A Novel Design for Polarization-Independent Single-Pump Fiber-Optic Parametric Amplifiers

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***Abstract*—We propose a novel scheme for designing fiber-optic parametric amplifiers whose operation does not depend on the input signal polarization. In our scheme, four-wave mixing takes place inside a birefringent fiber pumped at 45° from a principal axis. We show numerically that output power varies by < 0.1 dB for arbitrary states of input polarization under practical conditions.**

***Index Terms*—Fiber birefringence, four-wave mixing (FWM), parametric amplifiers, polarization-mode dispersion.**

ALTHOUGH dual-pump fiber-optic parametric amplifiers (FOPAs) have become important in recent years [1], the single-pump configuration remains attractive because of its relative simplicity [2]–[4]. A major drawback of the single-pump configuration is that the efficiency of the underlying four-wave mixing (FWM) process depends critically on the relative polarization states of the pump and signal. Several schemes have been proposed to solve this problem. They make use of polarization diversity [2], a Faraday rotator [3], or a pulsed-pump configuration [4], and are complex enough to require a large number of additional optical components.

In this letter, we provide details of a relatively simple scheme that we discussed recently in a conference [5] and which employs a birefringent fiber for FWM. In our scheme, the pump beam is polarized at 45° from the slow (or fast) axis of the fiber, while the signal polarization can vary over the entire Poincaré sphere. Under such conditions, the pump power is divided equally between the two principal axes of the fiber. We use the vector theory of FWM to show that each pump component amplifies only the copolarized signal component because the orthogonal FWM process is not phase-matched as a result of large birefringence. Our results show that gain variations with respect to signal state of polarization (SOP) can be reduced to < 0.1 dB with our proposed scheme.

The starting point of our analysis is the vectorial form of nonlinear Schrödinger (NLS) equation. In the Jones-vector notation, it takes the form [6]

$$\begin{aligned}\frac{\partial|A\rangle}{\partial z} &= i\beta_{0p}|A\rangle + \frac{i}{2}\left(B_{0p} + iB_1\frac{\partial}{\partial t}\right)\sigma_1|A\rangle \\ &\quad + i\sum_{m=2}^{\infty}\frac{i^m\beta_m}{m!}\frac{\partial^m|A\rangle}{\partial t^m} \\ &\quad + i\gamma\left(\langle A\,|\,A\rangle - \frac{1}{3}\langle A|\sigma_3|A\rangle\sigma_3\right)|A\rangle \qquad (1)\end{aligned}$$

where $|A\rangle$ is Jones vector associated with the total optical field and the Pauli spin matrices are defined as

$$\sigma_1 = \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}, \quad \sigma_2 = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}, \quad \sigma_3 = \begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}. \qquad (2)$$

Here, $\beta_{0p} = \omega_p(n_x + n_y)/(2c)$ is the average propagation constant at the pump frequency ω_p, n_x and n_y being the effective mode indices along the slow and fast axes, respectively. The two birefringence parameters are defined as $B_{0p} = \delta n\omega_p/c$ and $B_1 = \delta n/c$, where $\delta n = n_x - n_y$. The differential group delay (DGD) for a fiber of length L is related to B_1 as $\Delta\tau = B_1 L$. In physical terms, DGD represents the relative delay between the two polarization components of a field propagating along the fast and slow axes. The coefficient β_m represents the mth order dispersion of the fiber at the pump wavelength. The nonlinear parameter γ is responsible not only for FWM but also for the self- and cross-phase modulations (SPM and XPM). In general, the zero-dispersion wavelength (ZDWL) of the fiber may be different along the two principle axes [7]. However, when fiber birefringence is low ($\delta n < 10^{-5}$), the difference between the two ZDWLs becomes small ($<$ 0.5 nm). Noting that the ZDWL of a fiber varies randomly along its length by $\sim$1 nm [8], this difference in the two ZDWLs should be relatively less detrimental [9].

