Nonlinear interaction of two or more similaritons in loss- and dispersion-managed fibers

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We study the nonlinear interaction of two or more fundamental and higher-order similaritons. In particular, we consider the bright similaritons, propagating in nonlinear fibers whose gain–loss and anomalous dispersion vary along the fiber. We first focus on the situation such that the similaritons propagate with the same velocity but are spaced closely enough to overlap. Similar to the soliton case, the similariton interaction occurs through cross-phase modulation; the similaritons attract or repel each other depending on their relative phase difference. The main difference from the soliton case is that the similariton width, chirp, and the amplitude scale upon the interaction and the subsequent propagation along the fiber. We have found evidence of a substantial energy transfer between two similaritons when their relative phase lies between 0 and π. We also consider the case of three higher-order similaritons moving with different velocities and colliding inside the fiber. We show that the nonlinear interaction of closely spaced similaritons exhibits a variety of interesting features that are different from those typical of the soliton case. © 2008 Optical Society of America

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1. INTRODUCTION

Optical solitons, the waves keeping their structure and linear dimensions intact on propagation inside nonlinear media, have been studied in a number of settings [1–5]. Recently, a more general class of shape-preserving waves, known as similaritons, has attracted considerable attention [6–13]. The envelopes of such self-similar waves maintain their overall shape, but the field amplitude, width, and a phase chirp evolve on propagation inside nonlinear media [14]. Although temporal similaritons have so far attracted more attention, spatial similaritons have also been discovered in graded-index waveguide amplifiers [13,15]. Mathematically, all these cases are described by the inhomogeneous nonlinear Schrödinger (NLS) equation. The same equation governs the dynamics of Bose–Einstein condensates in atomic traps [16].

An interesting aspect of soliton physics is related to the nonlinear interaction of two or more solitons occurring when they are so close to each other that their tails begin to overlap [17–23]. An essential property of solitons is that two solitons propagating with different velocities remain intact upon a collision, acquiring only a small shift in their central positions [17]. Another interesting interaction scenario is that of the two closely spaced overlapping solitons propagating parallel to each other. In this case, the two solitons interact through cross-phase modulation (XPM) and they attract or repel each their depending on their relative phase difference [18]. In the case of in-phase solitons, two solitons close in on each other, collide, and move apart until their trajectories start bending toward each other again. The whole scenario then repeats itself in a periodic fashion. In contrast, the spacing between the solitons increases monotonically when their phases differ by 180°. The situation becomes more complex whenever more than two solitons interact nonlinearly [22].

Since similaritons are related to solitons through a rather complicated transformation [12], we expect the similariton collisions to display an even more intricate behavior. Indeed, we have recently shown that two similaritons moving with the opposite velocities can form a moleculelike bound state upon collision [12]. In this paper, we consider the case of two or more closely spaced similaritons. Similar to interacting solitons the closely spaced similaritons interact with each other through XPM and experience attractive and repulsive forces depending on their relative phases. However, due to a phase chirp the similariton evolution exhibits a variety of interesting features that can be quite different from the soliton case. Our similaritons form and propagate in the anomalous-dispersion regime of a fiber amplifier. We notice that parabolically shaped similaritons that can form asymptotically in the case of normal dispersion are not considered in this paper.

2. SIMILARITON SOLUTIONS

The evolution of similaritons inside a dispersion-managed fiber amplifier is governed by the generic inhomogeneous NLS equation in the form

$$i\frac{\partial U}{\partial z} - \frac{\beta(z)}{2}\frac{\partial^2 U}{\partial \tau^2} + \gamma(z)|U|^2 U = i\frac{g(z)}{2}U, \tag{1}$$

where $g(z)$, $\beta(z)$, and $\gamma(z)$ are distributed gain, dispersion, and nonlinearity of the amplifier, respectively. Equation

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(1) is known to have similariton solutions [7,9,12] provided a certain relation exists among $g(z)$, $\beta(z)$, and $\gamma(z)$.

