Interactions of chirped and chirp-free similaritons in optical fiber amplifiers

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Abstract: We obtain exact self-similar solutions to an inhomogeneous nonlinear Schrödinger equation, describing propagation of optical pulses in fiber amplifiers with distributed dispersion and gain. We show that there exists a one-to-one correspondence between such self-similar waves and solitons of the standard, homogeneous, nonlinear Schrödinger equation if a certain compatibility condition is satisfied. As this correspondence guarantees the stability of the novel self-similar waves, we refer to them as *similaritons*. We demonstrate that, the character of similariton interactions crucially depends on the sign of the similariton phase chirp. In particular, we show that the similariton interactions can under certain conditions lead to the formation of molecule-like bound states of two similaritons.

OCIS codes: (060.5530) Pulse propagation and solitons; (190.4370) Nonlinear optics, fibers

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1. Introduction

Optical solitons, the special wave envelops that maintain both their shape and size during propagation inside nonlinear media, have been studied in several different contexts [1–4]. Recently, optical solitons have been intensively studied in inhomogeneous nonlinear optical media, and a powerful generalization of the inverse scattering technique [1] has been developed to identify integrable inhomogeneous nonlinear systems and analytically determine the shape-preserving waves supported by such systems [5–8].

A more general class of shape-preserving waves includes self-similar waves, whose envelope maintains its overall shape but its parameters such as amplitude, width, and chirp evolve with propagation inside nonlinear media [9]. Such self-similar waves, often referred to as similaritons, have recently attracted much attention in the context of optical fiber amplifiers [10–15]. A spatial analog of similaritons has also been discovered in graded-index waveguide amplifiers [16]. The underlying equation in all cases is an inhomogeneous nonlinear Schrödinger equation (NLSE). The same equation governs the dynamics of Bose–Einstein condensation in atomic traps [17].

An intriguing unresolved aspect of similariton theory is concerned with the similariton interactions. In this context, a fundamental question arises: Do similaritons behave differently under collisions than solitons do? This question has recently been addressed numerically for parabolic-shape similaritons that form asymptotically in the normal-dispersion region of a homogeneous fiber amplifier [14]. Here, we develop an analytical approach to study similariton collisions governed by the generic inhomogeneous NLSE of the form

$$i\frac{\partial U}{\partial z} - i\frac{g(z)}{2}U - \frac{\beta(z)}{2}\frac{\partial^2 U}{\partial \tau^2} + \gamma(z)|U|^2 U = 0, \tag{1}$$

where $g(z)$, $\beta(z)$ and $\gamma(z)$ are arbitrarily distributed parameters. The model is sufficiently general to describe a multitude of physical systems in nonlinear optics and condensed matter physics. In the optical context, it describes the evolution of the slowly varying envelope $U(\tau,z)$ of an optical pulse propagating along the z axis in a fiber amplifier with nonuniform dispersion $\beta(z)$, gain $g(z)$, and nonlinearity $\gamma(z)$.

Equation (1) is known to have similariton solutions [11, 13], provided a certain relation exists among $\beta(z)$, $g(z)$, and $\gamma(z)$, but the relationship of these similaritons to the conventional solitons, if any, is not clear. It turns out that a simple transformation exists that reduces Eq. (1) to the standard homogeneous NLSE that is well known to be integrable by the inverse scattering method [18]. As a result, a one-to-one correspondence can be established between any soliton of the homogeneous NLSE and a stable self-similar wave of the inhomogeneous NLSE. Clearly, the stability of the latter follows from the stability of the former. We refer to such stable self-similar waves as *similaritons* and suggest that parabolic-shape similaritons that form asymptotically in fiber amplifiers [12], should be called *asymptotic similaritons*, because their

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stability is not guaranteed. We study both single- and multi-similariton solutions of Eq. (1). In particular, we show that the evolution of any multi-similariton solution can be interpreted as either an elastic similariton collision, or as the formation of a multi-similariton bound state. The latter scenario does not occur for solitons in exactly integrable systems, and hence it is *unique* for similaritons in such systems. The character of the similariton interaction is *universal* as it depends only on the sign of the overall chirp and asymptotic properties of the fiber's dispersion map, but not on the specific forms of $\beta(z)$ and $g(z)$.

