Effects of higher-order dispersion on resonant dispersive waves emitted by solitons

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Dispersive waves (DW) are generated owing to perturbation of solitons by higher-order dispersion (HOD) and nonlinearity during supercontinuum (SC) generation. The frequencies of these waves are governed by a phase-matching condition in the form of a polynomial whose coefficients depend on the numerical values of the properly normalized third- and HOD parameters. Our extensive numerical solutions show that all odd HOD terms generate a single peak on the blue or the red side of the carrier frequency, depending on the sign of the corresponding term. In contrast, positive even HOD terms create conjugate DW peaks, in both the blue and red sides. No radiation is observed for negative values of these parameters. The combination of all even and odd HOD coefficients may generate more than two DW peaks for some specific choice of parameters. The results predicted by the phase-matching condition agree well with extensive numerical simulations revealing interesting facts of SC generation. © 2009 Optical Society of America

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A higher-order soliton (HOS) breaks into its fundamental components through fission process [1]. Each individual soliton component emits resonant dispersive waves (DWs) (sometimes call nonsolitonic radiation) [2–4] that are of particular importance for blue-shifted supercontinuum (SC) generation. A special phase-matching (PM) condition between the propagating solitons and DWs determines the specific frequencies of these waves [3]. In this Letter, we study the influence of individual and collective higher-order dispersion (HOD) terms in generating DWs based on normalized PM condition. It is demonstrated that all positive even order dispersion (OD) terms (i.e., 4OD, 6OD, 8OD, etc.) emit conjugate radiations. No such radiation is observed when the numeric sign of the even OD coefficients is set to negative. It is also predicted that all positive odd OD terms (i.e., 3OD, 5OD, 7OD, etc.) are capable of generating blue radiation. The radiation falls on the red side when the sign of odd dispersion coefficient is reversed. A detailed analysis based on a numerical solution of the generalized nonlinear Schrödinger equation (GNLSE) confirms these features associated with DWs. The range of dispersion values that are used in our study are obtained from designs of realistic photonic crystal fibers (PCFs), indicating tremendous flexibility in dispersion tailoring.

Generally a DW is not phase matched with a fundamental soliton because the soliton's wavenumber lies in a range forbidden for a linear DW. The presence of HOD terms, however, leads to a PM situation in which energy is transferred from the soliton to a DW at specific frequencies. In the SC generation process by a HOS, HOD and intrapulse Raman scattering (IPRS) act as perturbations that split the Nth-order soliton into N fundamental solitons of different widths and amplitudes. The shortest and most energetic soliton, having a width $(2N-1)$ times smaller than the input pulse width T_0 and a peak power larger by a factor of $(2N-1)^2/N^2$ [5], is primarily responsible for generating resonant DWs. In a dimensionless notation, the frequencies of DWs can be calculated by using a relatively simple PM condition, which arises from the equality of the soliton and radiation propagation constant [3],

$$\sum_{m=2}^{\infty} \delta_m x^m = \frac{1}{2}(2N-1)^2, \tag{1}$$

where $\delta_m = \beta_m / m! |\beta_2| T_0^{m-2}$, $x = 2\pi(\nu_d - \nu_s)T_0$, and ν_s and ν_d are the carrier frequencies associated with the soliton and the DW, respectively. Here the mth OD coefficient is represented by β_m.

The real solutions of x for the polynomial in Eq. (1) can readily predict the exact frequencies of all radiated DWs. The number of real roots and their position depends critically on the relative values of dimensionless dispersion coefficients δ_m and their algebraic signs. To capture the impact of HOD effects on the DW generation in realistic condition, we use the GNLSE written in the following normalized form [6]:

$$\frac{\partial U}{\partial \xi} = \frac{i}{2} \frac{\partial^2 U}{\partial \tau^2} + \sum_{m \geq 3}^{\infty} i^{m+1} \delta_m \frac{\partial^m U}{\partial \tau^m} + iN^2 \left(1 + is\frac{\partial}{\partial \tau}\right)$$

$$\times \left(U(\xi,\tau) \int_{-\infty}^{\tau} R(\tau - \tau')|U(\xi,\tau')|^2 d\tau'\right), \tag{2}$$

