Effects of coherence and polarization on the coupling of stochastic electromagnetic beams into optical fibers

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We study the problem of coupling an electromagnetic beam of any state of coherence and polarization into a multimode optical fiber. Using the well-known concept of the cross-spectral density matrix, we derive a general expression for the coupling efficiency of a stochastic electromagnetic beam into a multimode fiber in terms of the cross-spectral density matrix of the incident beam and another matrix representing field distributions of fiber modes. We apply this result to a specific case in which the incident beam belongs to a broad class of so-called electromagnetic Gaussian Schell-model beams and obtain a simple analytical expression for the coupling efficiency in the case of single-mode fibers. We use this expression to study how coupling efficiency depends on the coherence and polarization properties of the incident beam. © 2009 Optical Society of America

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1. INTRODUCTION

Optical fibers are employed for a variety of applications, including optical communications, biomedical optics, and high-power lasers. Efficient coupling of light into an optical fiber at any stage of an optical system is very important, since increasing the coupled light improves the signal-to-noise ratio at the detection stage. The coupling process includes using suitable optical components required to match the spatial profile of the incident light with the field distribution of the desired mode to be launched into the fiber.

The coupling of light into optical fibers has been investigated, both theoretically and experimentally, since 1972, when attempts were made to couple light into a fiber by using the so-called butt joint [1]. Improved coupling techniques have been suggested since then. For example, Weidel described a coupling method using a cylindrical glass fiber as a focusing element [2]. Cohen and Schneider fabricated microlenses on the fiber endfaces to increase the coupling efficiency of GaAs semiconductor lasers into single-mode fibers [3]. Sakai and Kimura introduced a miniature optical lens tipped on one end of a single-mode fiber for improving the power coupled from a semiconductor laser [4]. Saruwatari and Sugie coupled light from a laser diode into a single-mode fiber by employing a combination of two lenses in the confocal configuration [5]. Shah *et al.* showed, both theoretically and experimentally, that efficient power coupling can be achieved between a laser and a single-mode fiber by using a wedge-shaped fiber endface that is butt coupled to the laser [6].

Several theoretical studies discuss issues relevant to the coupling efficiency of optical fibers. For example, Barrell and Pask studied the excitation of an optical fiber by a plane wave [7] and focused on the effects of misalignments of the fiber and the coupling optics on the coupled light. Lazaroni and Zocchi studied much later the coupling efficiency of a plane wave to a single-mode step-index fiber, taking into account the exact expression of the mode's field distribution [8]. Wagner and Tomlinson discussed in 1982 the effects of aberrations of the coupling optics on the coupling efficiency [9]. Shaklan and Roddier calculated the coupling efficiency when star light is coupled into a single-mode fiber and studied the effects of atmospheric turbulence [10]. Christodoulides *et al.* developed a theory to estimate the coupling efficiency of incoherent radiation from a light-emitting diode into a single-mode fiber [11]. Chen and Kerps analyzed the same problem and took into account the coherence properties of the source [12]. Winzer and Leeb studied the problem of coupling partially coherent light into a single-mode fiber [13], with emphasis on the effects of speckle on the coupling efficiency. They also discussed the implications of their results for a lidar system. Dikmelik and Davidson studied the effects of atmospheric turbulence on the coupling efficiency of laser light into single-mode fibers [14]. They also investigated the use of a coherent fiber array, instead of a single fiber, and pointed out the superiority of the array in such a situation. Mukhopadhyay *et al.* studied the coupling efficiency of a laser diode into an elliptical core, single-mode, step-index fiber via a hyperbolic microlens on the tip of the fiber [15]. These authors employed a simple theoretical formulation for the evaluation of the coupling efficiency, using an *ABCD* matrix for the coupling optics.

All these publications considered only the coupling of scalar beams, and the polarization state of the beam was not taken into account. Recently, the coherence and polarization properties of stochastic electromagnetic beams have been studied extensively [16] by using a unified theory of coherence and polarization, and this theory provides a theoretical framework for dealing with these two phenomena [17]. The theory revealed that these phenomena are intimately related and can be determined for any stochastic electromagnetic beam if the cross-spectral density matrix of that beam is known [18]. The cross-spectral density matrix defines an optical beam with any state of coherence and polarization.

In this paper we generalize the definition of the coupling efficiency, given previously for scalar beams, to stochastic electromagnetic beams. We derive an expression for the coupling efficiency in terms of the cross-spectral density matrix of the incident beam and another matrix related to the field distribution of each mode. We examine how the coupling efficiency varies with changes in the coherence and polarization properties of the incident beam.

The paper is organized as follows. In Section 2 we derive a general expression for the coupling efficiency of stochastic electromagnetic beams into an optical fiber. We apply it in Section 3 to find the coupling efficiency of a particular class of beams, namely, the electromagnetic Gaussian Schell-model (GSM) beams. An analytical expression for the coupling efficiency can be derived in this case. In Section 4 we show through numerical examples the effects of the coherence and the polarization properties of the incident beam on the coupling efficiency. We conclude with some remarks in Section 5.