Equation (1) can be used to describe all FWM processes induced inside a birefringent fiber by a single pump by writing the total field in the form $|A\rangle = |A_p\rangle + |A_s\rangle\exp(-i\Delta\omega t) + |A_i\rangle\exp(i\Delta\omega t)$, where $|A_s\rangle$ and $|A_i\rangle$ are the signal and idler fields at the frequencies ω_s and $\omega_i \equiv 2\omega_p - \omega_s$, respectively, and $\Delta\omega = \omega_s - \omega_p$ is the signal-pump detuning. If we neglect pump depletion and assume continuous-wave (CW) or quasi-CW conditions, the pump, signal, and idler fields are found to satisfy the following set of vector equations:

$$\begin{aligned}\frac{\mathrm{d}|A_p\rangle}{\mathrm{d}\,z} &= i\left(\beta_{0p} + \frac{B_{0p}}{2}\sigma_1\right)|A_p\rangle \\ &\quad + \frac{i\gamma}{3}\left(2\langle A_p\,|\,A_p\rangle + |A_p^*\rangle\langle A_p^*|\right)|A_p\rangle, \qquad (3)\\ \frac{\mathrm{d}|A_s\rangle}{\mathrm{d}\,z} &= i\left(\beta_{0s} + \frac{B_{0s}}{2}\sigma_1\right)|A_s\rangle \\ &\quad + \frac{2i\gamma}{3}(\langle A_p\,|\,A_p\rangle + |A_p\rangle\langle A_p| + |A_p^*\rangle\langle A_p^*|)|A_s\rangle \\ &\quad + \frac{i\gamma}{3}(\langle A_p^*\,|\,A_p\rangle + 2|A_p\rangle\langle A_p^*|)|A_i^*\rangle \qquad (4)\end{aligned}$$

where β_{0s} and $B_{0s} \equiv B_{0p} + \Delta\omega B_1$ are the propagation constant and fiber birefringence at the signal frequency, respectively. The equation for the idler field can be written by interchanging the subscripts s and i in (4). The signal and idler equations include the XPM effects (the first three terms), as well as different

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FWM processes through the last two terms. Some of these processes lead to polarization-sensitive gain but their impact can be reduced by noting that each FWM process has a different phase-matching condition.

Equation (3) shows that pump polarization changes because of fiber birefringence, as well as XPM-induced coupling between the two polarization components of the pump. However, when the beat length ($L_B = 2\pi/B_{0p}$) of the fiber is much shorter than the nonlinear length [$L_{\rm NL} = (\gamma P_0)^{-1}$], the XPM-induced polarization changes can be neglected. With this simplification, solution of the pump equation is given by

$$|A_p(z)\rangle = \exp\Big[i(B_{0p}z/2)\sigma_1 + \frac{i\gamma z}{6}(5\langle A_p\,|\,A_p\rangle + \langle A_p|\sigma_1|A_p\rangle\sigma_1)\Big]|A_p(0)\rangle. \quad (5)$$

In the Stokes space, pump polarization rotates around the x axis, leaving the x component of the Stokes vector unchanged and indicating that pump power along each principal axis remains constant.

To see the origin of various FWM processes explicitly, we introduce a simple change of variables for the signal and idler fields as

$$\begin{pmatrix} u_k \\ v_k \end{pmatrix} = \exp\left[\frac{iz}{2}B_{0s}\sigma_1 + \frac{2i}{3}\gamma z(2\langle A_p\,|\,A_p\rangle + \langle A_p|\sigma_1|A_p\rangle\sigma_1)\right]|A_k\rangle \quad (6)$$

where u_k and v_k ($k = s$ or i) are the polarization components along the fast and slow axes. Inserting (5) and (6) into (4), the individual polarization components of the signal are found to evolve as

$$\begin{aligned}\frac{du_s}{dz} &= i\gamma\left(u_p^2u_i^* + \frac{1}{3}v_p^2u_i^*e^{-2iz/L_B} + \frac{2}{3}u_pv_pv_i^*e^{i(\Delta\tau\Delta\omega)z/L}\right)e^{-i\kappa z} \\ &+ \frac{2i\gamma}{3}u_pv_p^*v_se^{i(\Delta\tau\Delta\omega)z/L} \\ &+ \frac{2i\gamma}{3}u_p^*v_pv_se^{-2iz/L_B}\end{aligned} \quad (7)$$

$$\begin{aligned}\frac{dv_s}{dz} &= i\gamma\left(v_p^2v_i^* + \frac{1}{3}u_p^2v_i^*e^{-2iz/L_B} + \frac{2}{3}v_pu_pv_i^*e^{-i(\Delta\tau\Delta\omega)z/L}\right)e^{-i\kappa z} \\ &+ \frac{2i\gamma}{3}v_pu_p^*u_se^{-i(\Delta\tau\Delta\omega)z/L} \\ &+ \frac{2i\gamma}{3}v_p^*u_pu_se^{-2iz/L_B}\end{aligned} \quad (8)$$

