Changes in the spectrum, in the spectral degree of polarization, and in the spectral degree of coherence of a partially coherent beam propagating through a gradient-index fiber

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Expressions are derived for the cross-spectral density matrix of an electromagnetic Gaussian Schell-model beam propagating through a paraxial $\mathcal{ABCD}$ system. Using the recently developed unified theory of coherence and polarization of electromagnetic beams and the $\mathcal{ABCD}$ matrix for gradient-index fibers, we study the changes of the spectral density, of the spectral degree of polarization, and of the spectral degree of coherence of such a beam as it travels through the fiber. Effects of material dispersion are also considered. © 2006 Optical Society of America

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1. INTRODUCTION

It has been known for some time that the spectral degree of polarization of an electromagnetic Gaussian Schell-model (EGSM) beam may change on propagation even in free space.[1–3] Using the recently formulated unified theory of coherence and polarization,[4] changes in the degree of polarization of an EGSM beam propagating through turbulence[5,6] and through random phase screens[7] has been investigated. In the present paper we apply the theory to examine the changes in the spectral, polarization, and coherence properties of an EGSM beam as the beam propagates through a gradient-index (GRIN) fiber.

We consider a GRIN fiber that is characterized by a parabolic refractive-index profile, symmetric about its axis. Such fibers exhibit lower pulse dispersion than their step-index counterparts and hence have been used extensively in optical communication systems. Plastic GRIN fibers are currently of interest for local area networks and for short-haul data communication. Due to its parabolic refractive-index profile, a GRIN fiber is an inhomogenous, isotropic medium. The cross-spectral density function of an optical field in the fiber, which characterizes the second-order correlation properties of the beam traveling through it, satisfies, in the scalar model, two inhomogenous Helmholtz equations. An integral equation for the cross-spectral density of a scalar wave propagating through a GRIN fiber was derived in Refs. 8 and 9 by the use of modal analysis. It is known that the modes of a parabolic-index medium are Hermite–Gauss functions.[10] A closed-form expression for the spectrum of the field generated by a *scalar* Gaussian Schell-model source at an arbitary distance within a GRIN fiber was derived in Ref. 11 on the basis of formalism developed in Ref. 8. Expressions for the spectrum may also be derived by using the generalized Huygen–Fresnel integral for paraxial wave propagation through an $\mathcal{ABCD}$ system.[12,13]

In the present paper, the generalized Huygens–Fresnel formalism is used to derive explicit expressions for the elements of the 2×2 cross-spectral density matrix of an *electromagnetic* Gaussian Schell-model beam propagating through a GRIN fiber. The beam we consider is characterized by a general set of parameters, which makes it possible to exploit the full richness of this broad class of beams. The knowledge of the cross-spectral density matrix in any plane orthogonal to the direction of propagation (the fiber axis) then makes it possible to determine the spectral density, the spectral degree of polarization, and the spectral degree of coherence of the field within the fiber.

2. ELECTROMAGNETIC GAUSSIAN SCHELL-MODEL BEAM

Consider a random electromagnetic beam propagating close to the z direction. Let $\{\mathbf{E}(\mathbf{r},\omega)\} \equiv \{E_x(\mathbf{r},\omega), E_y(\mathbf{r},\omega)\}$ be a statistical ensemble of the fluctuating component of the transverse electric vector in mutually orthogonal x and y directions, at a point $P(\mathbf{r})$, $[\mathbf{r} \equiv (\boldsymbol{\rho}, z)$, see Fig. 1] at frequency ω. The second-order correlation properties of the beam can then be characterized by the 2×2 electric cross-spectral density matrix[4]

$$\overleftrightarrow{\mathbf{W}}(\mathbf{r}_1,\mathbf{r}_2,\omega) \equiv [W_{ij}(\mathbf{r}_1,\mathbf{r}_2,\omega)] = [\langle E_i^*(\mathbf{r}_1,\omega)E_j(\mathbf{r}_2,\omega)\rangle], \quad (i=x,y;\ j=x,y), \tag{1}$$

where the asterisk denotes the complex conjugate and the angle brackets denote the average taken over the ensemble of the electric field.