The similariton solution is obtained by noting that a simple self-similar transformation reduces Eq. (1) to the standard homogeneous NLS equation that is known to be integrable by the inverse scattering method [17]. Such a transformation can be represented as [12]

$$U(\tau,z) = A(z)\Psi\left[\frac{\tau-\tau_c(z)}{w(z)},\zeta(z)\right]\exp[i\Phi(\tau,z)]. \tag{2}$$

Here $A(z)$, $w(z)$, and $\tau_c(z)$ are the amplitude, the width, and the center similariton position, respectively, and $\zeta(z)$ is an effective propagation distance, which is yet to be determined. For a similariton the phase front is parabolic (corresponding to a linearly chirped pulse) such that

$$\Phi(\tau,z) = c(z)\tau^2/(2w_0^2) + b(z)\tau + d(z), \tag{3}$$

where $c(z)$ and $b(z)$ specify the curvature and the position of the center of the wavefront, respectively, and $d(z)$ is independent of τ. We have introduced w_0 as an input pulse width parameter to make the chirp parameter $c(z)$ dimensionless. We stress that the chirp $c(z)$ is not an essential feature of similaritons—in fact, chirp-free similaritons with $c=0$ can exist.

The substitution from Eqs. (2) and (3) into the NLS equation (1) results in a set of differential equations for the parameters describing the evolution of the pulse such that Ψ obeys the homogeneous NLS equation

$$i\frac{\partial\Psi}{\partial\zeta} \pm \frac{1}{2}\frac{\partial^2\Psi}{\partial\chi^2} + |\Psi|^2\Psi = 0. \tag{4}$$

Here the upper (lower) sign corresponds to the case of anomalous (normal) dispersion, and the similarity variable is defined as

$$\chi(\tau,z) = [\tau-\tau_c(z)]/w(z). \tag{5}$$

The transformation of the inhomogeneous NLS equation into the homogeneous one is only possible if the parameters of the medium satisfy the condition [7,9,12]

$$g(z) = c(z)\frac{\beta(z)}{w_0^2} + \frac{d}{dz}\ln\left[\frac{\beta(z)}{\gamma(z)}\right]. \tag{6}$$

As long as condition (6) is satisfied, one obtains the following expressions for the effective propagation distance, the width, the amplitude, and the position of the pulse:

$$\zeta(z) = D(z)[1-c_0D(z)]^{-1}, \tag{7}$$

$$w(z) = w_0[1-c_0D(z)], \tag{8}$$

$$A(z) = w(z)^{-1}[|\beta(z)|/\gamma(z)]^{1/2}, \tag{9}$$

$$\tau_c(z) = \tau_0 - (c_0\tau_0 + b_0)D(z), \tag{10}$$

where we have introduced a dimensionless parameter $D(z)$ as

$$D(z) = \frac{1}{w_0^2}\int_0^z \beta(z)dz. \tag{11}$$

It represents the total dispersion accumulated over a fiber length z. The parameters related to the phase evolve with z as

$$c(z) = \frac{c_0}{1-c_0D(z)}, \quad b(z) = \frac{b_0}{1-c_0D(z)}, \tag{12}$$

$$d(z) = (b_0^2/2)D(z)[1-c_0D(z)]^{-1}. \tag{13}$$

Here c_0 is the input chirp parameter. It will be seen later that the value of c_0 strongly affects the similariton interaction scenario.

Equations (4)–(13) show that any soliton of the homogenous NLS equation is related to a similariton obeying the inhomogeneous NLS equation with the compatibility condition given in Eq. (6). It follows from the exact integrability of the NLS equation [1] that all such similaritons must be stable. They should survive mutual collisions just as the solitons do, even though their width, amplitude, and chirp will keep changing during and after the collision. Further, higher-order solitons of the NLS equation correspond to multisimilariton solutions of Eq. (1).

3. TWO-SIMILARITON INTERACTIONS

The similarity transformation, given by Eq. (2), can be used to study collisions of two closely spaced similaritons. Our approach consists of solving the standard NLS equation, Eq. (4), numerically with the split-step Fourier method [5] using the following initial condition:

$$\Psi(0,\chi) = (1+a_0)\mathrm{sech}(\chi+\chi_0/2)\exp(i\theta/2) + (1-a_0)\mathrm{sech}(\chi-\chi_0/2)\exp(-i\theta/2), \tag{14}$$

where θ is the initial phase difference between the two similaritons and a_0 is a measure of their amplitude difference. When $a_0=0$ and $\theta=0$, the similaritons are initially identical in all respects as they have the same amplitude and phase but their center positions are χ_0 away from each other. As the similaritons propagate and interact through XPM, their amplitudes, phases, and central positions change [18]. In this section, we fix the relative time delay to be $\chi_0=3$ but let a_0 and θ take on different values.

