreases, and the RIN level goes up. Analysis of Eq. (2d) shows that SPM of the signal develops at frequencies around $\omega_{\pm} = \beta_s (\sigma_s \tau_c^2)^{-1} [2\rho_s \mathcal{I}_s(L) + \rho_{ps} \mathcal{I}_p(L)]^{-1}$. For example, $\omega_{+} \approx 2, 10,$ and 90 MHz , respectively, for the red, blue, and black spectra in the upper panel of Fig. 1. Clearly, FCA can manifest its cumulative nature at frequencies much less than τ_c^{-1} .

Three important conclusions can be drawn from the peculiarities of the RIN spectra in Fig. 1. First, the presence of dense plasma generated by an intense pump reduces low-frequency RIN substantially in all SRAs. As is seen from the comparison of orange and blue spectra, this reduction can be achieved without decreasing the net Raman gain. Second, similar to linear losses in fiber

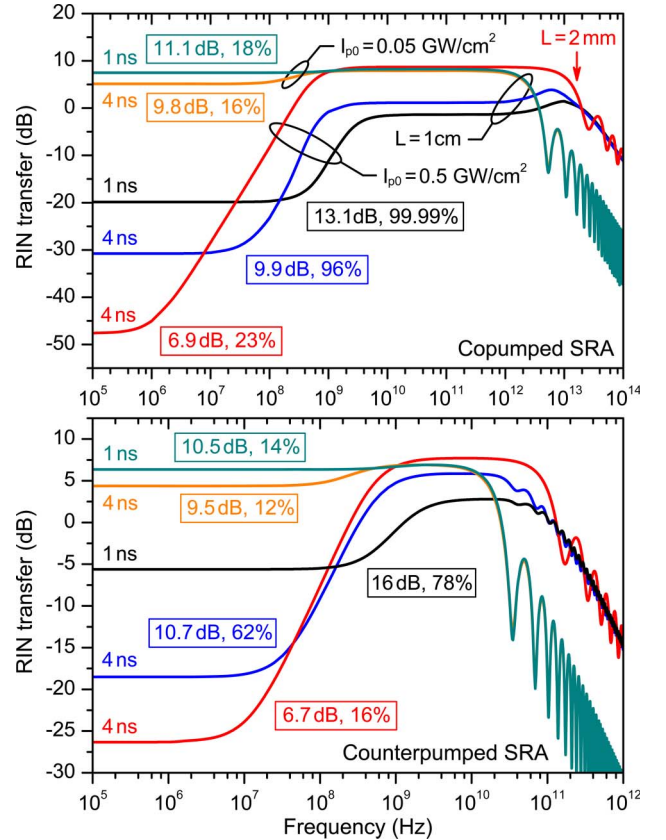


Fig. 1. (Color online) RIN-transfer spectra for different operating conditions of SRAs. The framed data represent the net signal gain and pump-depletion level. Free-carrier lifetimes, incident intensities, and amplifier lengths are varied for different curves as marked.

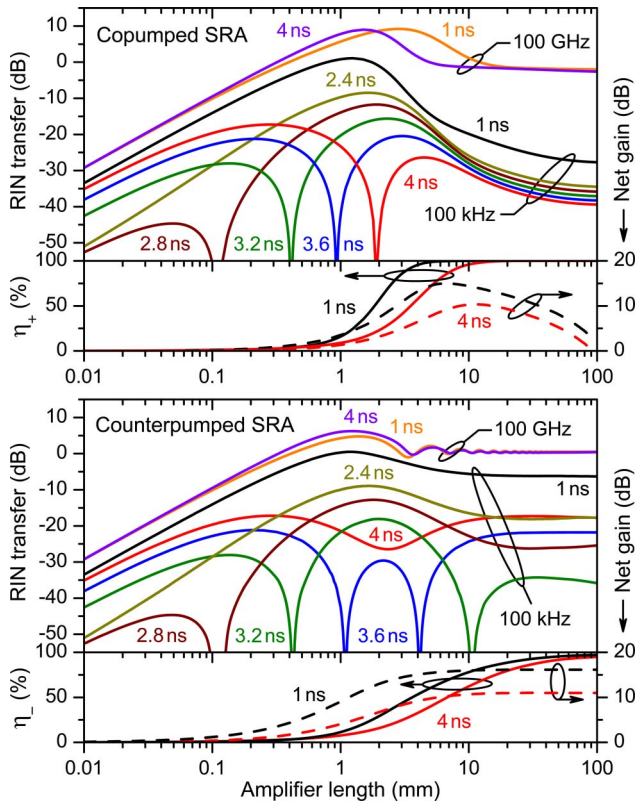


Fig. 2. (Color online) Dependence of low- (100 kHz) and high-frequency (100 GHz) RINs on amplifier length for $\mathcal{I}_{p0} = 0.5 \text{ GW/cm}^2$ for copumped (top) and counterpumped (bottom) SRAs. Small panels show net signal gain (dashed curves) and depletion coefficients (solid curves) for free-carrier lifetimes of 1 and 4 ns.