A scalar Gaussian Schell-model source (Ref. 14, Chap. 5) is one of the simplest and most widely employed secondary source models used for studying propagation-induced changes in correlation properties of the beam that the source generates (see, for example, Refs. 15–17). A vector generalization of the scalar GSM source was introduced in Ref. 18 and was discussed more fully in Ref. 19. Methods of generating such sources were proposed in Refs. 20 and 21. The electromagnetic GSM source considered in our analysis is characterized by the most general set of parameters.

The elements of the cross-spectral density matrix of the field generated by a planar, secondary electromagnetic Gaussian Schell-model source located in the plane $z=0$ (which we will refer to as the source plane), are given by the expressions

$$W_{ij}^{(0)}(\boldsymbol{\rho}_1',\boldsymbol{\rho}_2',\omega) = \sqrt{S_i^{(0)}(\boldsymbol{\rho}_1',\omega)}\sqrt{S_j^{(0)}(\boldsymbol{\rho}_2',\omega)}\,\mu_{ij}^{(0)}(\boldsymbol{\rho}_2'-\boldsymbol{\rho}_1',\omega), \quad (i=x,y;\ j=x,y). \tag{2}$$

Here $\boldsymbol{\rho}_1'$ and $\boldsymbol{\rho}_2'$ are two-dimensional position vectors of points in the source plane, $S_i^{(0)}$ are the spectral densities of the x and y components of the electric field, and $\mu_{ij}^{(0)}(\boldsymbol{\rho}_2'-\boldsymbol{\rho}_1')$ are coefficients that represent the correlation between the Cartesian components $E_i(\boldsymbol{\rho}_1',\omega)$ and $E_j(\boldsymbol{\rho}_2',\omega)$ of the electric field at the points $\boldsymbol{\rho}_1'$ and $\boldsymbol{\rho}_2'$. In Eq. (2), the spectral density functions and the correlation coefficients are taken to be Gaussian functions; i.e., they are of the form

$$S_i^{(0)}(\boldsymbol{\rho}',\omega) = A_i^2 \exp\left(-\frac{\boldsymbol{\rho}'^2}{2\sigma_i^2}\right), \quad (i=x,y), \tag{3}$$

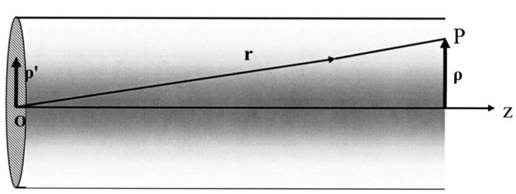

Fig. 1. Illustration of the notation.

$$\mu_{ij}^{(0)}(\boldsymbol{\rho}_2'-\boldsymbol{\rho}_1',\omega) = B_{ij}\exp\left(-\frac{|\boldsymbol{\rho}_2'-\boldsymbol{\rho}_1'|^2}{2\delta_{ij}^2}\right), \quad (i=x,y;\ j=x,y). \tag{4}$$

The parameters A_i, B_{ij}, σ_i, and δ_{ij} are independent of position but may depend on the frequency, and $B_{ii}=1$. Hence, the elements of the cross-spectral density matrix of the field generated by such an EGSM beam in the plane $z=0$ are given by

$$W_{ij}^{(0)}(\boldsymbol{\rho}_1',\boldsymbol{\rho}_2',\omega) = A_iA_jB_{ij}\exp\left(-\frac{\boldsymbol{\rho}_1'^2}{4\sigma_i^2} - \frac{\boldsymbol{\rho}_2'^2}{4\sigma_j^2}\right) \times \exp\left(-\frac{|\boldsymbol{\rho}_2'-\boldsymbol{\rho}_1'|^2}{2\delta_{ij}^2}\right), \quad (i=x,y;\ j=x,y). \tag{5}$$