2. General similariton solution

We begin by seeking a solution to Eq. (1) in the form

$$U(\tau,z) = A(z)\Psi\left[\frac{\tau-\tau_c(z)}{w(z)},\zeta(z)\right]\exp[i\Phi(\tau,z)], \tag{2}$$

where $A(z)$, $w(z)$, and $\tau_c(z)$ are the amplitude, width, and position of the pulse, respectively, and $\zeta(z)$ is an effective propagation distance yet to be determined. For a similariton, the phase front is generally parabolic (corresponding to a linearly chirped pulse) with the form

$$\Phi(\tau,z) = c(z)\tau^2/2 + b(z)\tau + d(z), \tag{3}$$

where $c(z)$ and $b(z)$ specify the curvature and the position of the center of the wavefront, respectively, and $d(z)$ is independent of τ. We stress that chirp is not an essential feature of similaritons considered in this paper, and chirp-free similaritons with $c = 0$ may exist.

Substituting from Eqs. (2) and (3) into the NLSE, Eq. (1), and gathering similar terms, we obtain a set of differential equations for the parameters describing the evolution of the pulse such that Ψ obeys the homogeneous NLSE

$$i\partial_\zeta\Psi \mp \frac{1}{2}\partial^2_{\chi\chi}\Psi + |\Psi|^2\Psi = 0, \tag{4}$$

where the upper (lower) sign corresponds to the case of normal (anomalous) dispersion and the similarity variable is defined as

$$\chi(\tau,z) = [\tau-\tau_c(z)]/w(z). \tag{5}$$

The differential equations governing the evolution of pulse parameters can be easily solved to obtain the following expressions for the effective propagation distance, width, amplitude, and the position of the pulse:

$$\zeta(z) = \frac{|D(z)|}{w_0^2[1-c_0D(z)]}, \qquad w(z) = w_0[1-c_0D(z)], \tag{6}$$

$$A(z) = w(z)^{-1}[|\beta(z)|/\gamma(z)]^{1/2}, \quad \tau_c(z) = \tau_0 - (c_0\tau_0 + b_0)D(z), \tag{7}$$

where $D(z) = \int_0^z ds\beta(s)$ represents the accumulated dispersion. The parameters related to the phase are given by

$$c(z) = \frac{c_0}{1-c_0D(z)}, \quad b(z) = \frac{b_0}{1-c_0D(z)}, \quad d(z) = \frac{(b_0^2/2)D(z)}{1-c_0D(z)}. \tag{8}$$

Further, the transformation of the inhomogeneous NLSE into the homogeneous one is only possible if the parameters of the medium satisfy the condition

$$g(z) = \beta(z)c(z) + \frac{d}{dz}\ln\left[\frac{\beta(z)}{\gamma(z)}\right]. \tag{9}$$

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This compatibility condition was first found in Ref. [11], and later in Ref. [13], where a few particular members of the similariton family were obtained using a different approach.

We note that a transformation from Eq. (1) to Eq. (4) with a structure similar to our Eq. (2), was previously applied in Ref. [19]. The difference, however, is that the transformation discovered by the authors of Ref. [19] using symbolic computer calculations is not optimal because it contains *six* free parameters, whose physical significance is rather opaque. By comparison, our transformation has only *four* free parameters with very clear physical meaning: c_0 and b_0 are the initial curvature and the position of the wavefront, τ_0 is the initial position of the pulse center, and w_0 is the initial similariton width.

Equations (4)–(8) show that any soliton of the homogenous NLSE is related to a similariton obeying the inhomogeneous NLSE with the compatibility condition given in Eq. (9). It follows from the exact integrability of the NLS equation [1] that all such similaritons must be stable. Moreover, they should survive mutual collisions just as solitons do, even though their width, amplitude, and chirp will keep changing during and after the collision. Further, higher-order solitons of the NLS equation correspond to multi-similariton solutions of Eq. (1).

In analogy with the standard solitons, we refer to similariton solutions of Eq. (1) as being bright or dark depending on whether the dispersion is anomalous or normal. It follows from the well-known single-soliton solution to the homogeneous NLSE [18] that fields of the fundamental bright and dark similaritons are given by

$$U_B(\chi,\zeta) = aA\,\mathrm{sech}[a(\chi - v\zeta)]e^{i(\Phi+\Theta_B)} \tag{10}$$

$$U_D(\chi,\zeta) = u_0[\cos(\phi)\tanh(\Theta_D) + i\sin(\phi)]e^{i(u_0^2\zeta+\Phi)}. \tag{11}$$

Here a and v are the amplitude and velocity of a bright soliton. In the case of a dark soliton, u_0 is the background amplitude obeying the homogeneous NLSE and ϕ corresponds to the total phase shift across the dark soliton. The corresponding phases of the bright (Θ_B) and dark (Θ_D) solitons are specified by the usual expressions [2]

$$\Theta_B(\chi,\zeta) = v\chi/2 + a^2\zeta - v^2/4, \tag{12}$$

$$\Theta_D(\chi,\zeta) = u_0\cos(\phi)[\chi - u_0\zeta\sin(\phi)]. \tag{13}$$

We note that, in the limiting case in which the similaritons are at rest ($v = 0$ and $\phi = 0$), our solution reduces to that previously reported in Ref. [13] with the particular initial condition $b_0 = -c_0\tau_0$.