2. COUPLING EFFICIENCY OF STOCHASTIC ELECTROMAGNETIC BEAMS

Light guided inside optical fibers can be decomposed into a set of modes that are linearly polarized in the weakly guiding approximation [19]. Each mode in an optical fiber is twofold degenerate. These two modes have the same transverse distribution and the same propagation constant, with the only difference being that they are polarized along two orthogonal directions, which we choose to coincide with the x and y axes of a Cartesian coordinate system. The optical fiber modes also have longitudinal components in the z direction, but their values are known to be small relative to the transverse components of the modes. In our analysis we ignore the longitudinal components because they are related to the transverse components through Maxwell's equations and can be found easily by using them.

We consider the arrangement shown in Fig. 1, used typically to couple the incident beam into an optical fiber by means of a lens of focal length f. The fiber supports multiple modes, and the field distributions at plane B (the entrance plane) of the mth mode in the two orthogonal directions x and y are given by F_{mxB} and F_{myB}. It is more convenient for us to use an equivalent field distribution of the mode at plane A (the aperture plane), as this step simplifies the following analysis. The field distribution of the fiber modes at plane A can be determined with a back-propagation technique [20]. Let us assume that the components of the field distribution of the mode at plane A in two orthogonal direction are given as F_{mxA} and F_{myA}. In view of the linear nature of the coupling process, we focus

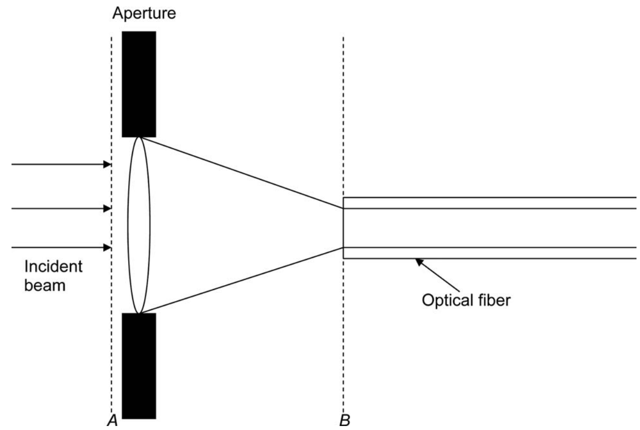

Fig. 1. Schematic and notation relating to the coupling of a stochastic electromagnetic beam into an optical fiber.

on one Fourier component of the electromagnetic beam incident on plane A and write it in the form

$$\mathbf{E}(\boldsymbol{\rho},\omega) = E_x(\boldsymbol{\rho},\omega)\hat{x} + E_y(\boldsymbol{\rho},\omega)\hat{y}, \tag{1}$$

where ω is the angular frequency, $\boldsymbol{\rho}$ is a two-dimensional transverse vector in plane A and $\hat{x}$ and $\hat{y}$ are unit vectors along the two transverse directions.

When an electromagnetic field is coupled into the optical fiber, it excites both the guided and the radiation modes of this fiber. Since these modes form a complete set, we can use the following expansion to express the electric field at any transverse position $\boldsymbol{\rho}$ of the aperture plane A:

$$\mathbf{E}(\boldsymbol{\rho},\omega) = \sum_m [C_{mxA}F_{mxA}(\boldsymbol{\rho},\omega)\hat{x} + C_{myA}F_{myA}(\boldsymbol{\rho},\omega)\hat{y}], \tag{2}$$

where C_{mxA} and C_{myA} are the coupling coefficients for the mth mode polarized along the x and y directions, respectively. The summation in Eq. (1) extends over all the guided modes and a continuum of radiation modes. Using Eqs. (1) and (2), we can write

$$E_j(\boldsymbol{\rho},\omega) = \sum_m C_{mjA}F_{mjA}(\boldsymbol{\rho},\omega) \quad (j = x,y). \tag{3}$$

Equation (3) can be used to express the components of the cross-spectral density matrix of the incident beam in terms of the mode-field distributions,

$$\begin{aligned} W_{ij}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\omega) &= \langle E_i^*(\boldsymbol{\rho}_1,\omega)E_j(\boldsymbol{\rho}_2,\omega)\rangle \\ &= \sum_m \sum_n \langle C_{miA}^* C_{njA}\rangle F_{miA}^*(\boldsymbol{\rho}_1,\omega)F_{njA}(\boldsymbol{\rho}_2,\omega), \end{aligned} \tag{4}$$

where i and j take values x or y and the angle brackets denote ensemble averaging. Noting that fiber modes are orthogonal to each other, we obtain from Eq. (4),

$$\begin{aligned} \langle C_{miA}^* C_{njA}\rangle = \int\int W_{ij}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\omega)F_{miA}(\boldsymbol{\rho}_1,\omega)F_{njA}^* \\ \times(\boldsymbol{\rho}_2,\omega)\mathrm{d}^2\rho_1\mathrm{d}^2\rho_2. \end{aligned} \tag{5}$$

In deriving Eq. (5) we used the following orthogonality relation for the fiber modes:

$$\int F^*_{mi}(\boldsymbol{\rho},\omega)F_{nj}(\boldsymbol{\rho},\omega)\mathrm{d}^2\rho = \delta_{nm}\delta_{ij}. \qquad (6)$$

