Continuous-wave Raman amplification in silicon waveguides: beyond the undepleted pump approximation

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We present approximate analytical expressions describing nonlinear interaction of two optical waves copropagating inside a silicon waveguide in the presence of linear losses, stimulated Raman scattering, and free-carrier absorption. Our approach avoids the undepleted-pump approximation, which we show to be inadequate to describe accurately the Raman-gain process. Based on our calculations, we propose a new generalized definition for the effective length and show that it provides better insight into the impact of nonlinear absorption on Raman amplification in silicon waveguides.

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Amplification of optical signals through stimulated Raman scattering (SRS) is one of the promising applications of silicon-on-insulator (SOI) waveguides [1]. Owing to a relatively large value of the Raman gain coefficient in silicon, SOI-based Raman amplifiers as short as a few centimeters long were able to exhibit net signal gains as high as 10 dB at moderate pump intensities in both the pulsed and continuous-wave (CW) regimes [2–4]. The main obstacle in achieving high operation efficiency of such amplifiers is the strong free-carrier absorption (FCA) caused by the generation of electrons and holes through the two-photon absorption (TPA) process [4–6].

A net positive gain in SOI-based Raman amplifiers has been made possible as a result of a great deal of effort in the experimental and theoretical investigations of SOI waveguides. Almost all the recent work on SOI waveguides has concentrated mainly on numerical simulations because of the inherent difficulty in constructing approximate analytical solutions of the underlying nonlinear problem [3,4,7,8]. Probably the only exception is the review [8], which provides some exact solutions and a lot of useful estimations. However, there is a greater need for better approximations giving clearer insight into the nonlinear process governing Raman amplification in SOI waveguides. Existing analytical results covering this phenomena [4,9] for CW amplification do not provide such an understanding since they utilize the undepleted-pump approximation, causing them to diverge. Moreover, these solutions do not take into account the free-carrier effects and are therefore unable to describe their influence on the amplification process. To clarify this and other crucial issues, we devote the present Letter to carrying out a comprehensive analytical study of CW Raman amplification in SOI waveguides. Specifically, we derive and analyze a simple solution of this problem after making a few well-justified approximations.

We consider the case in which a signal at the frequency ω_s is amplified inside an SOI waveguide by pumping it with a CW pump at the frequency ω_p that copropagates with the pump. The coupled differential equations governing the evolution of pump and signal (Stokes) intensities [$I_p(z)$ and $I_s(z)$, respectively] along the waveguide length are [8,10]

$$\frac{\mathrm{d}I_p}{\mathrm{d}z} = -\alpha_p I_p - \beta_p I_p^2 - \zeta_{ps} I_p I_s - \sigma_p \tau_c (\rho_p I_p^2 + \rho_s I_s^2 + \rho_{ps} I_p I_s) I_p, \tag{1a}$$

$$\frac{\mathrm{d}I_s}{\mathrm{d}z} = -\alpha_s I_s - \beta_s I_s^2 - \zeta_{sp} I_s I_p - \sigma_s \tau_c (\rho_p I_p^2 + \rho_s I_s^2 + \rho_{ps} I_p I_s) I_s. \tag{1b}$$

The notation and physical meaning of different terms on the right side of Eqs. (1) are as follows. First terms account for linear losses characterized by the parameters $\alpha_{p(s)}$. Coefficients $\beta_{p(s)}$ and their associated terms describe TPA. Third terms account for both the cross TPA and SRS. The corresponding coefficients are given by

$$\zeta_{ps} = 2\beta_{ps} + \frac{4 g_R \gamma_R^2 \Omega_R \Omega_{ps}}{(\Omega_R^2 - \Omega_{ps}^2)^2 + 4\gamma_R^2 \Omega_{ps}^2},$$

$$\zeta_{sp} = \frac{\omega_s}{\omega_p}\left(2\beta_{ps} - \frac{4 g_R \gamma_R^2 \Omega_R \Omega_{ps}}{(\Omega_R^2 - \Omega_{ps}^2)^2 + 4\gamma_R^2 \Omega_{ps}^2}\right),$$

where β_{ps} is the cross-TPA coefficient, g_R is the Raman gain coefficient, $\Omega_R = 15.6$ THz is the Raman shift, $\gamma_R = 105$ GHz is the Raman-gain bandwidth, and $\Omega_{ps} = \omega_p - \omega_s$. Finally, the last terms in Eq. (1) describe TPA-induced FCA whose magnitude is set by the empirical coefficients $\sigma_{p(s)} = \sigma_0(\lambda_{p(s)}/\lambda_0)^2$, where $\sigma_0 = 1.45 \times 10^{-21}$ m^2 and $\lambda_0 = 1550$ nm [8], by the effective carrier lifetime τ_c, and by the TPA-related coefficients $\rho_{p(s)} = \beta_{p(s)}/(2\hbar\omega_{p(s)})$ and $\rho_{ps} = 2\beta_{ps}/(\hbar\omega_p)$.