The parameters characterizing the source may not all be chosen arbitrarily. The nonnegative definiteness of the cross-spectral density matrix imposes constraints on the values of these parameters. There are additional constraints due to the requirement that the source generates a beam.[22] The necessary and sufficient conditions, that the parameters of the source must satisfy in order that the source generate a physically realizable beam have been recently derived.[19] A method for the synthesis of EGSM beams has also been proposed. It makes use of two mutually correlated phase-only liquid-crystal spatial light modulators.[21]

3. PROPAGATION OF THE CROSS-SPECTRAL DENSITY MATRIX THROUGH A GRIN FIBER

Consider a GRIN fiber with its axis of symmetry along the z axis, whose spatial dependence of the square of the index of refraction has a parabolic profile, i.e., is of the form

$$\tilde{n}^2(x,y;\omega) = \begin{Bmatrix} n_0^2(\omega)[1-\alpha^2(\omega)(x^2+y^2)], & x^2+y^2 \leqslant R_0^2 \\ n_0^2(\omega)[1-\alpha^2(\omega)R_0^2], & x^2+y^2 \geqslant R_0^2 \end{Bmatrix}. \tag{6a}$$

Here R_0 is the core radius, $n_0(\omega)$ is the refractive index at the center of the fiber, $\alpha(\omega)$ is the radial gradient of the refractive index, and $k=\omega/c$, c being the speed of light in vacuum. If $n_1(\omega)$ is the refractive index on the axis of the fiber and $n_2(\omega)$ is the refractive index at the boundary of the fiber, the radial gradient of the refractive index is given by the expression

$$\alpha(\omega) = \frac{1}{R_0}\left[1-\frac{n_2^2(\omega)}{n_1^2(\omega)}\right]^{1/2}. \tag{6b}$$

The $\mathcal{ABCD}$ matrix for paraxial ray propagation through a GRIN fiber between planes $z=0$ and $z=\text{const.}>0$ is given by[12]

$$\begin{bmatrix} \mathcal{A} & \mathcal{B} \\ \mathcal{C} & \mathcal{D} \end{bmatrix} = \begin{bmatrix} \cos(\alpha z) & \sin(\alpha z)/n\alpha \\ -n\alpha\sin(\alpha z) & \cos(\alpha z) \end{bmatrix}. \tag{7}$$

Let us consider an electromagnetic beam of any state of polarization and coherence propagating close to the z axis in the fiber, in the positive z direction. A typical member of the statistical ensemble of the electric field $\mathbf{E}(\boldsymbol{\rho},z,\omega)$ at any point P, specified by the position vector $\mathbf{r}\equiv(\boldsymbol{\rho},z)$, may be determined from the knowledge of the field $\mathbf{E}^{(0)}(\boldsymbol{\rho}',\omega)$ in the source plane $z=0$ by using the propagation law applicable to a paraxial $\mathcal{ABCD}$ system[12,13,23,24]:

$$\mathbf{E}(\boldsymbol{\rho},z;\omega)=-\frac{ik\exp(ikz)}{2\pi z}\int\int\mathbf{E}^{(0)}(\boldsymbol{\rho}';\omega)\times\exp\left[\frac{ik}{2\mathcal{B}}(\mathcal{D}\rho^2-2\boldsymbol{\rho}\cdot\boldsymbol{\rho}'+\mathcal{A}\rho'^2)\right]\mathrm{d}^2\rho'. \quad (8)$$

In Eq. (8), $\boldsymbol{\rho}'$ and $\boldsymbol{\rho}$ are, of course, the transverse position vectors that specify the location of the points in the plane $z=0$ and $z=\text{const.}>0$ respectively, $\mathcal{A},\mathcal{B},\mathcal{D}$ being the elements of matrix (7) which depend on z.