The power coupled into a specific mode can be obtained by setting $m=n$ in Eq. (5) and summing over the two values of i and j. Writing the resulting expression in a matrix form, the power coupled into the m mode of the fiber is given by

$$P_{cm} = \int\int Tr[\mathbf{W}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\omega)\cdot \mathbf{F}^{\dagger}_{mA}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\omega)]\mathrm{d}^2\rho_1\mathrm{d}^2\rho_2, \quad (7)$$

where a dagger denotes the Hermitian adjoint and the matrix $\mathbf{F}_{mA}$ depends on the transverse profiles of the mth mode at plane A as

$$\mathbf{F}_{mA}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\omega) = \begin{pmatrix} F^*_{mxA}(\boldsymbol{\rho}_1,\omega)F_{mxA}(\boldsymbol{\rho}_2,\omega) & F^*_{mxA}(\boldsymbol{\rho}_1,\omega)F_{myA}(\boldsymbol{\rho}_2,\omega) \\ F^*_{myA}(\boldsymbol{\rho}_1,\omega)F_{mxA}(\boldsymbol{\rho}_2,\omega) & F^*_{myA}(\boldsymbol{\rho}_1,\omega)F_{myA}(\boldsymbol{\rho}_2,\omega) \end{pmatrix}. \qquad (8)$$

The coupling efficiency is the fraction of the incident power that gets coupled into a specific fiber mode and is obtained by dividing Eq. (7) by the total power of the incident beam. With this normalization, the coupling efficiency can be expressed as follows:

$$\eta_{cm} = \frac{\int\int Tr[\mathbf{W}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\omega)\cdot \mathbf{F}^{\dagger}_{mA}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\omega)]\mathrm{d}^2\rho_1\mathrm{d}^2\rho_2}{2\int Tr[\mathbf{W}(\boldsymbol{\rho},\boldsymbol{\rho},\omega)]\mathrm{d}^2\rho}. \qquad (9)$$

Equation (9) constitutes our main result, and it reduces to a similar expression obtained previously in [13] in the scalar case, except for some notational differences. This equation shows explicitly the relationship between the coupling efficiency and the cross-spectral density matrix, which defines the coherence and polarization properties of the incident beam. In the next section we apply Eq. (9) to a specific situation in which the incident beam belongs to a class of stochastic beams, the so-called electromagnetic GSM, and the Gaussian approximation is used to represent the field distribution of a single-mode fiber.

3. APPLICATION TO ELECTROMAGNETIC GAUSSIAN SCHELL-MODEL BEAMS

When the incident beam belongs to a class of electromagnetic GSM beams [21], the elements of the cross-spectral density matrix have the form

$$W_{ij}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2;\omega) = \sqrt{S_i(\boldsymbol{\rho}_1;\omega)}\sqrt{S_j(\boldsymbol{\rho}_2;\omega)}\mu_{ij}(\boldsymbol{\rho}_2-\boldsymbol{\rho}_1;\omega)$$

$$(i=x,y;\ j=x,y), \qquad (10)$$

where S_i and S_j are the spectral densities of the i and j components of the electric field and μ_{ij} denotes the degree of correlation between the two components. Moreover, in this model the spectral densities S_x and S_y, as well as the spectral degree of correlation μ_{xy}, are Gaussian functions of positions, i.e.,

$$S_j(\boldsymbol{\rho};\boldsymbol{\omega}) = A_j^2 \exp\left[-\frac{\boldsymbol{\rho}^2}{2\sigma_j^2}\right] \quad (j=x,y), \qquad (11a)$$

$$\mu_{ij}(\boldsymbol{\rho}_2-\boldsymbol{\rho}_1;\omega) = B_{ij}\exp\left[-\frac{(\boldsymbol{\rho}_2-\boldsymbol{\rho}_1)^2}{2\delta_{ij}^2}\right]$$

$$(i=x,y;\ j=x,y). \qquad (11b)$$

The parameters A_j, B_{ij}, σ_j, and δ_{ij} are assumed to be independent of position but may depend on frequency.

Assuming a single-mode fiber with a circular or elliptical cross section, the field distribution of this mode can be approximated by a Gaussian function [22–24]. The field distribution of the mode polarized along the j direction is then given by (in plane B)

$$F_{jB} = \sqrt{2/\pi}w_j^{-1}\exp(-\boldsymbol{\rho}'^2/w_j^2) \quad (j=x,y), \qquad (12)$$

where $\boldsymbol{\rho}'$ is a transverse position vector in the fiber plane and w_j is a measure of the mode width. The field distribution of the mode at the aperture plane A is found by using the backpropagation technique and is given by

$$F_{jA} = \sqrt{2/\pi}w_{jA}^{-1}\exp(-\boldsymbol{\rho}^2/w_{jA}^2), \qquad (13)$$

where w_{jA} is the effective mode width at the aperture plane and is given by

$$w_{jA} = \frac{\lambda f}{\pi w_j}. \qquad (14)$$

In Eqs. (13) and (14), λ is the wavelength of the incident beam and f is the focal length of the coupling lens.