3. Specific dispersion profiles

We now apply our general results to a few cases of practical interest. Consider first a fiber with z-dependent gain but constant values of β and γ. This case was first studied in Ref. [10] and it leads to a simple compatibility condition $g(z) = \beta c(z)$. If the initial chirp $c_0 = 0$, the fiber should have no gain (or loss) to satisfy this condition. Since all pulse parameters then become independent of z, we recover the standard solitons. On the other hand, if $c_0 \neq 0$, the compatibility condition is satisfied if the gain (or loss) of the fiber varies with z as [10]

$$g(z) = \frac{c_0\beta}{1 - c_0\beta z}. \tag{14}$$

The gain is needed when $c_0\beta > 0$. Thus, a fiber amplifier whose gain increases along the fiber length, as indicated above, supports bright similaritons when $\beta < 0$, and dark similaritons when $\beta > 0$, with the opposite types of chirps. The first row of Figure 1 shows the evolution of the fundamental bright and dark similaritons after choosing $|c_0| = 0.1$, $b_0 = 2$, and $\tau_0 = -5$. In

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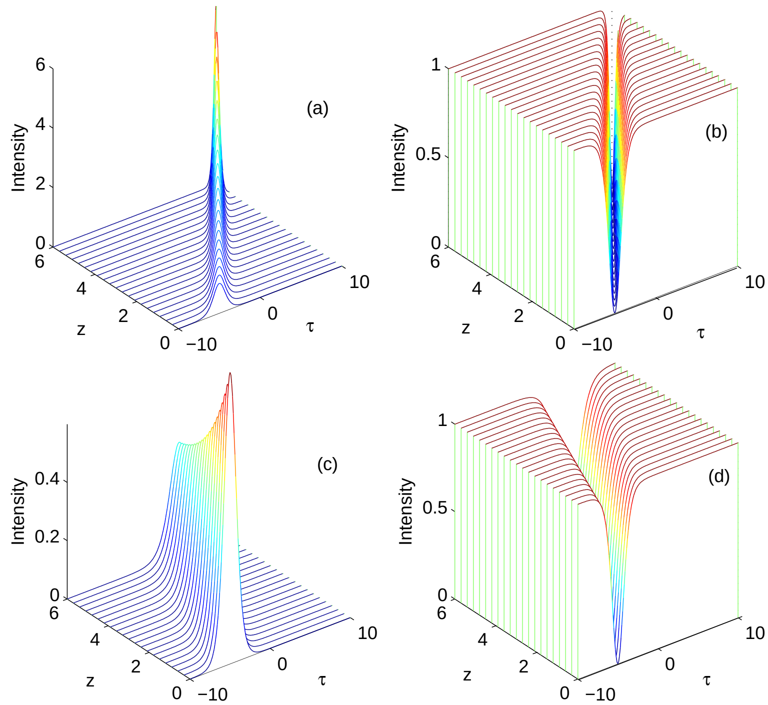

Fig. 1. Evolution of the fundamental (a) bright and (b) dark similaritons inside a fiber with constant dispersion. The sign of initial chirp of $|c_0| = 0.1$ ensures $c_0\beta > 0$. The initial chirp is reversed in parts (c) and (d) so that $c_0\beta < 0$ and the fiber exhibits loss instead of the gain.

the dark similariton case, we used $u_0 = 1$ and $\phi = 0$. In both cases, the pulse compresses as it is amplified, and its width decreases with z as $w(z) = w_0(1 - c_0\beta z)$. It is important to stress that the limit $c_0\beta z \to 1$ cannot be attained in practice not only because it requires infinite gain but also because the higher-order effects not included in Eq. (1) would stop the compression process well before this limit is reached.

