



to be the same for the pump and probe, although they may differ in some situations. The last term in the probe equations accounts for the birefringence resulting from different mode indices  $n_x$  and  $n_y$  for the TE and TM modes. Here,  $\bar{\beta}=(\beta_x+\beta_y)/2$  is the average value of  $\beta_x=n_xk_0$  and  $\beta_y=n_yk_0$ , with  $k_0=2\pi/\lambda_s$ ,  $\lambda_s$  being the probe wavelength. The walk-off effects result from different group velocities of the pump and probe components. Since only relative speeds matter, we work in a frame moving at the pump speed and introduce a reduced time  $T=t-z/v_{gp}$ . Using the notation,  $\beta_{1j}=d\beta_j/d\omega=1/v_{gj}$  with  $j=x,y,p$ , the two walk-off parameters of the probe are defined as  $d_x=\beta_{1x}-\beta_{1p}$  and  $d_y=\beta_{1y}-\beta_{1p}$ . Finally, the carrier density  $N_c(z, T)$  evolves as

$$\frac{\partial N_c}{\partial T} = \frac{\beta_{\text{TPA}}}{2h\nu_p} I_p^2(z, T) - \frac{N_c}{\tau_c}, \quad (4)$$

where  $h\nu_p$  is the energy of a pump photon and  $\tau_c$  is the carrier lifetime. To be consistent, we have neglected the terms containing probe intensity, assuming a relatively weak probe.

A numerical solution of Eqs. (1)–(4) provides a reasonable description of the pump-induced NPR effects. Before discussing the numerical results, we show that an approximate analytic solution is also possible and can provide considerable physical insight. The simplifying approximation is that free-carrier effects are negligible. This approximation is too drastic to be valid in general. However, we show later that it is reasonable for relatively short, not too intense, pump pulses that are widely separated. Since the pump equation (3) does not depend on the probe, it can be solved easily when  $N_c=0$ :

$$I_p(z, T) = \frac{\alpha_l I_0(T) e^{-\alpha_l z}}{\alpha_l + \beta_{\text{TPA}} I_0(T) (1 - e^{-\alpha_l z})}, \quad (5)$$

where  $I_0(T)$  is the intensity profile of the input pulse launched at  $z=0$ . The two probe equations can also be solved analytically because of their linear nature. Introducing  $T'_x=T-d_xz$  and the transformation  $E_x=A_x e^{-\alpha_l z/2} e^{i(\beta_x-\bar{\beta})z} e^{i\phi_x}$ , Eq. (1) leads to

$$\frac{\partial A_x}{\partial z} = -2k_0 n_2 a r I_p A_x, \quad \frac{\partial \phi_x}{\partial z} = 2k_0 n_2 a I_p. \quad (6)$$

Both of them can be solved to obtain

$$A_x(z, T'_x) = A_x(0, T'_x) \times \exp \left[ -a \int_0^z \frac{\alpha_l e^{-\alpha_l z'} \beta_{\text{TPA}} I_p(0, T'_x + d_x z')}{\alpha_l + \beta_{\text{TPA}} I_p(0, T'_x + d_x z') (1 - e^{-\alpha_l z'})} dz' \right], \quad (7)$$

$$\phi_x(z, T'_x) = \frac{a}{r} \int_0^z \frac{\alpha_l e^{-\alpha_l z'} \beta_{\text{TPA}} I_p(0, T'_x + d_x z')}{\alpha_l + \beta_{\text{TPA}} I_p(0, T'_x + d_x z') (1 - e^{-\alpha_l z'})} dz'. \quad (8)$$

The solution for  $A_y$  is obtained by replacing  $a$  with  $b$  and the subscript  $x$  with  $y$  in Eqs. (7) and (8).

The NPR of the probe beam by the pump beam is governed by the relative phase shift  $\Delta\phi=\phi_x-\phi_y$ . Notice that the linear birefringence of the waveguide also leads to a phase difference between the two polarization components. This birefringence-induced polarization rotation can be compensated by using a polarization compensator after the waveguide. Another polarizer is needed after the polarization compensator to block the probe in the absence of a pump pulse. Each pump pulse changes probe polarization through NPR such that some probe light is transmitted through the polarizer over the pump duration.

To test the range of validity of the analytic solution, we have solved Eqs. (1)–(4) numerically for a 5-mm-long SOI waveguide with a cross-section area of  $600 \times 450 \text{ nm}^2$ . The pump and probe wavelengths are  $\lambda_p=1.56 \mu\text{m}$  and  $\lambda_s=1.54 \mu\text{m}$ . The walk-off parameters are calculated using the dispersion curves for the fundamental TE and TM modes and are  $d_x=-0.78 \text{ ps/cm}$  and  $d_y=30.2 \text{ ps/cm}$ . For the nonlinear parameters, we use the values from our recent measurements [8]:  $n_2=2.5 \times 10^{-18} \text{ m}^2/\text{W}$ ,  $\beta_{\text{TPA}}=5 \times 10^{-12} \text{ m/W}$ . We note that  $n_2$  values in the literature differ by up to a factor of 2, but our results are not too sensitive to the exact value if the device length is adjusted to obtain the same nonlinear phase shift. The other parameters are chosen to be  $\sigma=1.45 \times 10^{-21} \text{ m}^2$ ,  $\mu=7.597$  [7],  $\tau_c=1.5 \text{ ns}$ ,  $\alpha_l=5 \text{ dB/cm}$ , and  $A_{\text{eff}}=0.1 \mu\text{m}^2$ . The peak power is 4 W for the sech pump pulses with  $P_p(T)=P_0 \text{ sech}(T/T_0)^2$ . The width  $T_0$  is varied over a wide range to study the shape and duration of the NPR-induced switching window. The input power of the weak cw probe is 1 mW.