Using Eq. (13) in Eq. (9), we can write the coupling efficiency as

$$\eta_C = \frac{\sum_{i,j=x,y} P_{C_{ij}}}{2\left[\sum_{j=x,y} P_{\mathrm{inc}_{jj}}\right]}. \qquad (15)$$

In Eq. (15) $P_{C_{ij}}$ represents the power coupled from the component W_{ij} into the fiber and is given by the expression

$$P_{C_{ij}} = \int\int_D W_{ij}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\omega)F_{iA}(\boldsymbol{\rho}_1)F^*_{jA}(\boldsymbol{\rho}_2)\mathrm{d}^2\rho_1\mathrm{d}^2\rho_2,$$

$$(i,j)=(x,y). \qquad (16)$$

$P_{\mathrm{inc}_{jj}}$ in Eq. (15) represents the power of the incident beam in the jth component,

$$P_{\mathrm{inc}_{jj}} = \int_D W_{jj}(\boldsymbol{\rho},\boldsymbol{\rho},\omega)\mathrm{d}^2\rho \quad (j=x,y). \qquad (17)$$

The symbol D in Eqs. (16) and (17) indicates that the integration extends over the area of the aperture of the coupling optics. In practice, it is a hard aperture of diameter D. To simplify the integration, following [25], we approximate this diameter by a Gaussian aperture of radius W related to the aperture diameter as

$$W^2 = D^2/8. \tag{18}$$

More specifically, we multiply the integrands on Eqs. (16) and (17) by the exponential cutoff factor $\exp[-(\rho_1^2+\rho_2^2)/W^2]$, extend the integration limits to infinity, and write them in the forms

$$Pc_{ij} = \int\int_W W_{ij}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\omega)F^*_{iA}(\boldsymbol{\rho}_1)F_{jA}(\boldsymbol{\rho}_2) \times\exp\left[-\frac{\rho_1^2+\rho_2^2}{W^2}\right]\mathrm{d}^2\rho_1\mathrm{d}^2\rho_2, \tag{19}$$

$$P_{\mathrm{inc}_{jj}} = \int_W W_{ii}(\boldsymbol{\rho},\boldsymbol{\rho},\omega)\exp\left[-\frac{2\rho^2}{W^2}\right]\mathrm{d}^2\rho. \tag{20}$$

With the Gaussian approximation given in Eq. (12) for the fundamental mode of a single-mode fiber with Eqs. (10) and (11) for a GSM beam, Eqs. (19) and (20) can be written in the following forms:

$$P_{C_{ij}} = \left(\frac{2}{\lambda f}\right)^2\int\int_W B_{ij}A_iA_j\exp\left[-\frac{\boldsymbol{\rho}_1^2}{4\sigma_i^2}\right] \times\exp\left[-\frac{\boldsymbol{\rho}_2^2}{4\sigma_j^2}\right]\exp\left[-\frac{(\boldsymbol{\rho}_2-\boldsymbol{\rho}_1)^2}{2\delta_{ij}^2}\right]\exp\left(-\frac{\boldsymbol{\rho}_1^2}{\sigma_{ig}^2}\right) \times\exp\left(-\frac{\boldsymbol{\rho}_2^2}{\sigma_{jg}^2}\right)\exp\left[-\frac{\rho_1^2+\rho_2^2}{W^2}\right]\mathrm{d}^2\rho_1\mathrm{d}^2\rho_2, \tag{21}$$

$$P_{\mathrm{inc}_{jj}} = \int_W A_i^2\exp\left[-\frac{\boldsymbol{\rho}^2}{2\sigma_i^2}\right]\exp\left[-\frac{2\rho^2}{W^2}\right]\mathrm{d}^2\rho \quad (i=x,y). \tag{22}$$

The multidimensional integrals in Eqs. (21) and (22) can be performed analytically, and the mathematical details are given in Appendix A. With the results from Appendix A, the analytical expressions for $P_{C_{ij}}$ are given by

$$P_{C_{ij}} = \frac{\pi^2 I_{ij}}{C_{ji}\left[C_{ji}-\frac{1}{4C_{ji}\delta_{ij}^4}\right]}, \tag{23}$$

where

$$C_{ij} = \frac{1}{4\sigma_i^2}+\frac{1}{2\delta_{ij}^2}+\frac{1}{\sigma_{ig}^2}+\frac{1}{W^2}, \tag{24}$$

$$I_{ij} = \frac{4}{(\lambda f)^2}B_{ij}A_iA_j. \tag{25}$$

The analytical expression for the denominator of Eq. (15) is

$$P_{\mathrm{inc}_{ii}} = \frac{\pi A_i^2}{\left[\frac{1}{2\sigma_i^2}+\frac{1}{W^2}\right]}. \tag{26}$$

Using Eqs. (23) through (26) and some algebra, we find the following analytical expression for the coupling efficiency in Eq. (15):

$$\eta_C = \frac{\sum_{i=x,y}\pi w_i^2\sum_{j=x,y}B_{ij}A_iA_j[C_{ij}C_{ji}-(1/4\delta_{ij}^4)]^{-1}}{(\lambda f W)^2\sum_{i=x,y}A_i^2\sigma_i^2(W^2+2\sigma_i^2)^{-1}}. \tag{27}$$

In the next section we use this expression to present some specific numerical examples and to discuss the effects of coherence and polarization on the coupling efficiency of an electromagnetic GSM beams into the fundamental mode of an optical fiber with a circular cross section.