where $\kappa = \Delta k + \gamma P_0$ is the phase-mismatch parameter and $\Delta k = 2\sum_{m=1}^{\infty}\beta_{2m}\Delta\omega^{2m}/(2m!)$ is its linear part originating from fiber dispersion. Equations for the idler components can be found by interchanging the subscripts s and i. In deriving (7) and (8), pump powers along the two principal axes are chosen to be equal at the input end, i.e., $|u_p|^2 = |v_p|^2 = P_0/2$.

We can easily identify different nonlinear processes in (7) and (8). The first three terms in the parentheses are the FWM terms; they transfer energy from the pump to the signal and idler as long as the phases associated with these terms do not vary considerably over one nonlinear length. The first FWM term couples the pump, signal, and idler components copolarized along the same axis. Since each component of signal is amplified independently by the same amount, this term yields polarization-insensitive gain. Its phase-matching condition $\kappa = 0$ is independent of fiber birefringence and leads to relatively broad gain bandwidth in the vicinity of the pump frequency. It would provide the dominant contribution and make the signal gain polarization-independent if one ensures that this FWM process is the only phase-matched process.

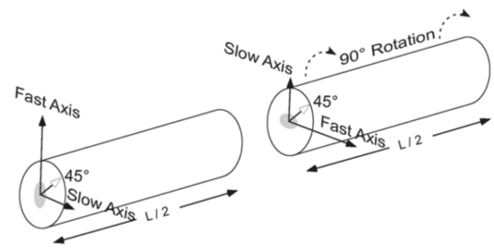

Fig. 1. Schematic of the proposed scheme. A linearly polarized pump is launched at 45° from each principal axis. The second half of the fiber is rotated by 90° and spliced to the first half.

The second FWM term transfers energy from the pump to the copolarized signal and idler components that are orthogonally polarized to the pump. The phase-matching condition for this process, $\kappa \pm 2/L_B = 0$, depends on fiber birefringence and cannot be satisfied in the vicinity of the pump frequency where the first FWM process is phase-matched. The third FWM term involves both polarization components of pump, and energy is transferred from them to the signal and idler components that are orthogonally polarized. Such a FWM process leads to polarization-dependent gain. The phase-matching condition for this process, $\kappa \pm (\Delta\omega\Delta\tau/L) = 0$, depends on birefringence through the DGD $\Delta\tau$. Both of these processes can be nearly eliminated by choosing a fiber with birefringence large enough that $\delta n \gg \gamma P_0 c/\Delta\omega$.

The last two terms in (7) involve only the pump and the signal, and they lead to polarization-dependent XPM. Combined with other FWM processes, these terms can also contribute to polarization-dependent gain by transferring energy from one component of the signal to the other one. However, when fiber has large DGD, these terms cannot be phase-matched. For the same reason, it is possible to realize polarization-insensitive XPM in a highly birefringent fiber [10]. When DGD is large, the only process that satisfies phase matching in the vicinity of the pump frequency is the first FWM term. Retaining only this term in (7) and (8), we follow a standard procedure [6] to find that the signal gain is given by:

$$G(\omega_s) = 1 + \frac{\sinh^2(gL/2)}{(gL_{\rm NL})^2}, \qquad g = \sqrt{(\gamma P_0)^2 - \kappa^2}. \quad (9)$$

One issue remains to be addressed. The large amount of DGD, required for polarization-insensitive gain, would split the polarization components of the signal (and idler) in the time domain. This may not be desirable in practice, especially when signal is in the form of pulses. However, in practice, this problem can be solved with a simple trick. The basic idea is shown in Fig. 1 and consists of cutting the birefringent fiber in half, rotating the second half by 90° so that its slow and fast axes are reversed,

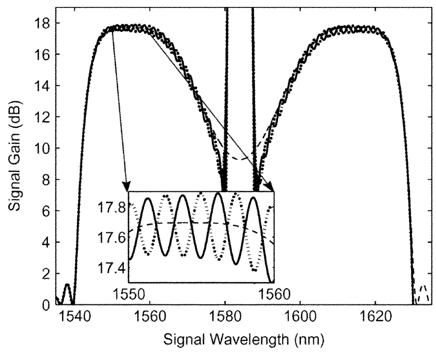

Fig. 2. Signal gain as a function of wavelength. The solid and dotted curves correspond to signal polarizations that are parallel and orthogonal to the pump, respectively. Dashed curve shows the gain predicted by (9). The inset shows the gain on a magnified scale.