Our numerical solution provides the soliton field $\Psi(\zeta,\chi)$ at any distance ζ starting from its initial value $\Psi(0,\chi)$ given in Eq. (14). We convert it into the corresponding similariton field $U(z,\tau)$ by inverting transformations Eqs. (5) and (7) with the aid of the relations

$$D(z) = \zeta/(1+c_0\zeta), \tag{15}$$

$$\tau(z) = \tau_c(z) + w(z)\chi(z). \tag{16}$$

To obtain an explicit expression for the distance z, we focus on a fiber whose dispersion decreases exponentially with the length such that $\beta(z)=\beta_0e^{-\sigma z}$. The parameter σ controls the rate of dispersion change inside the fiber. In particular, the constant dispersion case can be treated by

choosing $\sigma=0$. In the case of a dispersion-decreasing fiber (DDF), we obtain

$$z = -\frac{1}{\sigma}\ln\left(1 - \frac{\sigma L_D \zeta}{1 + c_0 \zeta}\right), \tag{17}$$

$$\tau(z) = \tau_c(z) + w_0\left[1 + c_0\frac{(1 - e^{-\sigma z})}{\sigma L_D}\right]\chi(z), \tag{18}$$

where $L_D = w_0^2/|\beta_0|$ is the dispersion length. It follows from these relations that σ is simply a scaling factor for the functions $z(\zeta)$ and $\tau(z)$. In the specific cases of $\sigma=0$ and $c_0=0$, we recover the soliton case because $g=0$ from Eq. (6) and the fiber has constant dispersion. In this specific case, $z=L_D\zeta$ and $\tau=\tau_c+w_0\chi$, as expected.

An interesting special case is of chirped similaritons, propagating inside a fiber with constant anomalous dispersion. This case was first studied in [6] and it leads to a simple compatibility condition

$$g(z) = -\frac{c_0/L_D}{1 + c_0 z/L_D}. \tag{19}$$

The gain is needed when $c_0<0$. Thus, a fiber amplifier whose gain increases along the fiber length, as indicated above, supports bright similaritons whose width decreases with propagation. In contrast, when $c_0>0$ similaritons exist if fiber losses decrease with z and the similariton width increases continuously. It is remarkable that stable similaritons can form in a lossy, constant-dispersion fiber.

Figure 1 shows the nonlinear interaction of two identical similaritons (the same amplitude and phase, $a_0=\theta=0$) inside a fiber with constant dispersion. The top part shows the case of two positively chirped similaritons ($c_0=0.1$) propagating inside a lossy fiber. As expected, similaritons spread as they "breathe" in a periodic manner. In contrast, when $c_0=-0.1$ and the fiber has a suitable gain profile, the two similaritons compress as they interact through XPM and breathe periodically. In a sense, such higher-order similaritons are reminiscent of the well-known soliton breathers [19].

As in the case of standard solitons, the relative phase of two similaritons has an enormous impact on the XPM-induced nonlinear coupling between them. As an example, Fig. 2 shows the evolution of two similaritons of the same amplitude ($a_0=0$) when their relative phase difference is changed from $\theta=\pi/2$ (top) to $\theta=\pi$ (bottom). The positive chirp of $c_0=0.1$ corresponds to propagation inside a lossy fiber. When the two similaritons are out of phase ($\theta=\pi$), they move away from each other without any energy transfer. In contrast, when $\theta=\pi/2$ the energy is transferred from one similariton to another. The direction of the energy transfer depends on the sign of θ and a mirror image of that seen in the top part of Fig. 2 is obtained for $\theta=-\pi/2$. Such a behavior is reminiscent of a fiber coupler in which a linear evanescent-wave coupling leads to the energy transfer.

One may ask what happens when the amplitudes of two similaritons are different. Figure 3 shows the evolution of two similaritons with different initial amplitudes of 1.2 and 0.8 ($a_0=0.2$) when their relative phase difference is changed from 0 (top image) to $\pi/2$ (bottom image). The other parameters are identical to those used for Fig. 2. To see the impact of amplitude mismatch, Fig. 3(a) should be compared with that in Fig. 1(a), while Fig. 3(b) should be compared with that in Fig. 2(a). In the in-phase case, periodic collisions still occur but only a part of the pulse energy of the weaker pulse is transferred to the intense similariton during each partial collision. When the two input similaritons also differ in phase by $\pi/2$, they repel each other but some energy is still transferred from the weaker to the stronger pulse before they separate far apart. Because of this energy loss the weaker pulse spreads rapidly because it is not able to maintain its similaritonlike nature. The stronger pulse also spreads and acquires a finite upward velocity, resulting in a continuous vertical shift in its position.