An exact analytical solution of Eq. (1) does not exist. To derive an approximate analytical solution, we need to simplify Eq. (1). First, we assume that, to a good approximation, linear losses are equal at the pump and signal wavelengths, i.e., $\alpha_p = \alpha_s \equiv \alpha$. Second, simple estimates have shown [8] that TPA is small compared with FCA in the case of CW pumping. This allows us to discard second terms on the right-hand side of Eq. (1). We also suppose that $\Omega_{ps} \approx \Omega_R \ll \omega_{p(s)}$, conditions that are often met in practice. This leads to several rough approximations: $\sigma_p \approx \sigma_s$, $\beta_p \approx \beta_s \approx \beta_{ps} \equiv \beta$, $\rho_p \approx \rho_s \approx \rho_{ps}/4$. In addition, for $|\Omega_{ps} - \Omega_R| \lesssim \gamma_R$, $g_R \gg \beta_{ps}$ so that $\zeta_{ps} \approx -\zeta_{sp}$. With these simplifications, Eqs. (1) become

$$\frac{\mathrm{d}I_p}{\mathrm{d}z} \approx -\alpha I_p - \kappa(I_p^2 + 4I_pI_s + I_s^2)I_p - \gamma I_sI_p, \quad (2a)$$

$$\frac{\mathrm{d}I_s}{\mathrm{d}z} \approx -\alpha I_s - \kappa(I_p^2 + 4I_pI_s + I_s^2)I_s + \gamma I_pI_s. \quad (2b)$$

To reduce the error in signal intensity, we use signal parameters in Eq. (2), i.e., $\kappa \approx \tau_c\sigma_s\rho_s$ and $\gamma \approx -\zeta_{sp}$.

As Eqs. (2) still cannot be solved analytically, we make one more simplification. It consists of replacing the quantity in the parenthesis with $(I_p + I_s)^2$ and amounts to omitting terms containing $2\kappa I_p^2 I_s$ and $2\kappa I_s^2 I_p$. With this replacement, we obtain

$$\frac{\mathrm{d}I_p}{\mathrm{d}z} \approx -\alpha I_p - \kappa(I_p + I_s)^2 I_p - \gamma I_sI_p, \quad (3a)$$

$$\frac{\mathrm{d}I_s}{\mathrm{d}z} \approx -\alpha I_s - \kappa(I_p + I_s)^2 I_s + \gamma I_pI_s. \quad (3b)$$

An important point is that, even though we drastically approximated Eq. (1) to arrive at Eqs. (3), they still qualitatively represent the interplay between FCA and SRS holding all the basic features of Eq. (2).

Equations (3) can be solved analytically by noting that the total intensity $I = I_p + I_s$ satisfies a simple equation whose solution is

$$I(z) = \frac{I_0 \exp(-\alpha z)}{\sqrt{1 + I_0^2(\kappa/\alpha)[1 - \exp(-2\alpha z)]}},$$

where $I_0 = I_{p0} + I_{s0}$ is the total input intensity with $I_{p0} = I_p(0)$ and $I_{s0} = I_s(0)$. By substituting $I_p = I - I_s$ in Eq. (3b), we can solve this equation as well. The resulting solution is

$$I_s(z) = \frac{I(z)}{1 + (I_{p0}/I_{s0})\exp[-\gamma I_0 L_{\mathrm{eff}}(z)]}, \quad (4a)$$

$$I_p(z) = I(z) - I_s(z). \quad (4b)$$

The effective waveguide length, L_{eff}, is defined as

$$L_{\mathrm{eff}}(z) = \frac{f(0) - f(z)}{\sqrt{\alpha\kappa}I_0}, \quad f(z) = \tan^{-1}\left[\sqrt{\frac{\kappa}{\alpha}}I(z)\right]. \quad (5)$$

