Coupling of stochastic electromagnetic beams into optical fibers

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We derive a general analytic expression for the coupling efficiency when a partially coherent, partially polarized beam is coupled into a multimode optical fiber. We adopt the Gaussian–Schell model for incident electromagnetic beams and use our general result to discuss the effects of the partial coherence and partial polarization on the coupling efficiency of an optical beam focused onto a step-index, single-mode fiber with a lens. Our results should be useful for any application requiring coupling of partially coherent beams into optical fibers. © 2009 Optical Society of America

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Optical fibers are routinely employed for a variety of applications, ranging from telecommunications to biomedical engineering. An important issue in all cases is the efficiency with which light can be coupled into an optical fiber. The coupling efficiency of completely coherent or completely incoherent light is by now well understood [1,2]. However, the incident light in practice may be partially coherent. The coupling of such electromagnetic beams is often studied within the scalar approximation that bypasses the polarization issue completely [3–6]. Recent work has shown that the coherence and the polarization properties of electromagnetic beams are intimately related, and both must be considered within a unified theoretical framework [7,8]. This Letter addresses the issue of coupling efficiency by using such a unified theory.

It is well known that optical fibers guide light through a set of optical modes that are linearly polarized within a weakly guiding approximation [9] that holds well for practical glass fibers in which the refractive indexes of the core and cladding materials differ by less than 1%. Each spatial mode in such fibers is twofold degenerate in the sense that the same transverse shape occurs for two orthogonally polarized modes that are linearly polarized along two directions that can be chosen to coincide with the x and y axes of a Cartesian coordinate system. Each of these modes also has electric and magnetic field components in the z direction, coinciding with the fiber axis. However, these components are relatively small, and they are also related to the transverse components through Maxwell's equations. We do not consider them further here.

Consider a multimode fiber whose input end is located in the plane $z=0$. In view of the linear nature of the coupling process, we focus on one Fourier component of the incident electromagnetic beam at the angular frequency ω and write its electric field in the form $\mathbf{E}(\boldsymbol{\rho},\omega)=E_x(\boldsymbol{\rho},\omega)\hat{x}+E_y(\boldsymbol{\rho},\omega)\hat{y}$, where $\boldsymbol{\rho}$ is a two-dimensional vector with components x and y, and $\hat{x}$ and $\hat{y}$ are unit vectors along the two transverse directions. The coupling of this electromagnetic beam into the fiber excites both the guided and the radiation modes of this fiber. Since these modes form a complete set, we can use the expansion

$$\mathbf{E}(\boldsymbol{\rho},\omega)=\sum_m [C_{mx}F_{mx}(\boldsymbol{\rho},\omega)\hat{x}+C_{my}F_{my}(\boldsymbol{\rho},\omega)\hat{y}], \quad (1)$$

where $F_{mx}(\boldsymbol{\rho},\omega)$ and $F_{my}(\boldsymbol{\rho},\omega)$ are the mode distributions and C_{mx} and C_{my} are the coupling coefficients for the mth mode polarized along the x and y directions, respectively. To simplify the notation, the sum extends over both the guided modes and a continuum of radiation modes [9]. It follows from Eq. (1) that the x and y components of the electromagnetic beam are coupled into the fiber modes polarized orthogonally such that

$$E_\mu(\boldsymbol{\rho},\omega)=\sum_m C_{m\mu}F_{m\mu}(\boldsymbol{\rho},\omega), \quad (\mu=x,y). \quad (2)$$

We use the preceding relation to calculate the components of the cross-spectral density matrix (see for example [8]) and obtain

$$W_{\mu\nu}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\omega)=\langle E_\mu^*(\boldsymbol{\rho}_1,\omega)E_\nu(\boldsymbol{\rho}_2,\omega)\rangle = \sum_m\sum_n \langle C_{m\mu}^*C_{n\nu}\rangle F_{m\mu}^*(\boldsymbol{\rho}_1,\omega)F_{n\nu}(\boldsymbol{\rho}_2,\omega), \quad (3)$$

where μ and ν take values x or y. Equation (3) can be inverted to obtain $\langle C_{m\mu}^*C_{n\nu}\rangle$ by noting that the fiber modes satisfy the orthogonality relation

$$\int F_{m\mu}^*(\boldsymbol{\rho},\omega)F_{n\nu}(\boldsymbol{\rho},\omega)\mathrm{d}^2\rho=\delta_{nm}\delta_{\mu\nu}. \quad (4)$$