4. RESULTS AND DISCUSSION

In this section we consider variations in the coupling efficiency of stochastic electromagnetic beams for different states of coherence and polarization with changes in the numerical aperture, NA=$D/2f$, of the coupling optics. To be specific, we consider an optical fiber with a circular core of radius $a=5$ μm. We assume that the fiber is weakly guiding with the refractive index profile

$$n(\rho') = \begin{cases} n_1\left[1-2\left(\frac{\rho'}{a}\right)^g\right]^{1/2} \\ n_2 = n_1(1-\Delta) \end{cases}, \tag{28}$$

where n_1 is the peak value of the refractive index of fiber at the core center, n_2 is the refractive index of the cladding, and $\Delta\approx(n_1-n_2)/n_1$. The normalized frequency or the V parameter of the fiber is defined as

$$V = n_1ka(2\Delta)^{1/2}, \tag{29}$$

where $k=\omega/c$ is the wavenumber. Assuming $V=2.4$ and $g=2$, we find the radius of the mode field of the fundamental propagating mode, by using the empirical formula given in [22], to be $w_x=w_y=5.15$ μm. In all cases, we assume that the focal length $f=10$ cm and that the wavelength $\lambda=1550$ nm. Even though we focus on a graded-index fiber, our analysis can be applied for step-index fibers as well ($g=\infty$) [26].

We begin our investigations by checking the effects of the degree of the correlations on the coupling efficiency of a stochastic beam. For this purpose, we consider the coupling of an unpolarized electromagnetic GSM beam into the optical fiber. We assume that the beam is circularly symmetric with $\sigma_x=\sigma_y=1$ cm, $A_x=A_y=1$, and $B_{xy}=B_{yx}=0$. Figure 2 shows how the coupling efficiency varies with NA for three different values of $\delta_{xx}=\delta_{yy}$. As the degree of the correlations decreases, the coupling efficiency decreases as a result of the mismatch between the incident beam and the distribution of the mode field. One can note also that the maximum coupling efficiency occurs at a smaller numerical aperture as the degree of the correla-

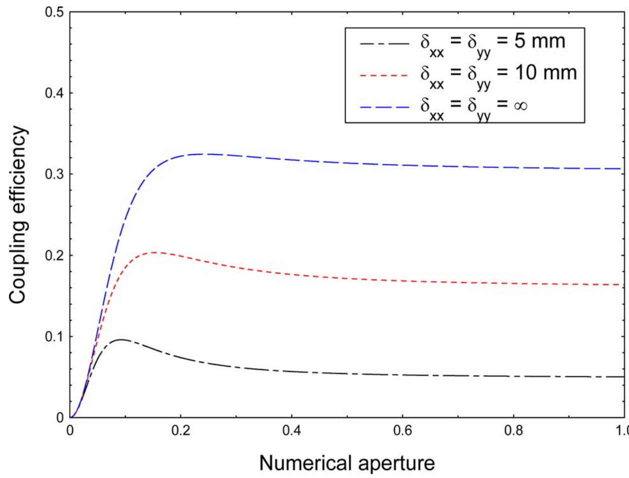

Fig. 2. (Color online) Variation of the coupling efficiency with numerical aperture for different correlation conditions. The parameters of the optical fiber are given within the text. The incident beam is assumed to be symmetric with the parameters $\sigma_x = \sigma_y = 1$ cm, $A_x = A_y = 1$, $B_{xy} = B_{yx} = 0$.

tions decreases; hence, matching between the incident beam and the mode field will exist only for a smaller area in this case. Finally, we note that the coupling efficiency has a maximum value of 0.5 in this unpolarized-beam case because of the absence of any off-diagonal correlations.

Next we consider the effects of asymmetry of the beam on the coupling efficiency. For this purpose, we assume that the incident beam is still an unpolarized, asymmetric GSM beam but has different widths in the two directions of polarization. We use $\sigma_x = 1$ cm, $A_x = A_y = 1$, $B_{xy} = B_{yx} = 0$, and $\delta_{xx} = \delta_{yy} = \infty$ and vary σ_y. Figure 3 shows how the coupling efficiency varies with NA for three different values of σ_y. As one can see in Fig. 3, the coupling efficiency varies with changes in the ratio σ_y / σ_x. The reason is basically that the power coupled from one correlation is different from the other in each case. Also, we can see that maximum coupling occurs in the $\sigma_y = 5$ mm case because of the dominance of one of the correlations to the coupled power in that case.