The opposite case for which $c_0\beta < 0$ in Eq. (14) is also interesting. In this case, similaritons exist if fiber losses decrease with z, and the similariton width increases continuously. It is remarkable that stable similaritons can form in a lossy, constant-dispersion fiber, a possibility that does not appear to have been noticed so far. In practice, fiber losses can be made z-dependent by pumping the fiber suitably so that the Raman gain compensates for a part of the total loss. The second row of Figure 1 shows the evolution of the fundamental bright and dark similaritons in this case. All parameters are the same, but the sign of initial chirp has been reversed to ensure that $c_0\beta < 0$. As expected from the relation $w(z) = w_0(1 - c_0\beta z)$, the widths of both types of similaritons broaden by a factor of 1.6 at $z = 6$ because $c_0\beta = -0.1$ in this example. Note also that the pulse center shifts as dictated by $b(z)$ in Eq. (8) because $b_0 = 2$ in all cases shown in Fig. 1.

As another example, consider a fiber whose dispersion decreases exponentially along its length such that $\beta(z) = \beta_0 e^{-\sigma z}$. The new feature in this case is that the accumulated dispersion $D(z)$ does not keep increasing with z but approaches a constant value of β/σ at distances such that $\sigma z \gg 1$. As a result, the width of the similariton inside such an amplifier decreases initially but eventually becomes constant with a value $w_m = w_0(1 - c_0\beta_0/w_0^2\sigma)$. Figure 2 shows the gain profile $g(z)$ for which the compatibility condition is satisfied, together with $w(z)$, for

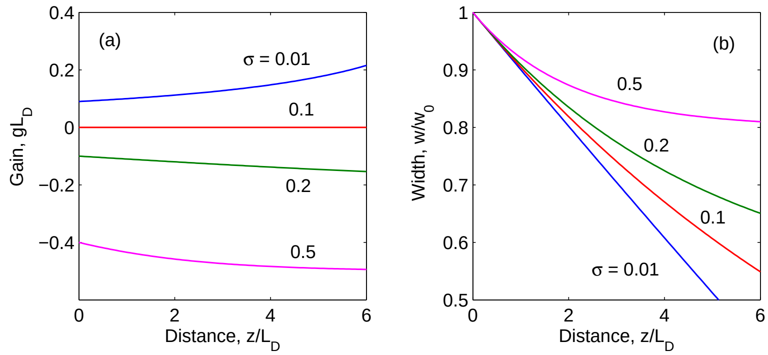

Fig. 2. (a). Amplifier gain $g(z)$ and (b) similariton width $w(z)$ as a function of z/L_D for several values of σ in the case $c_0 = -0.1$ and $\beta_2 < 0$.

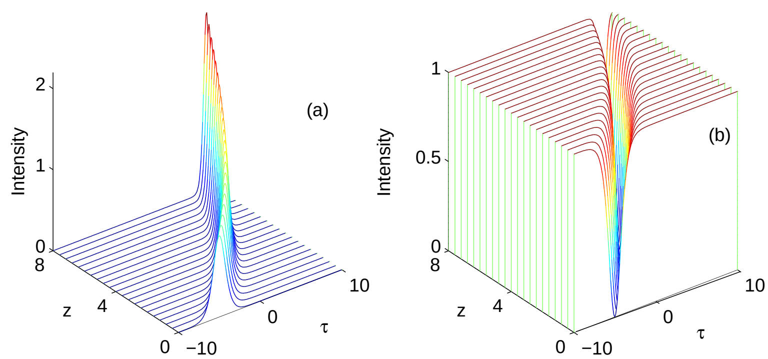

Fig. 3. Evolution of the fundamental (a) bright and (b) dark similaritons over $8L_D$ in the specific case of $\sigma = 0.1$.

$c_0 = -0.1$ and several values of σ with the dispersion length $L_D = w_0^2/|\beta_0|$ providing the length scale. The required gain or loss is relatively small and changes along the fiber length slowly. In particular, it vanishes for $\sigma = 0.1$. Note also that the required gain becomes negative for $\sigma > 0.1$, indicating again that similaritons can form in a lossy fiber. These features of dispersion-decreasing fibers are well known and have been discussed in Ref. [13]. Similaritons can undergo compression inside such a lossy fiber because decreasing dispersion provides an effective gain that can exceed fiber losses.

The evolution of the fundamental bright and dark similaritons over eight dispersion lengths is displayed in Fig. 3 for the specific case $\sigma = 0.1$ for which the required amplifier gain is zero. As before, pulses are chirped initially such that $|c_0| = 0.1$. All other parameters are identical to those used in Fig. 1. In both cases shown in Fig. 3, the fundamental similariton compresses and its position shifts because of the chirp imposed on it. A noteworthy feature is that the trajectory of the pulse center does not follow a straight line that was the case in Fig. 1. This observation can be understood from the expression for $b(z)$ in Eq. (8) and the fact that $D(z)$ tends to a constant for large values of z.