The result is given by

$$\langle C_{m\mu}^*C_{n\nu}\rangle = \int\int W_{\mu\nu}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\omega)F_{m\mu}(\boldsymbol{\rho}_1,\omega)F_{n\nu}^*(\boldsymbol{\rho}_2,\omega)\mathrm{d}^2\rho_1\mathrm{d}^2\rho_2. \quad (5)$$

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The power coupled into a specific mode (or the mode-power weight) is obtained by setting $m=n$ in Eq. (5) while summing over μ and ν to account for both polarization components. The final results can be written as

$$P_{cm} = \int\int Tr[\mathbf{W}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\omega) \,.\, \mathbf{F}_m^{\dagger}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\omega)]\mathrm{d}^2\rho_1\mathrm{d}^2\rho_2, \tag{6}$$

where a dagger denotes the Hermitian adjoint, and we have introduced a matrix $\mathbf{F_m}$ that takes into account the transverse profiles of the mth mode,

$$\mathbf{F_m}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\omega) = \begin{pmatrix} F_{mx}^*(\boldsymbol{\rho}_1,\omega)F_{mx}(\boldsymbol{\rho}_2,\omega) & F_{mx}^*(\boldsymbol{\rho}_1,\omega)F_{my}(\boldsymbol{\rho}_2,\omega) \\ F_{my}^*(\boldsymbol{\rho}_1,\omega)F_{mx}(\boldsymbol{\rho}_2,\omega) & F_{my}^*(\boldsymbol{\rho}_1,\omega)F_{my}(\boldsymbol{\rho}_2,\omega)\end{pmatrix}. \tag{7}$$

The coupling efficiency can be obtained by normalizing Eq. (6) with the incident power and is given by

$$\eta_{cm} = \frac{\int\int Tr[\mathbf{W}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\omega) \,.\, \mathbf{F}_m^{\dagger}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\omega)]\mathrm{d}^2\rho_1\mathrm{d}^2\rho_2}{2\int Tr[\mathbf{W}(\boldsymbol{\rho},\boldsymbol{\rho},\omega)]\mathrm{d}^2\rho}. \tag{8}$$

Equation (8) constitutes our main result. It applies to the general case of a multimode fiber and shows how the coupling efficiency depends on the coherence and polarization properties of the incident electromagnetic beam governed by the cross-spectral density matrix $\mathbf{W}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\omega)$.

As a simple application of our general result, we focus on the case of a step-index fiber that supports a single guided mode and drop the subscript m in Eq. (8). We further adopt the Gaussian–Schell model for the incident electromagnetic beam and use [7]

$$W_{ij}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2;\omega) = \sqrt{S_i(\boldsymbol{\rho}_1;\omega)}\sqrt{S_j(\boldsymbol{\rho}_2;\omega)}\mu_{ij}(\boldsymbol{\rho}_2-\boldsymbol{\rho}_1;\omega), \tag{9}$$

where i and j take values x and y. The spectral densities S_i and S_j and the degree of correlation μ_{ij} between the i and the j components of the electric field are taken to be Gaussian functions of the form

$$S_j(\boldsymbol{\rho};\boldsymbol{\omega}) = A_j^2 \exp\left[-\frac{\boldsymbol{\rho}^2}{2\sigma_j^2}\right], \quad (j=x,y), \tag{10a}$$

$$\mu_{ij}(\boldsymbol{\rho}_2-\boldsymbol{\rho}_1;\omega) = B_{ij}\exp\left[-\frac{(\boldsymbol{\rho}_2-\boldsymbol{\rho}_1)^2}{2\delta_{ij}^2}\right], \quad (i=x,y;\ j=x,y). \tag{10b}$$

The parameters A_j, B_{ij}, σ_j, and δ_{ij} may depend on the frequency. The spatial distribution of the fiber mode is also approximated by a Gaussian function [9,10], $F_j = \sqrt{2/\pi}w_j^{-1}\exp(-\boldsymbol{\rho}'^2/w_j^2)$, where $j=x$ or y and $\boldsymbol{\rho}'$ is a transverse vector in the plane of the fiber tip.