where the field amplitude $U(\xi,\tau)$ is normalized such that $U(0,0)=1$ and the other dimensionless variables are defined as

$$\xi = z/L_D, \quad \tau = (t - z/v_g)/T_0, \quad N = \sqrt{\gamma P_0 L_D}.$$

Here, P_0 is related to the peak power of the ultrashort pulse launched into the fiber, $L_D = T_0^2/|\beta_2|$ is the dispersion length, v_g is the group velocity, and γ

is the nonlinear parameter of the fiber. $s = (2\pi\nu_s T_0)^{-1}$ is the self-steepening parameter and $R(\tau)$ is the nonlinear response function of the optical fiber in the form

$$R(\tau) = (1 - f_R)\delta(\tau) + f_R h_R(\tau), \qquad (3)$$

where $f_R = 0.245$ and the first and the second terms correspond to the electronic and the Raman responses, respectively. The Raman response function can be expressed in the form [7]

$$h_R(\tau) = (f_a + f_c)h_a(\tau) + f_b h_b(\tau), \qquad (4)$$

where the functions $h_a(\tau)$ and $h_b(\tau)$ are defined as

$$h_a(\tau) = \frac{\tau_1^2 + \tau_2^2}{\tau_1 \tau_2^2}\exp\left(-\frac{\tau}{\tau_2}\right)\sin\left(\frac{\tau}{\tau_1}\right),$$

$$h_b(\tau) = \left(\frac{2\tau_b - \tau}{\tau_b^2}\right)\exp\left(-\frac{\tau}{\tau_b}\right), \qquad (5)$$

and the coefficients $f_a = 0.75$, $f_b = 0.21$, and $f_c = 0.04$ quantify the relative contributions of the isotropic and the anisotropic parts of the Raman response. In Eq. (5), τ_1, τ_2, and τ_b have values of 12, 32, and 96 fs, respectively. In our notation, they have been normalized by T_0.

We employ the standard split-step Fourier method [6] to solve the GNLSE numerically. The input sech-shape pulses are assumed to have a carrier wavelength of 835 nm and a width such that $T_0 = 50$ fs (FWHM of about 88 fs). Their peak power is chosen such that the soliton order N takes the value of 2. In the following simulations, the fiber length corresponds to two dispersion lengths (ξ varies from 0 to 2). We stress that the self-steepening effects are neg-

ligible in our simulations, because $s < 0.01$ for $T_0 = 50$ fs. Thus, IPRS is the major higher-order nonlinear process affecting the launched pulse.

The PM condition gives two real solutions for positive 4OD such that one DW frequency falls on the red side ($\nu_d < \nu_s$) and the other on the blue side ($\nu_d > \nu_s$) of the input spectrum. Dispersion tailoring of a PCF can produce two zero-dispersion wavelengths (ZDWs) that lead to conjugate radiations as a result of dominant 4OD [8,9]. A recent experiment [10] revealed the 4OD-mediated dual radiation in a specially designed hollow-core PCF. Physically, a soliton formed in the anomalous dispersion regime approaches ZDW by shifting its frequency through IPRS and subsequently transfers its energy to a linear wave at resonance frequency near other ZDW. This interesting phenomenon of soliton spectral tunneling has been discussed in [11] but it is not directly related to the objective of this Letter.

In Fig. 1(a) we represent the output spectrum of a launched second-order soliton ($N = 2$) for $\delta_3 = 0.01$ and $\delta_4 = 0.0015$ with the other HOD terms set to zero. Two distinct DW peaks on the blue and the red sides are observed under such conditions. The PM condition predicts the exact frequencies as shown in the bottom plot for the same set of parameters. The asymmetry in two resonant frequencies arises mainly owing to 3OD. We have verified that in the absence of 3OD and other nonlinear terms, 4OD always generates symmetric DW peaks. The most striking feature is observed in Fig. 1(b), where in the presence of 5OD ($\delta_5 = 0.0001$), the red peak generated by δ_4 disappears altogether and the blue peak shifts toward the central frequency with an enhanced power level.