We now briefly discuss the common case of dispersion management in which two fibers with constant but opposite kinds of dispersion are combined in a periodic fashion to form a dispersion map [20]. Our solution applies to this case as well. In fact, the situation is similar to the constant-dispersion case discussed earlier, because the compatibility condition in Eq.

(9) reduces to $g(z) = \beta c(z)$ in each fiber section. The main difference is that, as the sign of β changes from one section to the next, the gain of fiber should also change sign, i.e, if the section with anomalous dispersion exhibits gain, the normal-dispersion section must be lossy. In the specific case in which the magnitude of dispersion is the same for all sections, $D(z)$ is a periodic function such that it returns to zero after each map period. As a result, the pulse width and chirp also vary in a periodic manner.

4. Elastic collisions of self-compressing similaritons

Since our similaritons are related to standard solitons through a simple transformation, we expect them not only to be stable but also to survive mutual collisions. To study collisions of similaritons, we focus on the specific case of two identical bright similaritons [$\beta(z) < 0$], separated initially in time, but moving with opposite velocities so that they begin to overlap and interact with each other. It follows from the standard soliton theory based on the inverse scattering transform [18] that the combined intensity profile of the two such similaritons can be written as

$$I(\zeta,\tau) = 2A^2(z)\frac{\partial^2}{\partial\chi^2}\ln(\det M), \tag{15}$$

where M is a matrix whose determinant is given by

$$\det M = 1 + e^{2(\eta_1+\eta_1^*+\delta_{11})} + e^{2(\eta_2+\eta_2^*+\delta_{22})} - e^{2(\eta_1+\eta_2^*+\delta_{12})} - e^{2(\eta_1^*+\eta_2+\delta_{12}^*)} + Ke^{4\mathrm{Re}(\eta_1+\eta_2)}, \tag{16}$$

$$K = e^{2(\delta_{11}+\delta_{22})} + e^{2(\delta_{12}+\delta_{12}^*)} - 2e^{(\delta_{11}+\delta_{22}+\delta_{12}+\delta_{12}^*)}. \tag{17}$$

The parameters η_j and δ_{jk} $(j,k = 1,2)$ are defined as

$$\eta_j = i\lambda_j(\chi+\lambda_j\zeta), \qquad e^{\delta_{jk}} = \begin{cases} |r_j|/(2\nu_j), & j = k; \\ (r_j r_k^*)^{1/2}/(\lambda_j - \lambda_k^*), & j \neq k. \end{cases} \tag{18}$$

Here, r_j are the residues of direct scattering data and $\lambda_j = \mu_j + i\nu_j$ $(j = 1,2)$ are the discrete eigenvalues specifying the amplitude and velocity of single-soliton solutions of Eq. (4).

Consider the behavior of the two-similariton field in the reference frame moving with the jth similariton. The trajectory of this similariton, $\eta_j + \eta_j^* = \text{const.}$, can be expressed as

$$\frac{\tau - \tau_0 + (c_0\tau_0 + b_0 - 2\mu_j/w_0)D(z)}{1 - c_0 D(z)} = \text{const.} \tag{19}$$

When $c_0 < 0$, each similariton compresses on propagation in a fiber with anomalous dispersion [$\beta(z) < 0$]. It follows from Eqs. (5), (18), and (19) that in the interval $0 \le z \le z_*$, $\eta_{3-j} \to \infty$ as $z \to z_*$, where z_* is a solution of the equation, $1 - |c_0 D(z_*)| = 0$. From Eqs. (15) and (16), the intensity profile of the jth similariton in the limit $z \to z_*$ has the asymptotic form

$$I_j(\chi,\zeta) \sim 4\nu_j^2 A^2(z)\,\mathrm{sech}^2[2\nu_j(\chi + 2\mu_j\zeta) + \Delta_j], \tag{20}$$

where the shift Δ_j of the similariton center position is given by

$$\Delta_j = -\frac{1}{2}\ln[e^{2\delta_{jj}} + e^{2(\delta_{12}+\delta_{12}^*-\delta_{3-j,3-j})} - 2e^{(\delta_{12}+\delta_{12}^*+\delta_{jj}-\delta_{3-j,3-j})}]. \tag{21}$$

Thus, similar to the soliton case, the self-compressing multi-similariton solutions can be interpreted in terms of elastic collisions of continuously evolving fundamental similaritons.