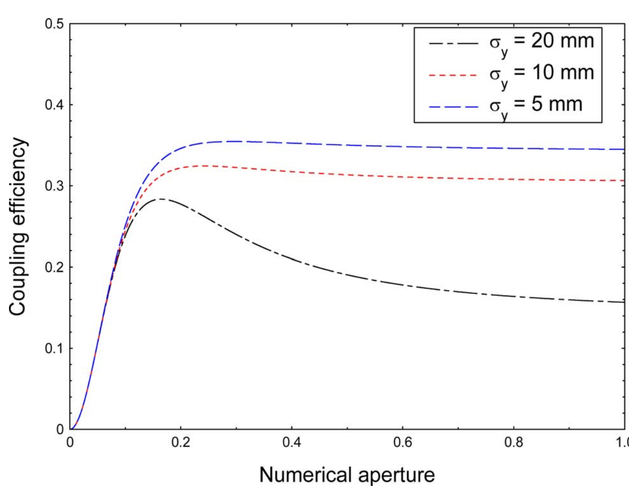

Fig. 3. (Color online) Variation of the coupling efficiency with numerical aperture for three values of σ_y. The parameters of the optical fiber are given within the text. The incident beam is assumed to be asymmetric with parameters $\sigma_x = 1$ cm, $A_x = A_y = 1$, $B_{xy} = B_{yx} = 0$.

In Fig. 4 we show that changes in the degree of correlations affect the coupling efficiency of a fully polarized beam. We assume that the beam is symmetric with that parameter values $\sigma_x = \sigma_y = 1$ cm, $A_x = A_y = 1$, and $B_{xy} = B_{yx} = 1$. We also assumed that $\delta_{xx} = \delta_{yy} = \delta_{xy}$ and considered three values of the resulting single parameter. As expected, the coupling efficiency is larger in this polarized case, but the qualitative behavior is similar to the unpolarized case seen in Fig. 2. As before, the coupling efficiency decreases with a decrease in the degree of the correlations. The only difference is that the coupling efficiency can reach a value of 100%, as all four correlations contribute to it.

Finally, we focus on the effects of changing the polarization of the incident beam on the coupling efficiency. We considered two limiting cases in Figs. 2 and 4, where we illustrate the cases of unpolarized and fully polarized beams, respectively. The degree of polarization of an optical beam increases as we increase the amplitudes of the cross-correlations terms. For example, the degree of polarization DOP of the beam at the source plane $z = 0$ is given by [21] (assuming $\sigma_x = \sigma_y$)

$$\mathrm{DOP} = \frac{\sqrt{(A_x^2 - A_y^2)^2 + 4A_x^2 A_y^2 |B_{xy}|^2}}{A_x^2 + A_y^2}. \tag{30}$$

In Fig. 5 we show the coupling efficiency as a function of NA for three values of DOP, using $B_{xy} = B_{yx}$. We assume that the incident beam is symmetric with parameters $\sigma_x = \sigma_y = 1$ cm, $A_x = A_y = 1$, and $\delta_{xx} = \delta_{yy} = \delta_{xy} = \infty$ (spatially uniform correlations). As one can see from Fig. 5, as the degree of polarization of the incident beam increases, the coupling efficiency increases too, mainly because matching between the incident beam and the mode field improves. Polarization changes do not affect the optimum value of the numerical aperture at which coupling efficiency becomes maximal, although this value varies with the degree of correlations and the widths of the two polarization components.

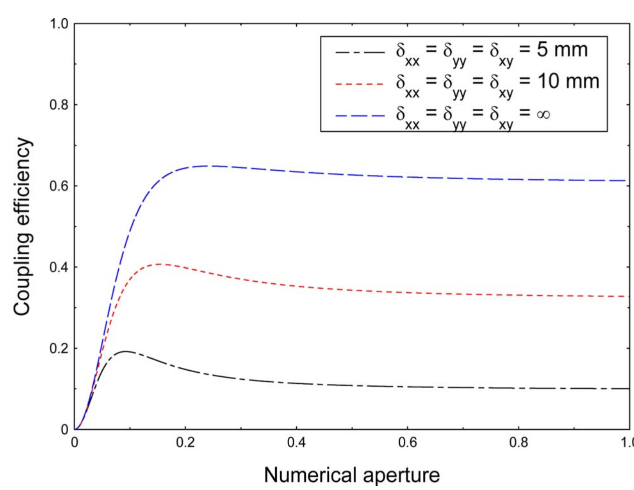

Fig. 4. (Color online) Variation of the coupling efficiency with numerical aperture under conditions identical to those of Fig. 2, except that the input beam is taken to be fully polarized with $B_{xy} = B_{yx} = 1$.