One more practical issue should be addressed. In practice, the incident electromagnetic beam is often focused onto the input end of the fiber with a suitable lens of focal length f (see Fig. 1). We account for such a focusing geometry by backpropagating the fiber mode from plane B to plane A, where the lens is located. The backpropagated fiber mode is given by

$$F_{jA} = \sqrt{2/\pi}w_{jA}^{-1}\exp(-\boldsymbol{\rho}^2/w_{jA}^2), \tag{11}$$

where $w_{jA}=\lambda f/\pi w_j$. The finite size of the lens is included by replacing the hard aperture of diameter D with a Gaussian aperture of radius W such that $W^2 = D^2/8$ [11]. Because of our use of Gaussian functions, all integrals in Eq. (8) can be performed analytically. After considerable algebra, the coupling efficiency is found to be

$$\eta_C = \frac{\sum_{i=x,y}\pi w_i^2\sum_{j=x,y}B_{ij}A_iA_j[C_{ij}C_{ji}-(1/4\delta_{ij}^4)]^{-1}}{(\lambda f W)^2\sum_{i=x,y}A_i^2\sigma_i^2(W^2+2\sigma_i^2)^{-1}}, \tag{12}$$

where

$$C_{ij} = \frac{1}{4\sigma_i^2}+\frac{1}{2\delta_{ij}^2}+\frac{1}{w_{iA}^2}+\frac{1}{W^2}. \tag{13}$$

Finally, we illustrate our results by calculating how the coupling efficiency of stochastic electromagnetic beams of different states of coherence and polarization into a single-mode fiber changes with the NA of the coupling optics (NA$=D/2f$) for a lens of focal length f with aperture diameter D. We consider a step-index fiber with a 5 μm core radius and assume that the normalized frequency parameter, the V number, of the fiber equals 2 at the operating wavelength of 1550 nm. Using the empirical formulae given in [12], the mode-field radius of the fiber is found to be $w_x=w_y=6.34$ μm. We also assume that a lens of focal length $f=10$ mm is employed to couple light into the fiber.

Figure 2 shows the effects of changing the degree of coherence of a fully polarized beam on the coupling efficiency η_C by using $\sigma_x=\sigma_y=0.7$ mm, $A_x=A_y=1$, $B_{xy}=B_{yx}=1$, and $\delta_{xx}=\delta_{yy}=\delta_{xy}$. One can see that η_C decreases as the launched beam becomes less coherent.

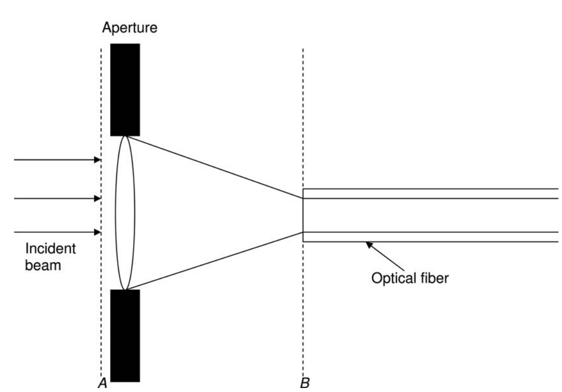

Fig. 1. Illustrating notation related to the coupling of stochastic electromagnetic beam into an optical fiber.

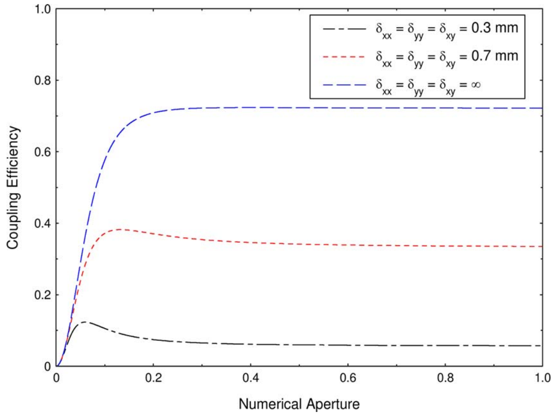

Fig. 2. (Color online) Variation of the coupling efficiency with the NA. The parameters of the single-mode fiber are given in the text. The incident beam is assumed to be symmetric and has the following parameters: $\sigma_x=\sigma_y=0.7$ mm, $A_x=A_y=1$, $B_{xy}=B_{yx}=1$, and $\delta_{xx}=\delta_{yy}=\delta_{xy}$ and taking a different value for each curve.