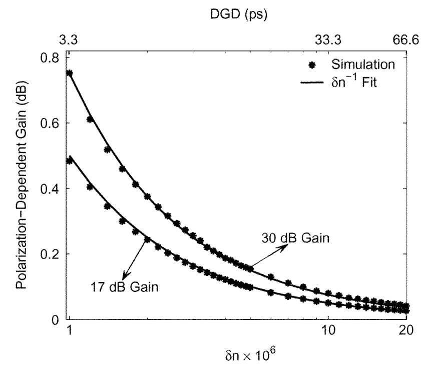

Fig. 3. Polarization-dependent gain as a function of fiber birefringence for two vales of peak gain. The top scale shows the corresponding differential group delay. The solid curves show a fit based on the inverse dependence of PDG on δn.

and splicing it back to the first half. In the second section, signal still experiences polarization-insensitive gain (because pump is still oriented at 45° with respect to the principal axes), but the DGD induced in the first section is completely compensated inside this section. Of course, one can also employ a precompensation or postcompensation scheme in which the DGD is compensated, before or after the FOPA, by sending the signal (and idler) pulses through an unpumped birefringent fiber that has the same magnitude of DGD but its principal axes are rotated by 90° from the FOPA fiber.

To test the validity of our simple analytical expression given in (9), we solved (1) numerically. The FOPA parameters used in the simulations correspond to a realistic, highly nonlinear fiber and are $L = 1$ km, $\gamma = 17$ W^{-1}km^{-1}, $\beta_3 = 0.055$ ps^3/km, and $\beta_4 = 2.35 \times 10^{-4}$ ps^4/km. The zero-dispersion wavelength of the fiber is at 1583.5 nm and its birefringence $\delta n = 10^{-6}$ corresponds to a beat length of 1.5 m. Birefringence is kept low intentionally so that residual effects of unwanted FWM processes are visible. The pump is in the form of quasi-CW pulses with a peak power of 320 mW at a wavelength of 1583.7 nm.

Fig. 2 shows the FOPA gain as a function of signal wavelength. The central peak is at the location of the pump. The solid and dotted curves show the maximum and the minimum gain occurring when signal polarization is parallel or perpendicular to the pump polarization, respectively. Signal gain lies in between these two curves for other polarization states. The dashed curve shows, for comparison, the signal gain predicted by (9). Clearly, a nearly polarization-independent gain is achieved even for a fiber birefringence as low as $\delta n = 10^{-6}$.

The inset in Fig. 2 shows that a small amount of polarization dependence remains in the form of small-scale ripples. Its magnitude can be quantified through a quantity, called the polarization-dependent gain (PDG) and defined as the ratio of the maximum to the minimum signal gain. As seen in the inset, PDG remains below 0.5 dB across the gain peak. To study the dependence of PDG on birefringence, we changed δn from 10^{-6} to 10^{-5}. Our results show that the amplitude of ripples (hence PDG) decreases and their frequency increases as fiber birefringence is increased. Fig. 3 shows PDG as a function of fiber birefringence. The upper curve corresponds to the same FOPA pumped with 500 mW of pump power, producing 30 dB peak gain. We find that PDG scales inversely with fiber birefringence (on a dB scale) and reduces to < 0.05 dB when fiber birefringence exceeds 10^{-5}. Solid curves show the fit obtained using an inverse dependence of PDG on δn.

In conclusion, we have presented a novel and simple scheme for making a single-pump FOPA exhibiting negligible PDG (<0.1 dB). It makes use of only a birefringent fiber, and thus is not affected by the polarization-mode dispersion effects [11]. Although the single-pump configuration proposed here can provide the same gain as the dual-pump one, for the same total pump power, it would suffer from idler broadening induced by pump phase modulation. However, our scheme can also be applied to the dual pump-configuration that does not suffer from idler broadening.

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