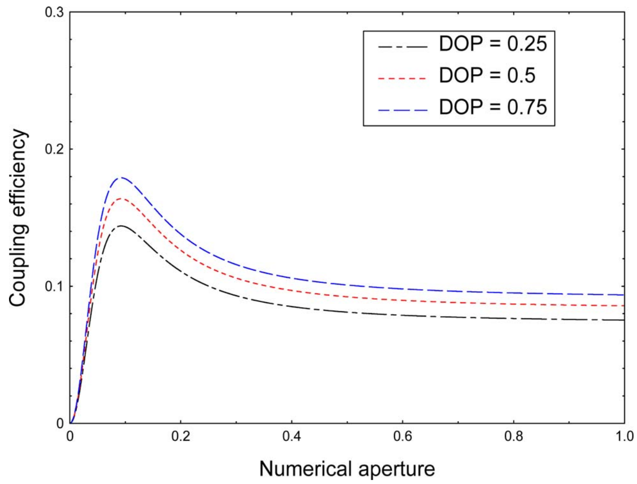

Fig. 5. (Color online) Variation of the coupling efficiency with numerical aperture for three partially polarized beams. The parameters of the optical fiber are given within the text. The incident beam is assumed to be symmetric with parameters $\sigma_x=\sigma_y=1$ cm, $A_x=A_y=1$, $\delta_{xx}=\delta_{yy}=\delta_{xy}=\infty$.

5. CONCLUSIONS

In this paper we derived a general expression for the coupling efficiency of stochastic electromagnetic beams into multimode optical fiber in terms of its cross-spectral density matrix. This expression involves multidimensional spatial integrations in the transverse plane containing the coupling lens. We were able to derive an analytical expression for the coupling efficiency when the incident beam satisfies an electromagnetic Gaussian Schell model (GSM) and the field distribution of the fundamental mode of an optical fiber is approximated by a Gaussian function. We studied, through some numerical examples, the effects of the coherence and the polarization of the incident beam on the coupling efficiency. The results showed that both the value of the coupling efficiency and the maximum numerical aperture vary considerably with changes in the incident beam parameters.

APPENDIX A

In this Appendix we provide mathematical details leading to the analytical solution for the coupling efficiency of electromagnetic GSM beams into a single-mode optical fiber.

Let us start by evaluating the coupled power of the correlation $\mu\nu$ of an electromagnetic GSM beam into optical fiber, by using Eq. (21):

$$P_{C_{\mu\nu}}=\left(\frac{2}{\lambda f}\right)^2\int\int_W B_{\mu\nu}A_\mu A_\nu \exp\left[-\frac{\boldsymbol{\rho}_1^2}{4\sigma_\mu^2}\right]\exp\left[-\frac{\boldsymbol{\rho}_2^2}{4\sigma_\nu^2}\right] \times\exp\left[-\frac{(\boldsymbol{\rho}_2-\boldsymbol{\rho}_1)^2}{2\delta_{\mu\nu}^2}\right]\exp\left(-\frac{\boldsymbol{\rho}_1^2}{\sigma_{\mu g}^2}\right)\exp\left(-\frac{\boldsymbol{\rho}_2^2}{\sigma_{\nu g}^2}\right) \times\exp\left[-\frac{\rho_1^2+\rho_2^2}{W^2}\right]\mathrm{d}^2\rho_1\mathrm{d}^2\rho_2. \quad \text{(A1)}$$

By using the cylindrical coordinates, we can write Eq. (A1) as

$$P_{C_{\mu\nu}}=\int_{\phi_2=0}^{2\pi}\int_{\phi_1=0}^{2\pi}\int_{\rho_2=0}^{\infty}\int_{\rho_1=0}^{\infty} I_{\mu\nu}\exp[-C_{\mu\nu}\boldsymbol{\rho}_1^2]\exp[-C_{\nu\mu}\boldsymbol{\rho}_2^2] \times\exp\left[\frac{-\rho_1\rho_2\cos(\phi_1-\phi_2)}{\delta_{\mu\nu}^2}\right]\rho_1\rho_2\mathrm{d}\rho_1\mathrm{d}\rho_2\mathrm{d}\phi_1\mathrm{d}\phi_2, \quad \text{(A2)}$$

where

$$I_{\mu\nu}=\left(\frac{2}{\lambda f}\right)^2 B_{\mu\nu}A_\mu A_\nu, \quad \text{(A3)}$$

$$C_{\mu\nu}=\frac{1}{4\sigma_\mu^2}+\frac{1}{2\delta_{\mu\nu}^2}+\frac{1}{\sigma_{\mu g}^2}+\frac{1}{W^2}, \quad \text{(A4)}$$

$$C_{\nu\mu}=\frac{1}{4\sigma_\nu^2}+\frac{1}{2\delta_{\mu\nu}^2}+\frac{1}{\sigma_{\nu g}^2}+\frac{1}{W^2}. \quad \text{(A5)}$$

Let us perform the integration over ϕ_1:

$$P_{C_{\mu\nu}}=\int_{\phi_2=0}^{2\pi}\int_{\rho_2=0}^{\infty}\int_{\rho_1=0}^{\infty} I_{\mu\nu}\exp[-C_{\mu\nu}\boldsymbol{\rho}_1^2] \times\exp[-C_{\nu\mu}\boldsymbol{\rho}_2^2]\rho_1\rho_2\mathrm{d}\rho_1\mathrm{d}\rho_2\mathrm{d}\phi_2 \times\int_{\phi_1=0}^{2\pi}\exp\left[\frac{-\rho_1\rho_2\cos(\phi_1-\phi_2)}{\delta_{\mu\nu}^2}\right]\mathrm{d}\phi_1. \quad \text{(A6)}$$