In Fig. 1(c) we show that the red peak reappears when δ_4 is increased to 0.004 while keeping δ_5 fixed at 0.0001. In fact, Eq. (1) predicts three real roots for $\delta_4 = 0.004$ corresponding to two red peaks and one blue peak. A small peak is observed at the far end of the red side whose frequency matches exactly with one of the roots associated with the PM condition. However, this red peak is highly sensitive to the 5OD coefficient and again vanishes with a small increment of δ_5. Clearly, the relative values of δ_4 and δ_5 play a major role in determining the number of peaks. It seems that a contest is inevitable between 4OD and 5OD for the generation of DWs. A positive δ_4 always tries to generate blue and red peaks simultaneously, whereas δ_5 tends to suppress the red peak. Observing this fact, we gradually increase the value of δ_5, while keeping δ_4 fixed, and find a critical value of δ_5 (roughly 0.000 15) beyond which both red peaks vanish completely. At this inversion point, the spectrum broadens considerably even for $N = 2$, indicating the significance of this critical value in SC generation.

The signs of the dispersion coefficients represent another critical factor in resonant DW generation. By changing the sign of the 3OD coefficient, the resonant frequency can be shifted from the blue to the red side [3,12]. A similar effect is observed when we invert the sign of the 5OD coefficient. Figure 2(a) shows a single red peak, instead of the blue peak for negative δ_5
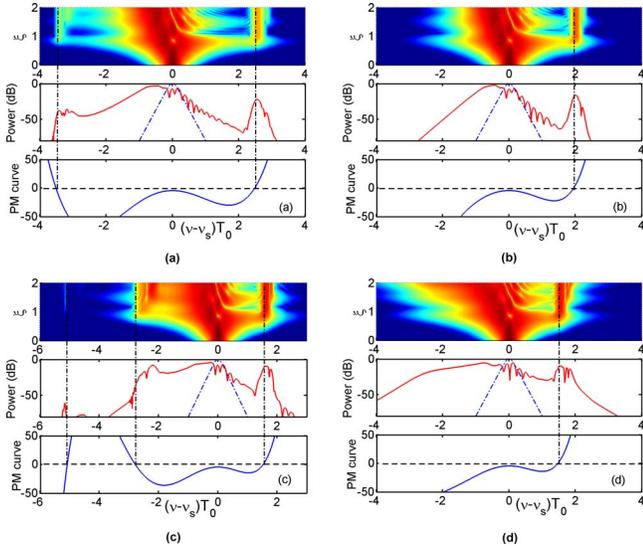
Fig. 1. (Color online) Contour and output spectra of a second-order soliton ($N = 2$) at two dispersion lengths. The PM curve is plotted simultaneously. The dimensionless dispersion coefficients used in the plots are (a) $\delta_3 = 0.01$, $\delta_4 = 0.015/10$, $\delta_5 = 0$, $\delta_6 = 0$; (b) $\delta_3 = 0.01$, $\delta_4 = 0.015/10$, $\delta_5 = 0.01/100$, $\delta_6 = 0$; (c) $\delta_3 = 0.01$, $\delta_4 = 0.04/10$, $\delta_5 = 0.01/100$, $\delta_6 = 0$; and (d) $\delta_3 = 0.01$, $\delta_4 = 0.04/10$, $\delta_5 = 0.015/100$, $\delta_6 = 0$.
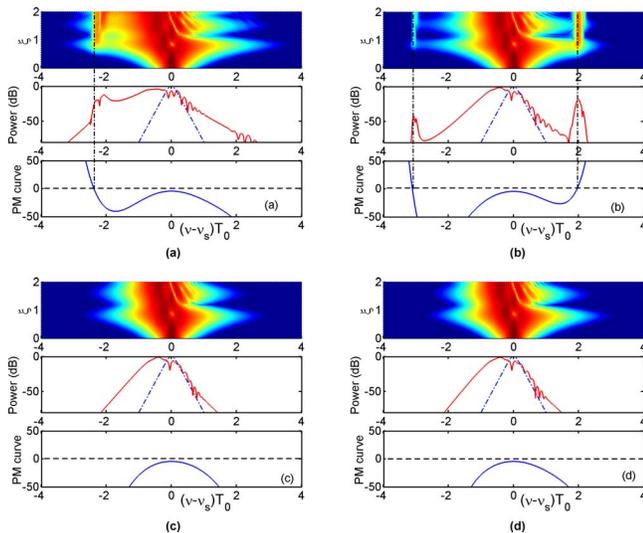
Fig. 2. (Color online) Contour and output spectra of a second-order soliton ($N=2$) at two dispersion length. The PM curve is plotted simultaneously. The dimensionless dispersion coefficients used in the plots are (a) $\delta_3=0.01$, $\delta_4=0.015/10$, $\delta_5=-0.01/100$, $\delta_6=0$; (b) $\delta_3=0.01$, $\delta_4=0$, $\delta_5=0.01/100$, $\delta_6=0.01/1000$; (c) $\delta_3=0.01$, $\delta_4=-0.015/10$, $\delta_5=0$, $\delta_6=0$; and (d) $\delta_3=0.01$, $\delta_4=0$, $\delta_5=0.01/100$, $\delta_6=-0.01/1000$.