The propagation kernel in Eq. (8), with the substitution of the matrix elements from Eq. (7) is formally identical with Eq. (17) of Ref. 8, which was derived on the basis of modal expansion of the field supported by an infinite square-law medium [i.e., with $R_0\to\infty$ in Eq. (6)]. For such a medium, the index of refraction assumes unrealistic values at points sufficiently far from the axis. However, for a multimode fiber, when a large number of low-order modes are excited (a requirement satisfied for paraxial propagation), the analysis of the infinitely extended refractive-index profile gives results in close agreement with those obtained by using the truncated profile.[24]

With $\mathbf{E}(\boldsymbol{\rho},z,\omega)$ given by Eq. (8), one obtains from Eq. (1) the following expression for the elements of cross-spectral density matrix of the electric field at points $\mathbf{r}_1=(\boldsymbol{\rho}_1,z)$ and $\mathbf{r}_2=(\boldsymbol{\rho}_2,z)$ in the plane $z=\text{const.}$:

$$W_{ij}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,z;\omega)=\left(\frac{k}{2\pi B}\right)^2\int\int\mathrm{d}^2\rho_1'\int\int\mathrm{d}^2\rho_2'W_{ij}^{(0)}(\boldsymbol{\rho}_1',\boldsymbol{\rho}_2';\omega)\times\exp\left[-\frac{ik}{2\mathcal{B}}(\mathcal{A}(\rho_1'^2-\rho_2'^2)\ -2(\boldsymbol{\rho}_1\cdot\rho_1'-\boldsymbol{\rho}_2\cdot\boldsymbol{\rho}_2')+\mathcal{D}({\rho_1}^2-{\rho_2}^2)\right]. \quad (9)$$

Here $W_{ij}^{(0)}(\boldsymbol{\rho}_1',\boldsymbol{\rho}_2',\omega)=\langle E_i^{(0)*}(\boldsymbol{\rho}_1',\omega)E_j^{(0)}(\boldsymbol{\rho}_2',\omega)\rangle$, $(i=x,y;\ j=x,y)$, are, of course, elements of the cross-spectral density matrix in the plane $z=0$. For the field generated by an EGSM source, they are given by Eq. (5).

The spectral density (sometimes called spectral intensity) $S(\mathbf{r};\omega)$ at any point P, specified by the position vector $\mathbf{r}$ in the half-space $z>0$, at frequency ω, is given by the formula

$$S(\mathbf{r};\omega)\equiv\mathrm{Tr}\,\overleftrightarrow{\mathbf{W}}(\mathbf{r},\mathbf{r};\omega)=W_{xx}(\mathbf{r},\mathbf{r};\omega)+W_{yy}(\mathbf{r},\mathbf{r};\omega), \quad (10)$$

where $\mathrm{Tr}\,\overleftrightarrow{\mathbf{W}}$ denotes trace of the $\overleftrightarrow{\mathbf{W}}$ matrix. Knowledge of the spectral density makes it possible to study the broadening of a beam and the spectral shifts that may occur on propagation through the GRIN fiber. The spectral degree of coherence of the electric field at any two points $P(\mathbf{r}_1)$ and $P(\mathbf{r}_2)$ may be determined by the use of the formula [Ref. 4, Eq. 8]

$$\eta(\mathbf{r}_1,\mathbf{r}_2,\omega)\equiv\frac{\mathrm{Tr}\,\overleftrightarrow{\mathbf{W}}(\mathbf{r}_1,\mathbf{r}_2,\omega)}{\sqrt{\mathrm{Tr}\,\overleftrightarrow{\mathbf{W}}(\mathbf{r}_1,\mathbf{r}_1,\omega)}\sqrt{\mathrm{Tr}\,\overleftrightarrow{\mathbf{W}}(\mathbf{r}_2,\mathbf{r}_2,\omega)}}=\frac{W_{xx}(\mathbf{r}_1,\mathbf{r}_2,\omega)+W_{yy}(\mathbf{r}_1,\mathbf{r}_2,\omega)}{\sqrt{S(\mathbf{r}_1,\omega)}\sqrt{S(\mathbf{r}_2,\omega)}}. \quad (11)$$