By using the identity in [27,28], we obtain

$$\int_{\phi=0}^{2\pi} e^{[\alpha\cos\phi+\beta\sin\phi]}\mathrm{d}\phi=2\pi I_0(\sqrt{\alpha^2+\beta^2}), \quad \text{(A7)}$$

where I_0 is the modified Bessel function of zero order; the integration over ϕ_1 gives

$$P_{C_{\mu\nu}}=\int_{\phi_2=0}^{2\pi}\int_{\rho_2=0}^{\infty}\int_{\rho_1=0}^{\infty} 2\pi I_{\mu\nu}\exp[-C_{\mu\nu}\boldsymbol{\rho}_1^2] \times\exp[-C_{\nu\mu}\boldsymbol{\rho}_2^2]I_0\left(\frac{\rho_1\rho_2}{\delta_{\mu\nu}^2}\right)\rho_1\rho_2\mathrm{d}\rho_1\mathrm{d}\rho_2\mathrm{d}\phi_2. \quad \text{(A8)}$$

The integration over ϕ_2 yields only a factor of 2π, resulting in

$$P_{C_{\mu\nu}}=\int_{\rho_2=0}^{\infty}\int_{\rho_1=0}^{\infty}(2\pi)^2 I_{\mu\nu}\exp[-C_{\mu\nu}\boldsymbol{\rho}_1^2] \times\exp[-C_{\nu\mu}\boldsymbol{\rho}_2^2]I_0\left(\frac{\rho_1\rho_2}{\delta_{\mu\nu}^2}\right)\rho_1\rho_2\mathrm{d}\rho_1\mathrm{d}\rho_2. \quad \text{(A9)}$$

The integration over ρ_2 can be performed by using the following two identities [29]:

$$I_m(x_1)=j^{-m}J_m(x), \quad j=\sqrt{-1}, \quad \text{(A10)}$$

$$\int_{x=0}^{\infty} x^{\nu+1}\exp(-\alpha x^2)J_\nu(\beta x)\mathrm{d}x=\frac{\beta^\nu}{(2\alpha)^{\nu+1}}\exp\left(-\frac{\beta^2}{4\alpha}\right),$$

$$[\mathrm{Re}\,\alpha > 0, \mathrm{Re}\,\nu > -1]. \tag{A11}$$

After performing the integration over ρ_2, we obtain

$$P_{C_{\mu\nu}} = \frac{(2\pi)^2 I_{\mu\nu}}{2C_{\nu\mu}} \int_{\rho_1=0}^{\infty} \exp[-C_{\mu\nu}\boldsymbol{\rho}_1^2]\exp\left[\frac{\boldsymbol{\rho}_1^2}{4C_{\nu\mu}\delta_{\mu\nu}^4}\right]\rho_1 \mathrm{d}\rho_1. \tag{A12}$$

Finally, the integration over ρ_1 can be performed by using the identity [29]

$$\int_{x=0}^{\infty} x\exp(-\alpha x^2)\mathrm{d}x = \frac{1}{(2\alpha)}. \tag{A13}$$

Performing this last integration over ρ_1, we obtain the final result:

$$P_{C_{\mu\nu}} = \frac{\pi^2 I_{\mu\nu}}{C_{\nu\mu}\left[C_{\mu\nu} - \dfrac{1}{4C_{\nu\mu}\delta_{\mu\nu}^4}\right]}. \tag{A14}$$

Next we evaluate $P_{\mathrm{inc}_{\mu\mu}}$, given in Eq. (22) as

$$P_{\mathrm{inc}_{\mu\mu j}} = \int_W A_\mu^2 \exp\left[-\frac{\boldsymbol{\rho}^2}{2\sigma_\mu^2}\right]\exp\left[-\frac{2\rho^2}{W^2}\right]\mathrm{d}^2\rho. \tag{A15}$$

Using the cylindrical coordinates, we can write Eq. (A15) as

$$P_{\mathrm{inc}_{\mu\mu j}} = \int_{\phi=0}^{2\pi}\int_{\rho=0}^{\infty} A_\mu^2 \exp\left[-\frac{\boldsymbol{\rho}^2}{2\sigma_\mu^2}\right]\exp\left[-\frac{2\rho^2}{W^2}\right]\rho \mathrm{d}\rho \mathrm{d}\phi. \tag{A16}$$

Integrating over ϕ, we find that

$$P_{\mathrm{inc}_{\mu\mu j}} = 2\pi\int_{\rho=0}^{\infty} A_\mu^2 \exp\left[-\frac{\boldsymbol{\rho}^2}{2\sigma_\mu^2}\right]\exp\left[-\frac{2\rho^2}{W^2}\right]\rho \mathrm{d}\rho. \tag{A17}$$

The integration over ρ can be performed by using the identity given in (A13), and the result is

$$P_{\mathrm{inc}_{\mu\mu j}} = \frac{\pi A_\mu^2}{\left[\dfrac{1}{2\sigma_\mu^2} + \dfrac{1}{W^2}\right]}. \tag{A18}$$

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