Figure 2 compares the switching windows for values of  $T_0$  ranging from 1 ns to as short as 0.1 ps obtained numerically (thin curves) with those predicted analytically (thick curves) by neglecting the free-carrier effects. These results show that free carriers play a significant role for pulses much wider than 1 ps but can be neglected for shorter pump pulses. In particular, the switching window is not affected much by free carriers for 100 fs pulses. In the case of 1 ps pump pulses, the switching windows almost coincide except for a low-amplitude tail that results from the interaction between the free-carrier effects and the walk-off effect. Free-carrier effects break the temporal symmetry of the probe field. In the presence of group-velocity mismatch, the free-carrier-induced phase shift is not the same for  $A_x$  and  $A_y$ , resulting in polarization changes that produce some leakage of the probe power. With this in mind, we can understand the appearance of a second peak for 10 ps or wider pulses. Since the index and phase changes produced by the XPM and free carriers occur with opposite signs, they cancel each other occasionally. The

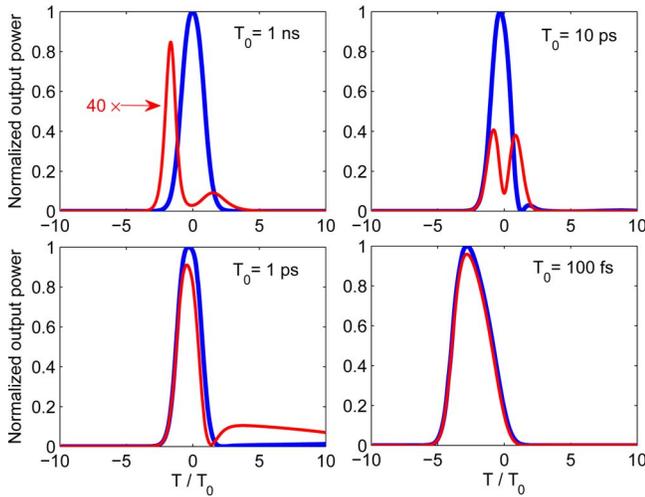


Fig. 2. (Color online) Switching windows for four different pump-pulse widths. The thinner (red) curves mark the numerical results, while the thicker (blue) curves show the analytical prediction neglecting the free-carrier effects.

case of very long pulses ( $T_0 > 100$  ps) is interesting, because the switching window can be considerably shorter than the pump pulse but at the expense of a much-reduced power (note the magnification by a factor of 40, in Fig. 2 for this case).

We note from Fig. 2 that the performance of a silicon-based Kerr shutter suffers considerably (compared with the case of silica fibers) from a combination of the free-carrier and walk-off effects. The use of a square-shape waveguide may help because it will reduce the birefringence effects. Figure 3 shows the results under conditions identical to those of Fig. 2, except that a square-shape waveguide ( $450 \times 450$  nm<sup>2</sup>) is employed with the modified values  $d_x = -1.98$  ps/cm,  $d_y = -1.16$  ps/cm, and  $A_{\text{eff}} = 0.09$   $\mu\text{m}^2$ . As expected, the switch performance is improved con-

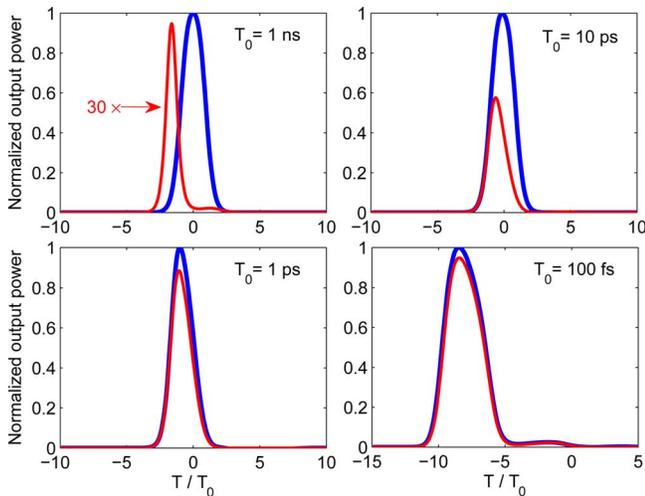


Fig. 3. (Color online) Switching windows for the same four pump-pulse widths and the same operating conditions used in Fig. 2. The only difference is that a square-shape silicon waveguide is employed to minimize walk-off effects.

siderably because of reduced walk-off effects. In particular, the long tail seen in Fig. 2 for 1 ps pump pulses has disappeared completely. We should point out that the performance of a Kerr switch also depends on the peak power of pump pulses. In the case of Kerr shutter based on silica, one may want to choose the power level such that  $\Delta\phi \approx \pi$ . However in the case of silicon waveguides, nonlinear absorptions set the limit on peak pump power. Serious distortion on switching window starts even before  $\Delta\phi$  approaches  $\pi$ .

In conclusion, we have shown theoretically that the NPR, induced by a pump pulse on a cw probe inside a silicon waveguide, can be used to realize a fast Kerr shutter. We show that free carriers generated by the pump pulse through TPA affect the switching process because of a combination of the free-carrier and walk-off effects. However, their impact is not detrimental for pump pulses shorter than 5 ps or so, especially when birefringence effects are minimized by employing a square-shape waveguide. For such short pulses, an approximate analytical solution predicts the shape and duration of the switching window with reasonable accuracy. Even though a switching window of  $<5$  ps duration can be realized using SOI waveguides, we stress that the repetition rate of such a switch is invariably limited by the carrier lifetime. It is unlikely to exceed 1 GHz unless the effective carrier lifetime is reduced substantially using a suitable technique such as carrier sweeping by an external electric field [9] or enhanced recombination through ion implantation [10].

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