Evidently, to determine the behavior of the spectral density and the spectral degree of coherence of the electric field in the half-space $z>0$, we need to calculate the diagonal elements of the cross-spectral density matrix from Eq. (9). The off-diagonal elements do not contribute to the degree of coherence, as is clear from the formula (11). They do, however, contribute to the spectral degree of polarization $P(\mathbf{r},\omega)$, which is given by the expression [see Ref. 4, Eq. 11]

$$P(\mathbf{r},\omega)=\sqrt{1-\frac{4\,\mathrm{Det}\,\overleftrightarrow{\mathbf{W}}(\mathbf{r},\mathbf{r},\omega)}{[\mathrm{Tr}\,\overleftrightarrow{\mathbf{W}}(\mathbf{r},\mathbf{r},\omega)]^2}}, \quad (12)$$

where $\mathrm{Det}\,\overleftrightarrow{\mathbf{W}}$ denotes the determinant of the $\overleftrightarrow{\mathbf{W}}$ matrix.

4. CROSS-SPECTRAL DENSITY MATRIX OF A GENERAL ELECTROMAGNETIC GAUSSIAN SCHELL-MODEL BEAM PROPAGATING THROUGH A GRIN FIBER

We will now apply some of the preceding formulas to the situation when the beam that propagates through the fiber is an EGSM beam. The elements of the cross-spectral density matrix of an EGSM beam propagating through a GRIN fiber may be determined by substituting from Eq. (5) into Eq. (9) and then substituting from Eq. (7) the values of the matrix elements $\mathcal{A},\mathcal{B},\mathcal{C}$, and $\mathcal{D}$ for the GRIN fiber.

On substituting from Eq. (5) into Eq. (9) we obtain for the elements of the cross-spectral density matrix the expression

$$W_{ij}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,z;\omega)=\left(\frac{k}{2\pi\mathcal{B}}\right)^2A_iA_jB_{ij}\exp\left[-\frac{ik}{2\mathcal{B}}(\mathcal{D}\ (\rho_1^2-\rho_2^2)\right]\times\int\int\mathrm{d}^2\rho_1'\int\int\mathrm{d}^2\rho_2'\times\exp\left(-\frac{\boldsymbol{\rho}_1^2}{4\sigma_i^2}-\frac{\boldsymbol{\rho}_2^2}{4\sigma_j^2}\right)\times\exp\left(-\frac{|\boldsymbol{\rho}_2-\boldsymbol{\rho}_1|^2}{2\delta_{ij}^2}\right)\exp\left[-\frac{ik}{2\mathcal{B}}(\mathcal{A}(\rho_1'^2-\rho_2'^2)-2(\boldsymbol{\rho}_1\cdot\boldsymbol{\rho}_1'-\boldsymbol{\rho}_2\cdot\boldsymbol{\rho}_2'))\right]. \quad (13)$$

On introducing the variables

$$\boldsymbol{\xi} = \frac{\boldsymbol{\rho}'_1 + \boldsymbol{\rho}'_2}{2}; \quad \boldsymbol{\xi}' = \boldsymbol{\rho}'_2 - \boldsymbol{\rho}'_1;$$

$$\boldsymbol{\rho} = \frac{\boldsymbol{\rho}_1 + \boldsymbol{\rho}_2}{2}; \quad \boldsymbol{\rho}' = \boldsymbol{\rho}_2 - \boldsymbol{\rho}_1, \tag{14}$$