Let us now consider similariton interactions inside the DDFs. The main feature of such fibers is that the accu-

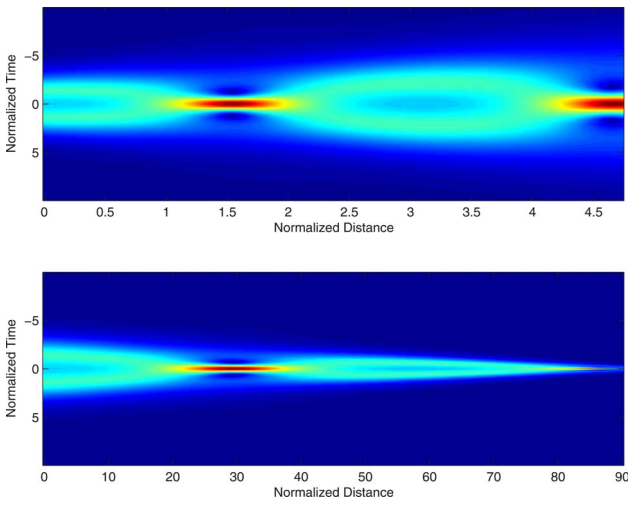

Fig. 1. (Color online) Nonlinear interaction of two identical similaritons inside a fiber with constant anomalous dispersion. The initial chirp is positive ($c_0=0.1$) in the top image but is changed to a negative value $c_0=-0.1$ in the bottom one.

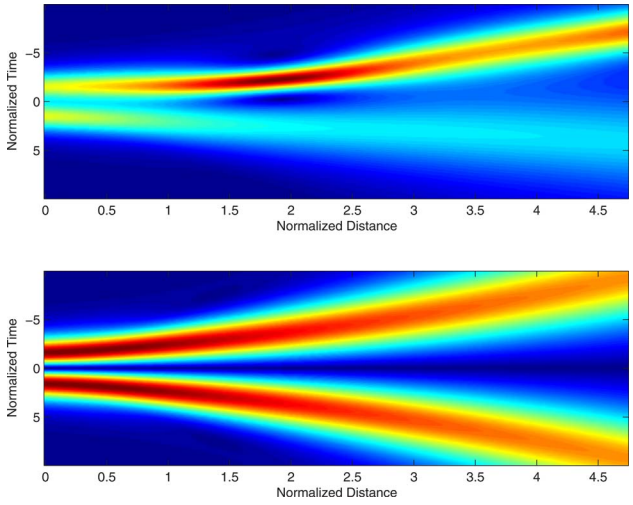

Fig. 2. (Color online) Nonlinear interaction of two identical chirped similaritons ($c_0=0.1$) inside a fiber with constant anomalous dispersion. The relative phase difference between them is $\pi/2$ in the top image but is changed to π in the bottom one.

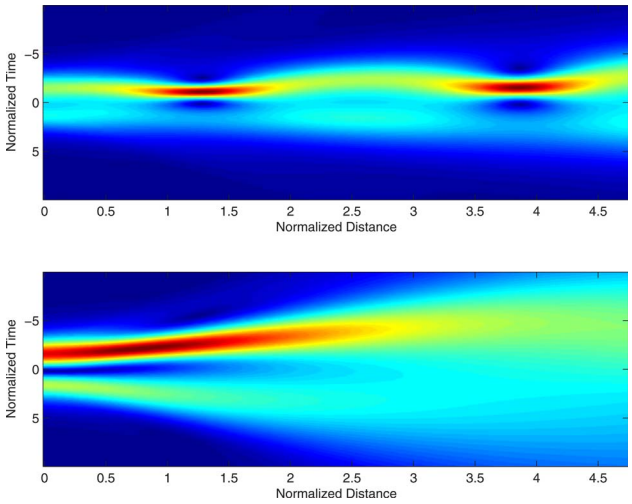

Fig. 3. (Color online) Nonlinear interaction of two chirped similaritons ($c_0=0.1$) with different amplitudes ($a_0=0.2$) inside a fiber with constant dispersion. The relative phase difference between them is 0 in the top image but is changed to $\pi/2$ in the bottom one.