Equation (4a) shows that changes in the signal intensity result from two sources with different physical origin. The steadily decreasing function $I(z)$ is responsible for linear losses and FCA, whereas the denominator of Eq. (4a) arises from SRS and exhibits a saturable character. The structure of the denominator shows that the effective length has direct influence on the signal gain. In the ideal situation, when this length becomes sufficiently large, signal approaches the total intensity $I(z)$. To estimate the position of the most efficient energy transfer from pump to signal, z_0, we note that for $I_p + I_s \approx \text{const}$, the Raman terms in Eq. (3) peak near $I_p = I_s$ (we assume $I_{p0} > I_{s0}$). Using $I_s = I/2$ in Eq. (4b), we are led to the relation $\gamma I_0 L_{\mathrm{eff}}(z_0) = \ln(I_{p0}/I_{s0})$. Hence for a given I_0, z_0 is determined by the ratio I_{p0}/I_{s0}. Once this distance is reached and the Raman interaction is strong enough, the energy will keep on passing from pump to the signal until the conversion efficiency is substantially reduced by the pump depletion. Therefore, each time we get considerable amplification of signal, the pump loses its power completely. This is the main reason why the undepleted-pump approximation fails to describe the operation of a silicon Raman amplifier correctly.

In the case of low input intensities, FCA is negligible. Rather than simplifying Eq. (5) in the limit $\kappa \to 0$, we note that Eq. (1) has the following closed-form solution when $\alpha_p = \alpha_s$, $\beta_{p(s)} = 0$, and $\tau_c = 0$:

$$I_p(z) = I_s(z)(I_{p0}/I_{s0})\exp[-\zeta_{ps}\mathcal{I}_0\mathcal{L}_{\mathrm{eff}}(z)], \quad (6a)$$

$$I_s(z) = \frac{\mathcal{I}_0 \exp(-\alpha z)}{1 - (\zeta_{sp}/\zeta_{ps})(I_{p0}/I_{s0})\exp[-\zeta_{ps}\mathcal{I}_0\mathcal{L}_{\mathrm{eff}}(z)]}, \quad (6b)$$

where $\mathcal{I}_0 = I_{s0} - (\zeta_{sp}/\zeta_{ps})I_{p0}$ and $\mathcal{L}_{\mathrm{eff}}(z) = [1 - \exp(-\alpha z)]/\alpha$, which is the standard effective length used in the literature [6,8,11]. As one can easily check, $L_{\mathrm{eff}}(z) \to \mathcal{L}_{\mathrm{eff}}(z)$ as κ tends to zero. In this limit, Eq. (4) becomes a special case of Eq. (6) with $\zeta_{ps} = -\zeta_{sp} = \gamma$. Comparison of these solutions offers more insight into the influence of FCA on CW Raman amplification than the numerical approach in [4]. This influence manifests itself in two ways. First, FCA leads to an overall attenuation of the signal as indicated by the radical in the function $I(z)$. Second, it leads to a decrease in the effective length compared to the linear-loss situation only and, what is more important, makes it intensity dependent. The influence of the total input intensity on the effective length $L_{\mathrm{eff}}(z)$ is illustrated by solid curves in Fig. 1. It is seen that one can substantially increase L_{eff} by decreasing I_0. It is worth noting that similar increases in the effective length may be achieved by reducing τ_c because κ scales linearly with τ_c. Indeed, τ_c is reduced often in practice by employing techniques such as ion-implantation or by using a reverse biased p-n junction [3,5,12].

It should be emphasized that for relatively small input intensities Eq. (4b) usually overestimates the pump near the input end and underestimates it at the output end of a SOI waveguide, while Eq. (4a) always overestimates the signal. Clearly, these errors

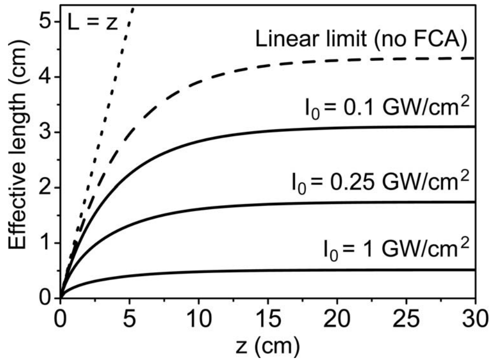

Fig. 1. Effective length, L_{eff}, versus propagation distance for different input intensities I_0 (solid curves). Dashed curves shows the linear-loss limit, $\mathcal{L}_{\text{eff}}(z)$, realized in the absence of TPA and FCA. Dotted line corresponds to the lossless case. In the calculations the following parameters were used: $\alpha=1$ dB/cm, $\beta=0.5$ cm/GW, $\tau_c=1$ ns, $g_R=76$ cm/GW, and $\lambda_s=1686$ nm.