Eq. (12) becomes

$$\begin{aligned} W_{ij}(\boldsymbol{\rho},\boldsymbol{\rho}',z;\omega) = {} & \left(\frac{k}{2\pi\mathcal{B}}\right)^2 A_i A_j B_{ij} \exp\left[-\frac{ik}{\mathcal{B}}(\mathcal{D}\boldsymbol{\rho}\cdot\boldsymbol{\rho}')\right] \\ & \times \int\int \mathrm{d}^2\xi \exp\left[-\xi^2\left(\frac{1}{4\sigma_i^2}+\frac{1}{4\sigma_j^2}\right)\right] \\ & \times \exp\left(-\frac{ik}{\mathcal{B}}\boldsymbol{\xi}\cdot\boldsymbol{\rho}'\right)\int\int \mathrm{d}^2\xi' \\ & \times \exp\left[-\xi^2\left(\frac{1}{16\sigma_i^2}+\frac{1}{16\sigma_j^2}+\frac{1}{2\delta_{ij}^2}\right)\right] \\ & \times \exp\left[\boldsymbol{\xi}'\cdot\left(\frac{\boldsymbol{\xi}}{4\sigma_i^2}-\frac{\boldsymbol{\xi}}{4\sigma_j^2}+\frac{ik\mathcal{A}}{\mathcal{B}}\boldsymbol{\xi}-\frac{ik}{\mathcal{B}}\boldsymbol{\rho}\right)\right]. \end{aligned} \tag{15}$$

After long calculations, which are outlined in Appendix A, the elements of the cross-spectral density matrix in any plane $z>0$ are found to be given by the formula

$$\begin{aligned} W_{ij}(\boldsymbol{\rho},\boldsymbol{\rho}',z;\omega) = {} & A_i A_j B_{ij}\frac{1}{\Delta_{ij}}\exp\left[-\frac{4\alpha_{ij}\rho^2}{\Delta_{ij}}\right] \\ & \times \exp\left\{\left[\frac{ik\mathcal{D}}{\mathcal{B}} - \left(\beta_{ij}+\frac{ik\mathcal{A}}{\mathcal{B}}\right)\frac{1}{\Delta_{ij}}\right]\boldsymbol{\rho}\cdot\boldsymbol{\rho}'\right\} \\ & \times \exp\left[-\frac{\gamma_{ij}\rho'^2}{\Delta_{ij}}\right], \end{aligned} \tag{16}$$

where

$$\alpha_{ij} = \frac{1}{16}\left(\frac{1}{\sigma_i^2}+\frac{1}{\sigma_j^2}\right), \quad \beta_{ij} = \frac{1}{4}\left(\frac{1}{\sigma_i^2}-\frac{1}{\sigma_j^2}\right), \quad \gamma_{ij} = \alpha_{ij} + \frac{1}{2\delta_{ij}^2}, \tag{17a}$$

and

$$\Delta_{ij} = \left(\frac{\mathcal{B}}{k}\right)^2\left[16\alpha_{ij}\gamma_{ij} - \left(\beta_{ij}+\frac{ik\mathcal{A}}{\mathcal{B}}\right)^2\right]. \tag{17b}$$

We note that for the diagonal elements of the cross-spectral density matrix $W_{ii}(\boldsymbol{\rho},\boldsymbol{\rho}',\omega)$, $\beta_{ii}=0$ and consequently formula (17b) becomes

$$\Delta_{ii} = \left(\frac{\mathcal{B}}{k}\right)^2\left[16\alpha_{ii}\gamma_{ii} + \left(\frac{k\mathcal{A}}{\mathcal{B}}\right)^2\right], \tag{17c}$$

which is purely real. The parameter Δ_{ij} may be interpreted as an expansion coefficient. However, compared with its free-space counterpart, it is not a monotonically increasing function of z; rather, it oscillates with z because the matrix elements $\mathcal{A},\mathcal{B},\mathcal{C}$, and $\mathcal{D}$ vary sinusoidally with z [see Eq. (7)].