mulated dispersion $D(z)$ asymptotically approaches a constant value of $(\sigma L_D)^{-1}$. The width of each similariton may increase, decrease, or remain constant depending on the initial chirp. A chirp-free similariton can exist inside a DDF. As seen from Eq. (13), the pulse width does not change for chirp-free similaritons. The compatibility condition (6) yields $g(z)=-\sigma$, i.e., such similaritons form in fibers with a constant loss. Figure 4(a) shows the XPM-induced attraction between two such similaritons for $\sigma=0.1$. If a small chirp is initially imposed, the two similaritons may expand or compress as they collide, depending on whether c_0 is initially positive or negative. Figure 4(b) shows the case of compressing similaritons with $c_0=-0.1$. The two similaritons are in phase in both cases. The interaction dynamics are similar to those shown in Fig. 2 when two similaritons have different phases.

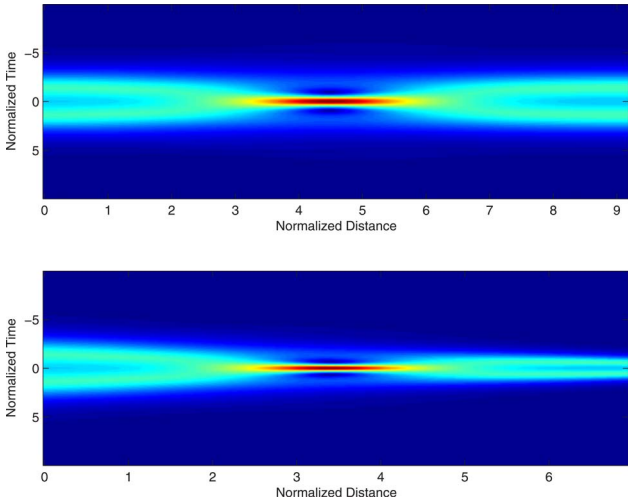

Fig. 4. (Color online) Nonlinear interaction of two identical in-phase similaritons inside a DDF with $\sigma=0.1$. The initial chirp is zero ($c_0=0$) in the top image but is changed to $c_0=-0.1$ in the bottom one.

4. THREE-SIMILARITON INTERACTIONS

We next focus on the interaction of three closely spaced similaritons. This case is studied by solving the standard NLS equation (4) with the following initial condition:

$$\Psi(0,\chi)=\mathrm{sech}(\chi+\chi_0)\exp(i\theta)+\mathrm{sech}(\chi)+\mathrm{sech}(\chi-\chi_0)\exp(-i\theta), \quad (20)$$

Figure 5 (top image) shows the XPM-induced attraction between three chirp-free similaritons inside a DDF using $\sigma=0.1$, $\chi_0=2.5$, and $\theta=0$. As mentioned earlier the width of each similariton does not change under such conditions. The three similaritons first merge into two similaritons that are shifted half-way from the original positions. These two then merge to form a single intense pulse which subsequently separates into two and then into three original similaritons. The whole scenario then repeats in a periodic fashion. In the chirped case, the collision scenario remains unchanged except that individual similaritons spread for positive values of the chirp and compress for negative values of the chirp. Figure 5 (bottom image) shows the case of three positively chirped ($c_0=0.1$) expanding similaritons.

The effect of different relative phases for three chirped similaritons ($c_0=0.1$) is shown in Fig. 6. In the top image, the relative phase difference is $\theta=\pi/2$ whereas it is π in the bottom image. All other parameters are identical to those used in Fig. 5. The width of each positively chirped similariton increases. As expected, the three similaritons repel when they are out of phase ($\theta=\pi$). The repulsion takes place even for $\theta=\pi/2$, but the energy is asymmetrically redistributed among the similaritons.