are due to our simplifications that were used to get Eq. (2) and the terms that we discarded in Eq. (3). We can substantially reduce the errors of the second type by introducing correcting multipliers. Assuming the corrected solution of the form $\tilde{I}_s(z)=\xi(z)I_s(z)$, $\tilde{I}_p(z)=\xi(z)I_p(z)$, we find from Eqs. (2) that $\xi(z)=(1+2\kappa\int_0^z I_p(z')I_s(z')\mathrm{d}z')^{-1/2}$, where $I_p(z')$ and $I_s(z')$ are given by Eq. (4). Obviously such a correction does not change the qualitative features of our analysis or the solution [Eq. (4)]. Once corrected with this multiplier, Eqs. (4) provide an analytical solution of Eq. (2) that is quite close to the exact numerical solution of Eq. (1).

To estimate the applicability range of our solution, we note that the TPA terms in Eq. (1) are small compared with the linear-loss or FCA terms when $I_0 \ll \alpha/\beta$ or $I_0 \gg \beta/\kappa$. For $\alpha=1$ dB/cm, $\beta=0.5$ cm/GW, and $\lambda_s=1686$ nm we obtain $\alpha/\beta\approx 0.5$ GW/cm^2 and $\beta/\kappa\approx 0.01$ GW/cm^2. Thus, Eqs. (4) give a very good approximate solution of Eq. (1) for all input intensities. Figure 2 demonstrates this point by comparing the exact numerical solution (solid curves) of Eq. (1) with the analytical solutions when $\lambda_p=1550$ nm and

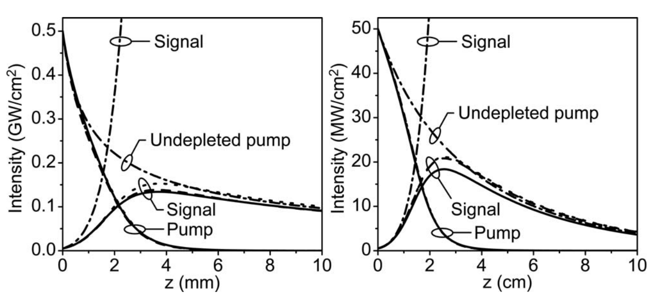

Fig. 2. Pump and signal intensity evolution for the exact numerical solution of Eq. (1) (solid curves) compared with the corrected (dashed curves) and uncorrected (dotted curves) solutions [Eq. (4)] as well as with the solution that corresponds to the undepleted pump approximation (dashed–dotted curves). In both cases $I_{s0}=0.01I_{p0}$. For other parameters see the text.

$\lambda_s=1686$ nm. The left panel shows the uncorrected (dotted curves) and corrected (dashed curves) solution [Eq. (4)] for $I_{p0}=0.5$ GW/cm^2. We used the same parameter values used for Fig. 1. One can see that the uncorrected signal intensity overestimates the exact solution by about 10%, but the corrected solution nearly coincides with the exact one. The right panel in Fig. 2 demonstrates that such a good agreement between numerical and analytical solutions persists also at a relatively low value of pump intensity, $I_{p0}=0.05$ GW/cm^2. For reference, the solution that corresponds to the undepleted pump approximation is also plotted in the same figure using dashed–dotted curves. According to Eq. (4a), Raman gain depends on the product $I_0L_{\text{eff}}(z,I_0)$. Comparison of signal intensity profiles for different input intensities I_0 (see Fig. 2) reveals that variation of Raman gain maximum with I_0 is predominantly governed by the effective length but not by the multiplier I_0.

To summarize, we have presented an approximate analytical solution of nonlinearly interacting copropagating waves in SOI-based Raman silicon amplifiers while incorporating both the saturation and depletion effects. This solution enables one to gain valuable insight into the amplifier operation and gauge amplifier performance without numerical simulations. We showed that the analytically calculated pump and Stokes intensity profiles are in excellent agreement with the detailed numerical results.

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