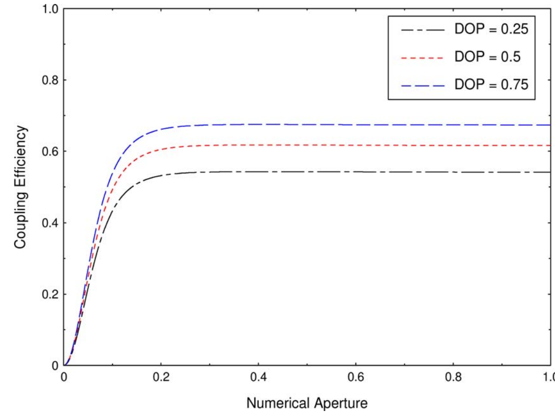

Fig. 3. (Color online) Effect of degree of polarization on the coupling efficiency of a beam with $\delta_{xx}=\delta_{yy}=\delta_{xy}=\infty$. Other parameters are the same as in Fig. 2.

Note also that the coupling efficiency becomes maximum for an optimum NA and decreases with a further increase in the NA.

In Fig. 3 we consider the impact of varying the magnitude of cross correlations ($B_{xy}=B_{yx}$), which affect the polarization properties of the beam. As a reminder, the degree of polarization (DOP) of the beam at the source plane $z=0$ is given by [13] (assuming $\sigma_x=\sigma_y$)

$$\mathrm{DOP}=\frac{\sqrt{(A_x^2-A_y^2)^2+4A_x^2A_y^2|B_{xy}|^2}}{A_x^2+A_y^2}. \qquad (14)$$

We take $\sigma_x=\sigma_y=0.7$ mm, $A_x=A_y=1$, and $\delta_{xx}=\delta_{yy}=\delta_{xy}=\infty$. As the DOP of the incident beam increases, the coupling efficiency increases. The reason for increase in the coupling efficiency can be explained as follows. Since the deterministic part of the incident stochastic beam increases as the DOP increases, the incident beam can be matched better with the mode field, leading to improved coupling into the mode. Changes in the polarization properties of the beam do not affect the NA that corresponds to the maximum coupling efficiency.

In conclusion, we have derived a general analytic expression for the coupling efficiency when a partially coherent, partially polarized electromagnetic beam is coupled into a multimode optical fiber. We used it to discuss the effects of the partial coherence and partial polarization on the coupling efficiency into a step-index single-mode fiber by adopting the Gaussian–Schell model for the incident electromagnetic beam. Our results should be useful for any application requiring coupling of a partially coherent beam into optical fibers.

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References

1. R. Wagner and W. Tomlinson, Appl. Opt. **21**, 2671 (1982).
2. D. N. Christodoulides, L. A. Reith, and M. A. Saifi, J. Lightwave Technol. **5**, 1623 (1987).
3. K. Chen and D. Kerps, J. Lightwave Technol. **5**, 1600 (1887).
4. P. Winzer and W. Leeb, Opt. Lett. **23**, 986 (1998).
5. S. Withington and G. Yassin, J. Opt. Soc. Am. A **18**, 3061 (2001).
6. C. Zhao, Y. Cai, F. Wang, X. Lu, and Y. Wang, Opt. Lett. **33**, 1389 (2008).
7. E. Wolf, Opt. Lett. **28**, 1078 (2003).
8. E. Wolf, *Introduction to the Theory of Coherence and Polarization of Light* (Cambridge U. Press, 2007).
9. D. Marcuse, *Theory of Dielectric Optical Waveguides*, 2nd ed. (Academic, 1991), Chap. 2.
10. D. Marcuse, J. Opt. Soc. Am. A **68**, 103 (1978).
11. L. C. Andrews, R. L. Phillips, and C. Y. Hopen, *Laser Beam Scintillation with Applications* (SPIE Press, 2001).
12. G. P. Agarwal, *Fiber-Optic Communication Systems*, 3rd ed. (John Wiley and Sons, 2002).
13. O. Korotkova, M. Salem, and E. Wolf, Opt. Commun. **233**, 225 (2004).