We briefly consider the case of three identical similaritons moving with different velocities and interacting inside a DDF with $\sigma=0.1$. The standard NLS equation in this case is solved with the initial condition

$$\Psi(0,\chi)=\mathrm{sech}(\chi+\chi_0)\exp(iv\chi)+\mathrm{sech}(\chi)+\mathrm{sech}(\chi-\chi_0)\exp(-iv\chi), \quad (21)$$

where v is the initial velocity and χ_0 is the initial separa-

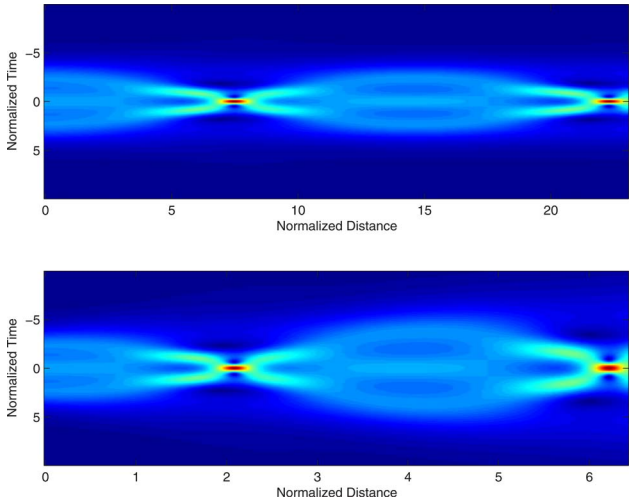

Fig. 5. (Color online) Nonlinear interaction of three identical in-phase similaritons inside a DDF with $\sigma=0.1$. The initial chirp is zero ($c_0=0$) in the top image but is changed to $c_0=0.1$ in the bottom one.

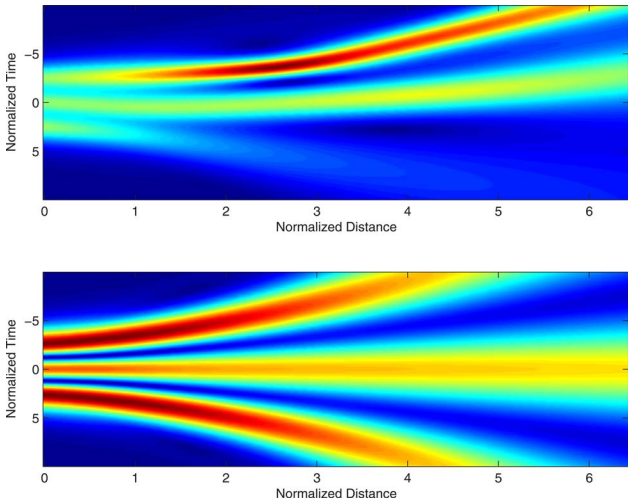

Fig. 6. (Color online) Nonlinear interaction of three identical chirped similaritons ($c_0=0.1$) inside a DDF with $\sigma=0.1$. The relative phase θ is $\pi/2$ in the top image but is changed to π in the bottom one.

tion of each similariton from its neighbor. Figure 7 (top) shows the case of positively chirped similaritons with $c_0=0.1$ using $\chi_0=8$ and $v=3$. In this case, each pulse spreads upon propagation. After the collision, each pulse accelerates away from the others. The case of negatively chirped similaritons compressing similaritons ($c_0=-0.1$) is shown in Fig. 7(b). Notice that each pulse now decelerates as it moves away from the others after the collision and the similaritons appear to be moving at nearly the same speeds.

5. DIFFERENCES BETWEEN SIMILARITON AND SOLITON COLLISIONS

In this section we focus on the main differences between soliton and similariton collisions by focusing on three pulses moving with different velocities such that they collide at a certain point within the fiber. In both cases, we use the initial condition given in Eq. (21) and assume fiber dispersion to be anomalous with a constant value. Thus the only difference is that pulses are chirped in the similariton case but $c_0=0$ in the soliton case.