complementing the PM solution. It turns out that this feature remains unchanged for all odd higher-order dispersion terms. More specifically, odd positive dispersion terms always create blue radiation, and inversion of their sign switches the radiation to the red side. Another important observation is that all positive even OD terms create two conjugate resonant DWs. In principle, the frequencies of these radiated waves will be symmetric around the carrier frequency if they are generated solely from a single even term. In practice, the DW peaks are located asymmetrically because of the simultaneous presence of odd HOD and higher-order nonlinear terms. Figure 2(b) shows, like 4OD, how 6OD generates two conjugate resonant DWs. Another striking feature we observe is that negative even HOD terms never create any DWs. Since for negative even HOD terms there are no real roots of Eq. (1), generation of DW is prohibited in this condition. In Figs. 2(c) and 2(d) we show this effect, respectively, for negative 4OD and 6OD terms, while setting the other parameters the same. The spectra predicted through direct numerical simulations of GNLSE also reciprocate positively

by exhibiting no DW peak generation under such conditions.

The frequencies of the DW radiation is governed by a PM condition in the form of a polynomial whose coefficients depend on the soliton order N and on the numerical values of third-order and HOD parameters. Extensive numerical solutions reveal that all odd HOD terms (i.e., 3OD, 5OD, etc.) generate a single DW peak on the blue or the red side of the carrier frequency, depending on whether the odd OD has a positive or negative sign. On the other hand, the positive even HOD terms (i.e., 4OD, 6OD, etc.), create conjugate DW peaks, one on the blue and other on the red side. Interestingly, for negative values of the even HOD coefficients, no real solution of the PM equation is possible, and hence no DW radiation is emitted under such conditions. The combination of all even and odd HOD coefficients may generate more than two DW peaks for some specific choice of parameters. We believe that the results and the conclusions of this work will provide new insights for understanding the complex process of SC generation.

References

1. J. M. Dudley, G. Genty, and S. Coen, Rev. Mod. Phys. **78**, 1135 (2006).
2. V. I. Karpman, Phys. Rev. E **47**, 2073 (1993).
3. N. Akhmediev and M. Karlsson, Phys. Rev. A **51**, 2602 (1995).
4. A. V. Husakou and J. Herrmann, Phys. Rev. Lett. **87**, 203901 (2001).
5. A. Hasegawa and M. Matsumoto, *Optical Solitons in Fibers* (Springer, 2002).
6. G. P. Agrawal, *Nonlinear Fiber Optics*, 4th ed. (Academic, 2007).
7. Q. Lin and G. P. Agrawal, Opt. Lett. **31**, 3086 (2006).
8. A. V. Husakou and J. Herrmann, Appl. Phys. B **77**, 227 (2003).
9. K. Hilligsøe, T. Andersen, H. Paulsen, C. Nielsen, K. Mølmar, S. Keiding, R. Kristiansen, K. Hansen, and J. Larsen, Opt. Express **12**, 1045 (2004).
10. F. Benabid, F. Biancalana, P. S. Light, F. Couny, A. Luiten, P. J. Roberts, J. Peng, and A. V. Sokolov, Opt. Lett. **33**, 2680 (2008).
11. E. Tsoy and C. De Sterke, Phys. Rev. A **76**, 043804 (2007).
12. S. Roy, S. K. Bhadra, and G. P. Agrawal, Phys. Rev. A **79**, 023824 (2009).