Raman amplifiers [10], FCA increases the -3 dB corner frequency and enhances high-frequency noise transfer. For example, if $\tau_c = 1 \text{ ns}$, a rise in the pump intensity from 0.05 to 0.5 GW/cm^2 shifts the -3 dB frequency (measured from the upper plateau) from 2.7 to 50 THz in a copumped SRA, and from 20 to 200 GHz in a counterpumped SRA. Such high values of the cutoff-noise frequency make RIN engineering in SRAs of prime importance. Finally, the impact of FCA on RIN transfer depends more on the level of pump depletion than on the net gain (compare blue and orange spectra). We characterize the extent of pump-power depletion by the depletion coefficient $\eta_{\pm} = 1 - \mathcal{I}_p(\delta_{\mp})/\mathcal{I}_p^{(0)}(L)$, where $\mathcal{I}_p^{(0)}(z)$ is the solution of Eqs. (2a) and (2b), with the boundary conditions $\mathcal{I}_p(\delta_{\pm}) = \mathcal{I}_{p0}$ and $\mathcal{I}_s(0) = 0$. As the pump is depleted, SRS becomes less efficient, and the relative importance of FCA increases.

The dependence of RIN transfer on the length of SRA, shown in Fig. 2, indicates that FCA can totally suppress the low-frequency noise. As seen from this figure, for a given pump power, RIN vanishes for a specific value of amplifier length in the case of copumping, and for two values in the case of counterpumping. The existence

of such zero-RIN points is related to the evolution of the phase shift $\Delta\phi$ between intensity fluctuations of the pump and signal [8]. For instance, near the input end of the counterpumped SRA, $\Delta\phi = 0$, because pump and signal are weak and signal gain grows with pump intensity. As the intensity of the pump increases, FCA comes into effect and causes signal gain to decay with pump intensity. As a result, $\Delta\phi$ turns to $-\pi$. Near the output end of SRA, the intensities of the pump and signal may be high enough for SRS to dominate again and $\Delta\phi$ may become zero for the second time. At the points where $\Delta\phi$ abruptly changes from 0 to $-\pi$ and from $-\pi$ to 0 , the noise transfer reduces to zero. If the incident power is fixed, FCA-induced RIN compensation cannot be realized for free-carrier lifetimes below a minimal value because FCA stays too weak, irrespective of the distance inside the SRA. However, one can always achieve zero RIN in a given amplifier by increasing pump intensity. Most importantly, in the regime of strong pump depletion, the reduction of RIN in a broad frequency range can be achieved at the cost of a small decrease in the net gain (see small panels in Fig. 2).

In summary, we studied theoretically the impact of free carriers on pump-to-signal noise transfer in SRAs. We showed that FCA reduces the RIN transfer at low frequencies and increases it at high frequencies, in both copumped and counterpumped SRAs. In a given frequency range, signal noise can be minimized by varying incident powers and the free-carrier lifetime, or by adjusting the length of the amplifier.

This work was supported by the Australian Research Council through its Discovery Grant scheme under grant DP0877232. The work of G. P. Agrawal is also supported by the U.S. National Science Foundation (NSF) through award ECCS-0801772.

References

1. R. Claps, D. Dimitropoulos, V. Raghunathan, Y. Han, and B. Jalali, *Opt. Express* **11**, 1731 (2003).
2. A. Liu, H. Rong, M. Paniccia, O. Cohen, and D. Hak, *Opt. Express* **12**, 4261 (2004).
3. B. Jalali, V. Raghunathan, D. Dimitropoulos, and O. Boyraz, *IEEE J. Sel. Top. Quantum Electron.* **12**, 412 (2006).
4. R. Claps, V. Raghunathan, D. Dimitropoulos, and B. Jalali, *Opt. Express* **12**, 2774 (2004).
5. T. K. Liang and H. K. Tsang, *Appl. Phys. Lett.* **84**, 2745 (2004).
6. I. D. Rukhlenko, C. Dissanayake, M. Premaratne, and G. P. Agrawal, *Opt. Express* **17**, 5807 (2009).
7. X. Sang, D. Dimitropoulos, B. Jalali, and O. Boyraz, *IEEE Photon. Technol. Lett.* **20**, 2021 (2008).
8. I. D. Rukhlenko, M. Premaratne, and G. P. Agrawal, *IEEE J. Sel. Top. Quantum Electron.* **16**, 200 (2010).
9. Q. Lin, O. J. Painter, and G. P. Agrawal, *Opt. Express* **15**, 16604 (2007).
10. C. R. S. Fludger, V. Handerek, and R. J. Mears, *IEEE J. Lightwave Technol.* **19**, 1140 (2001).