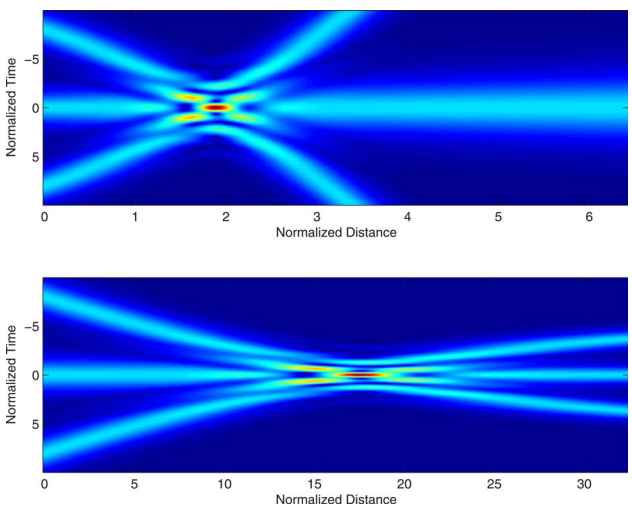

Fig. 7. (Color online) Nonlinear interaction of three identical similaritons moving at different speeds ($v=3$) and colliding inside a DDF with $\sigma=0.1$. The initial chirp is positive ($c_0=0.1$) in the top image but is changed to a negative value of $c_0=-0.1$ for the bottom one.

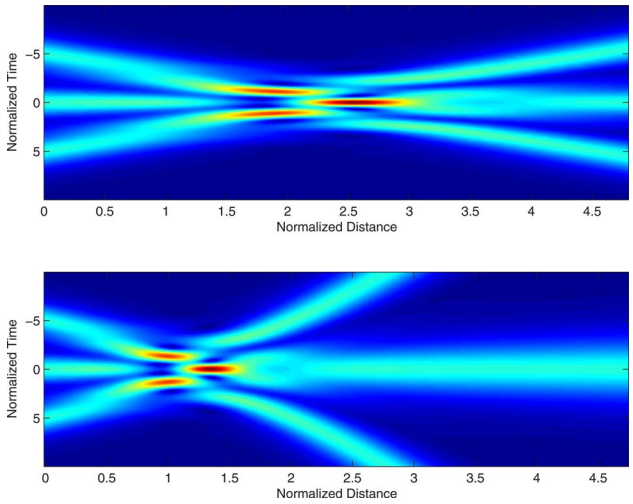

Fig. 8. (Color online) Nonlinear interaction of three identical solitons (top) and similaritons (bottom) moving at different speeds ($v=2$) and colliding inside a constant-dispersion fiber. The initial chirp is zero in the soliton case but $c_0=0.1$ in the similariton case.

The top part in Fig. 8 shows the soliton case ($c_0=0$) using $\chi_0=5$ and $v=2$. In the bottom part, the chirp is introduced using $c_0=0.1$ but all other parameters remain the same. One can clearly see the similarities as well as differences between soliton and similariton collisions. The main differences are that the similaritons collide sooner, their widths change continuously, and their trajectories curve after collision so much that collisions are far from being symmetric. An interesting feature worth commenting is that collision of even three solitons does not exhibit a perfect symmetry as one may have expected naively.

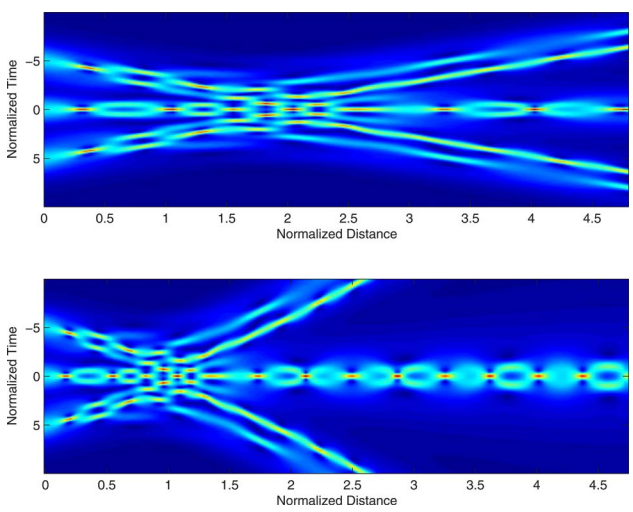

Fig. 9. (Color online) Nonlinear interaction of three third-order solitons (top) and similaritons (bottom) moving at different speeds ($v=2$) and colliding inside a constant-dispersion fiber. The initial chirp is zero in the soliton case but $c_0=0.1$ in the similariton case.