To illustrate numerically the behavior of similaritons upon collisions, we consider a simple nontrivial example of two colliding single-similariton pulses of equal amplitudes ($\nu_1 = \nu_2 =$

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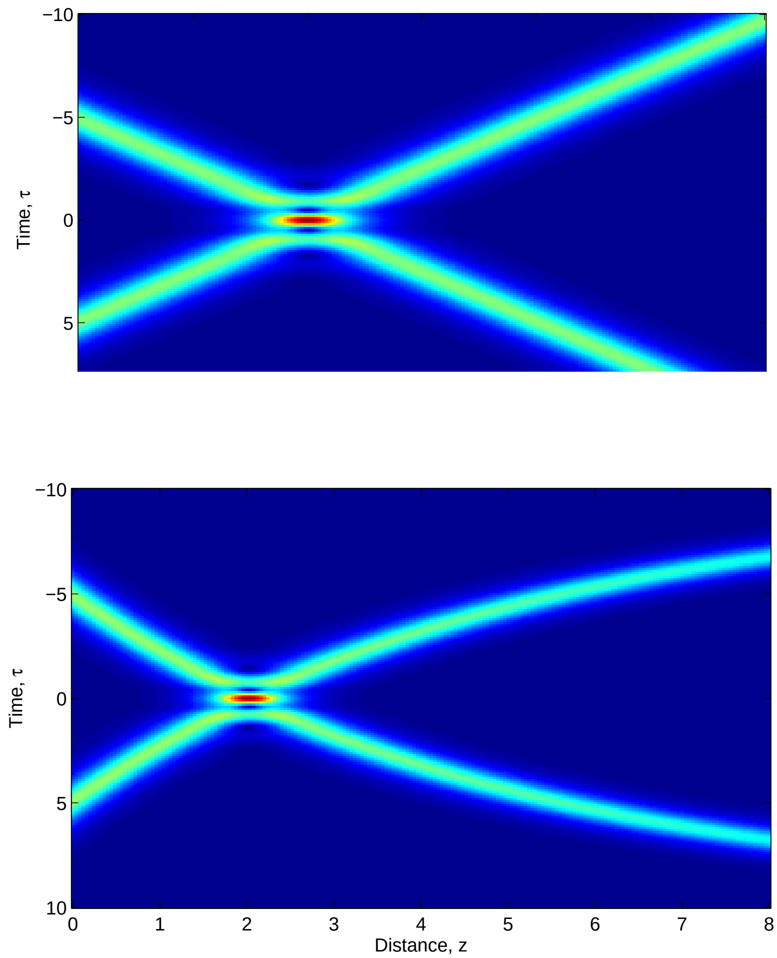

Fig. 4. Collision of two bright similaritons inside a constant-dispersion fiber (top) and a dispersion-decreasing fiber (bottom) with $\sigma = 0.2$. In both cases $c_0 = -0.1$.

ν) and phases traveling with the opposite velocities ($\mu_1 = -\mu_2 = \mu$). The initial positions of the similaritons are chosen such that $r_1 = r_2^* = (2\nu/\mu)(\mu + i\nu)$. Under such conditions, we have obtained an explicit analytical expression for the combined intensity profile of the two colliding similaritons, but it is too cumbersome to present here. We only point out that, under such conditions, the center position of each similariton suffers a shift, $\Delta = \ln(\sqrt{\mu^2+\nu^2}/\mu)$, during collision.

We now illustrate elastic collisions of similaritons for the two specific dispersion profiles of Section 3. Figure 4 displays the collision of two bright similaritons inside a constant-dispersion (top) and inside a dispersion-decreasing fiber with $\sigma = 0.2$ (bottom). Notice from Fig. 2 that the required gain is negative for the dispersion-decreasing fiber, i.e., the similaritons collide inside a lossy fiber. In both cases, we use $\mu = 1.2,\ \nu = 0.8$ and $c_0 = -0.1$. The collision is elastic in both cases and displays qualitative features that are well-known for standard soliton theory with only minor differences. In the case of constant dispersion, solitons follow a straight trajectory, before as well as after the collision. In contrast, the trajectories are curved for the dispersion-decreasing fiber.