Finally, we consider the case of three higher-order similaritons moving with different velocities and colliding together inside a constant-dispersion fiber. Figure 9 shows the soliton (top) and similariton (bottom) collisions when soliton order $N=3$. Except for changing the soliton order, we keep all other parameters identical to those used in Fig. 8. To the best of our knowledge, these kinds of collisions have not attracted much attention in the past. As one would expect, each third-order soliton or similariton compresses, splits into two pulses, and then recovers its initial shape as it approaches the other solitons. After their nonlinear interaction during the collision, only the center soliton (or similariton) survives nearly intact. The other two are perturbed enough that they separate into three individual pulses, as dictated by the inverse scattering theory. Other difference between the soliton and similariton collisions are similar to the $N=1$ case in the sense that similaritons collide sooner, their widths change continuously, and their trajectories curve after collision.

6. CONCLUSIONS

In summary, we have studied in detail the nonlinear interaction of two or more similaritons occurring when they are so close to each other that their fields overlap. Our approach makes use of a similarity transformation that establishes a one-to-one correspondence between the similaritons that are solutions of an inhomogeneous NLS equation and the standard solitons that are solutions of the standard homogeneous NLS equation, provided a certain comparability condition is satisfied. This correspondence shows that similaritons can form in lossy fibers as well as inside fiber amplifiers with the main difference being that in the latter case the similaritons are often chirped. If fiber dispersion is allowed to change with distance, similaritons can also be chirp-free. In this paper, we have assumed that anomalous dispersion of the fiber and focused exclusively on bright similaritons. In general, whenever the similaritons are chirped, their initial chirp c_0 plays an important role. If the chirp is negative, the existence of similaritons requires gain (i.e., they form inside a fiber amplifier), and the similariton width decreases with propagation. In contrast, if c_0 is positive similaritons can form even in a lossy fiber, but they spread on propagation along the fiber.

In this paper we have chiefly focused on the situation in which similaritons propagate with the same velocity but are spaced close enough to overlap. Similar to the case of standard solitons the similariton interaction is attractive or repulsive depending on their relative phase difference. In the case of two in-phase similaritons, they mover closer, collide, and then separate from each other in a periodic fashion. In contrast, the similariton spacing increases monotonically when their phases differ by π. The main difference from the soliton case is that the width, chirp, and amplitude of such similaritons scale on propagation. We have found the evidence of a substantial energy transfer between the two similaritons when their relative phase lies between 0 and π.

We have also studied the nonlinear interaction of three closely spaced similaritons inside a DDF and have found some interesting features. In the in-phase case, three pulses are first transformed into two pulses, followed by their merger into a single intense pulse. The process then reverses itself, resulting in the formation of three separate similaritons. The whole scenario repeats periodically. In the chirp-free case, the similariton width does not change. However, if the initial chirp is positive, the pulse width increases monotonically. Whenever the phases of three similaritons are not the same, the similaritons redistribute energy among themselves and scatter away upon collision.

We have also studied the case of three similaritons, which are initially separated far apart and move with different velocities as they collide inside the fiber. Although, prior to collision, the energy transfer among three pulses still occurs, a substantial part of the energy resides in the outermost pulses and a single intense pulse never forms. An even more interesting behavior occurs when the initial peak powers of the pulses are large enough for higher-order similaritons to form. In the case of third-soliton similaritons, each similariton undergoes a periodic sequence of compression and splitting into. However, after the collision is over, the two outer solitons as well as similaritons are perturbed enough that each splits into three first-order components in accordance with the inverse scattering theory. The main point to note is that the nonlinear interaction of closely spaced similaritons exhibits a variety of interesting features that can be different from the soliton case.

Since the formation of similaritons hinges crucially on the compatibility condition given in Eq. (6), we discuss briefly how practical it is to satisfy this condition. Two approaches can be used for this purpose. If the fiber parameters β_2 and γ remain constant along the fiber length, the last term in Eq. (6) vanishes, and this equation can be satisfied by doping the fiber such that the dopant concentration varies along the fiber in a prescribed manner. By pumping the doped fiber suitably, it is possible to realize an axially varying gain $g(z)$ such that Eq. (6) is satisfied. We stress that fiber does not have to provide a net gain for similaritons to form but the loss should vary with z to ensure that the compatibility condition is satisfied. The second approach makes use of DDFs in which the core diameter of the fiber is changed along its length such that the magnitude of the dispersion parameter decreases exponentially [24]. In this case Eq. (6) can be satisfied even in passive fibers with constant loss. Such fibers have been fabricated [25] and employed recently to observe the formation of parabolically shaped similaritons [26–28]. An exponential dispersion profile can also be realized approximately by using several fibers exhibiting constant but different dispersion characteristics [29].

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