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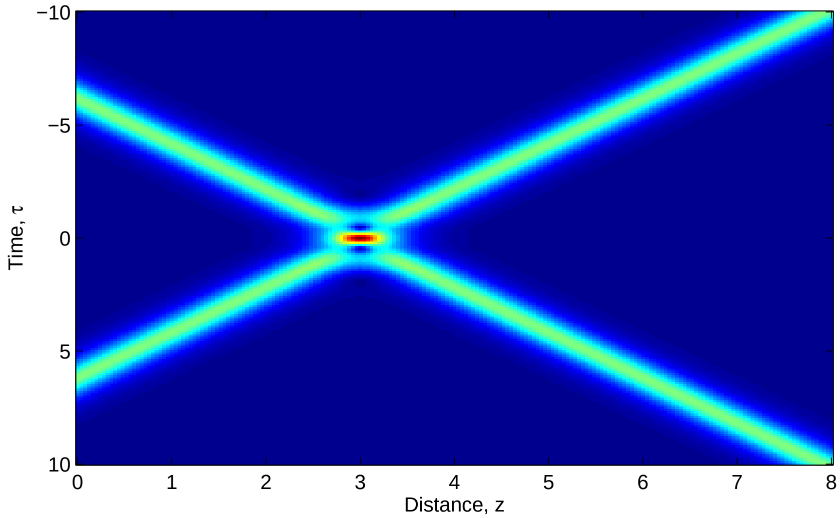

Fig. 5. Collision of two bright similaritons inside a constant dispersion fiber. The similariton parameters are $\mu = 1,\ \nu = 1$, and $c_0 = 0$.

5. Interactions of chirp-free similaritons

So far, we have focused on chirped similaritons. In the case of a chirp-free similariton ($c_0 = 0$), the similariton width does not change on propagation. The analysis of Eqs. (16) and (18) for this situation leads to the conclusion that, when the dispersion satisfies the asymptotic condition

$$\lim_{z\to\infty} D(z) = \infty, \tag{22}$$

the two-similariton field breaks into two single similaritons whose asymptotic intensity profiles are of the form

$$I_j(\tau,z) \sim 4\nu_j^2 A^2(z)\,\mathrm{sech}^2\left\{\frac{2\nu_j}{w_0}[\tau+\kappa_j D(z)]+\tilde{\Delta}_j\right\}, \tag{23}$$

where $\kappa_j = b_0 - 2\mu_j/w_0$ and $\tilde{\Delta}_j = \Delta_j - 2\nu_j\tau_0/w_0$. In this case, the similariton interaction can be interpreted as an elastic collision, and a specific case is illustrated in Fig. 5 for a fiber with constant dispersion using the parameters $\mu = 1$ and $\nu = 1$ with $c_0 = 0$. Recall that the similaritons reduce to standard solitons under such conditions and their width does not change during propagation.

If the asymptotic condition (22) is not met, the similariton interaction leads to the formation of a bound state of two similaritons. To illustrate such a situation, we consider again the case of a dispersion-decreasing fiber but choose a relatively large value $\sigma = 0.8$. In this case, $\lim_{z\to\infty} D(z) = \beta_0/\sigma$, and the asymptotic condition in Eq. (22) is not satisfied. The collision of two chirp-free similaritons in this case is shown in Fig. 6 using $\mu = 3,\ \nu = 1$, $\sigma = 0.8$, and $c_0 = 0$. This figure shows clearly that, after the two pulses collide, they become trapped and move together as one unit. We refer to such a state as a “similariton molecule,” bound together through the nonlinear interaction mediated by the cross-phase modulation. Notice that the compatibility condition now corresponds to $g(z) = -\sigma$, implying a constant loss along the fiber. This is generally the case in practice. Thus, we have discovered that stable similariton bound states can propagate inside a lossy fiber, provided fiber dispersion decreases along its length.

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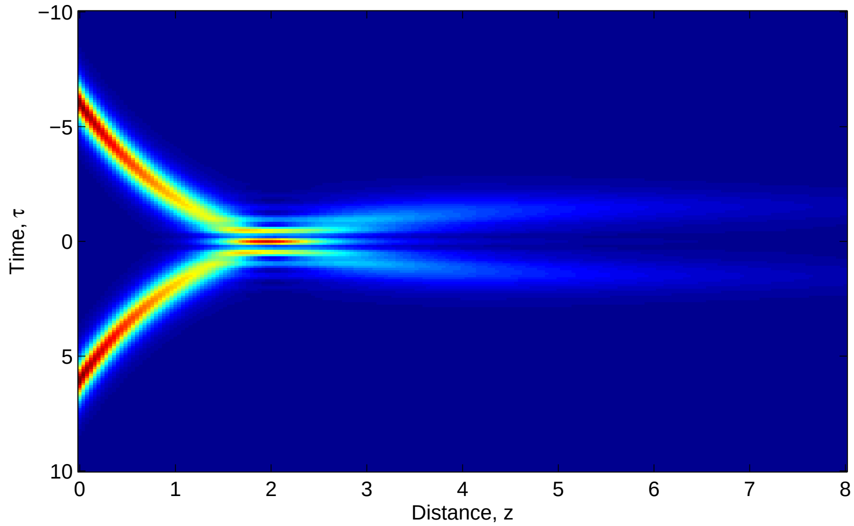

Fig. 6. Formation of a two-similariton bound state upon collision of two bright similaritons inside a dispersion-decreasing fiber with $\mu = 3,\ \nu = 1$, $\sigma = 0.8$, and $c_0 = 0$.

6. Bound states of two spreading similaritons

In the case of collision shown in Fig. 6, the tails of two emerging similaritons overlap, but each similariton still maintains its own identity. A very different situation occurs when two spreading similaritons collide inside a fiber such that the asymptotic condition (22) is not met. Indeed, our analysis of Eqs. (16) and (18) shows that a multi-similariton solution never breaks up into single similaritons in the case $c_0\beta(z) < 0$ in which the similariton width increases with distance, regardless of a particular form of the dispersion map.

We illustrate the bound-state formation of two similaritons in the case of a dispersion-decreasing fiber in Fig. 7 using the same parameter values as those used in Fig. 6, except that $c_0 = 0.2$. Since the fiber is assumed to exhibit anomalous dispersion, $c_0\beta(z) < 0$, and the width of similaritons increases as they collide. As is clearly seen in the figure, after the two similaritons collide, they fail to separate and form a bound state. The fringe pattern across the temporal profile of the two pulses results from their temporal overlapping. We stress that this behavior holds irrespective of a particular functional form of $\beta(z)$. In this sense, the evolution scenarios discussed in this paper are universal for all similaritons of Eq. (1).

7. Conclusions

In summary, we have modeled the propagation of optical pulses inside a fiber with nonuniform dispersion and gain by an inhomogeneous NLSE. We show through a similarity transformation that a one-to-one correspondence exists between the similaritons that are analytical solutions of this equation and the standard solitons that are analytical solutions of the homogeneous NLSE, provided a certain comparability condition is satisfied. This correspondence guarantees the stability of the novel bright and dark similaritons. As an example, we focus on two specific dispersion maps and discuss the different similariton features associated with them. In general, whenever the similaritons are chirped, their initial chirp c_0 plays an important role through the product $c_0\beta$. If this quantity is positive, the similaritons are self-compressed and amplified. In contrast, if $c_0\beta$ is negative, similaritons can form even in a lossy fiber, but they spread on propagation along the fiber.

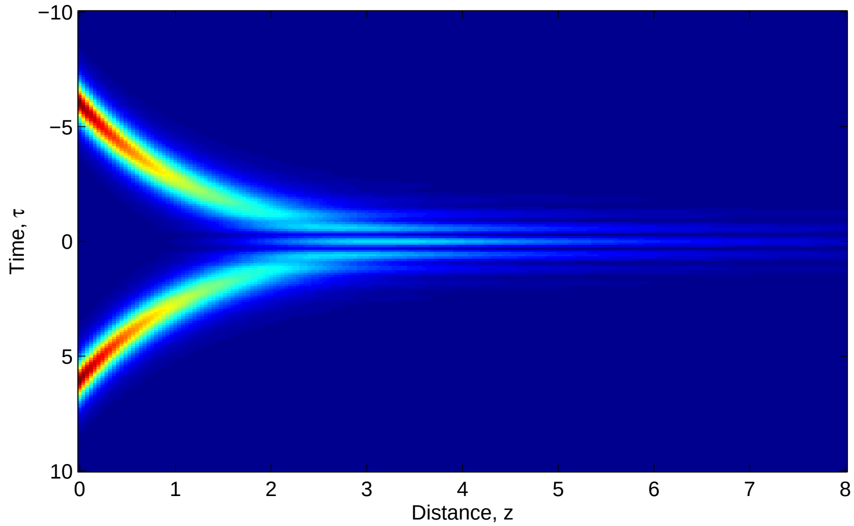

Fig. 7. Formation of a two-similariton bound state upon collision of two bright similaritons inside a dispersion decreasing fiber with $c_0 = 0.2$. All other parameters are identical to those used in Fig.6.

Our results indicate deep similarities as well as qualitative differences between solitons in homogeneous media and similaritons in inhomogeneous media. We show that, despite the exact integrability of the nonlinear system, the interaction between two such similaritons can result in either an elastic collision in which the two similaritons experience a temporal shift but separate from each other after the collision, or in the formation of a bound state of two similaritons representing a similariton molecule. Which scenario takes place in a specific case depends on the sign of the overall similariton chirp and the asymptotic properties of the medium dispersion map.

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