Equation (16) together with Eqs. (17) is the general expression for the elements of the 2×2 cross-spectral density matrix of an electromagnetic Gaussian Schell-model beam propagating through an $\mathcal{ABCD}$ system. On substituting for the values of the $\mathcal{ABCD}$ matrix elements from Eq. (7) into Eqs. (16) and (17), one obtains the following expressions for the elements of the cross-spectral density matrix of an EGSM beam propagating through a GRIN fiber:

$$\begin{aligned} W_{ij}(\boldsymbol{\rho},\boldsymbol{\rho}',z;\omega) = {} & A_i A_j B_{ij}\frac{1}{\Delta_{ij}} \\ & \times \exp\left[-\frac{4\alpha_{ij}\rho^2}{\Delta_{ij}}\right]\exp\left\{\left[\frac{ik\cot(\alpha z)}{n\alpha} - \left(\beta_{ij}+\frac{ik\cot(\alpha z)}{n\alpha}\right)\frac{1}{\Delta_{ij}}\right]\boldsymbol{\rho}\cdot\boldsymbol{\rho}'\right\} \\ & \times \exp\left[-\frac{\gamma_{ij}\rho'^2}{\Delta_{ij}}\right], \end{aligned} \tag{18}$$

where

$$\Delta_{ij} = \left(\frac{\sin(\alpha z)}{kn\alpha}\right)^2\{16\alpha_{ij}\gamma_{ij} - [\beta_{ij} + ikn\alpha\cot(\alpha z)]^2\}, \tag{19}$$

and the parameters α_{ij}, β_{ij}, and γ_{ij} are given by Eq. (17)(a). The expressions are valid at any distance $z>0$ from the source and for any real-valued $n(\omega)$ and $\alpha(\omega)$.

5. SPECTRAL CHANGES OF AN ELECTROMAGNETIC GAUSSIAN SCHELL-MODEL BEAM ON PROPAGATION THROUGH A GRIN FIBER

There have been many studies of propagation-induced changes of the spectrum of light generated by a scalar source, the changes depending on the coherence properties of the source (for a review of the subject see Ref. 17). In Ref. 11, frequency shifts of the spectrum of the field generated by a scalar Gaussian Schell-model beam on propagation through a gradient-index fiber were reported, taking into account the effect of material dispersion. It is worth mentioning that not only may the spectrum of the propagated field suffer a shift of its peak frequency, but, in general, the spectral profile will also undergo a distortion. In this section we will study changes of the spectrum of an EGSM beam propagating through a GRIN fiber.

As is evident from Eq. (10), the spectrum at a point $\mathbf{r} \equiv (\boldsymbol{\rho},z)$ depends only on the diagonal elements of the cross-spectral density matrix. With the choice of $\boldsymbol{\rho}_1=\boldsymbol{\rho}_2 \equiv \boldsymbol{\rho}$ in Eq. (13), the diagonal elements of the cross-spectral density matrix at the point $P(\mathbf{r})$ become

$$W_{ii}(\boldsymbol{\rho},\boldsymbol{\rho},z;\omega) = A_i\frac{1}{\Delta_{ii}}\exp\left(-\frac{4\alpha_{ii}\rho^2}{\Delta_{ii}}\right), \quad (i=x,y) \tag{20}$$

where $\boldsymbol{\rho}$ represents the transverse position vector of the observation point. In Eq. (20),

$$\Delta_{ii} = \left[\cos^2(\alpha z) + \frac{\sin^2(\alpha z)}{(2nk\alpha\sigma_i^2)^2}\left(1 + \frac{4\sigma_i^2}{\delta_{ii}^2}\right)\right], \tag{21a}$$

and

$$\alpha_{ii} = \frac{1}{8\sigma_i^2}, \quad (i = x, y). \tag{21b}$$

The spectral density of the field $S(\boldsymbol{\rho}, z; \omega)$ may be calculated by substituting from Eqs. (20) and (21) into Eq. (10). It may be verified that in the limit as $z \to 0, S(\boldsymbol{\rho}, z; \omega)$ reduces to the expression for the spectrum $S^{(0)}(\boldsymbol{\rho}; \omega)$ of the field in the plane $z=0$; it may be calculated by substituting from Eq. (5) into Eq. (10).

As an example, let us consider the case when the normalized spectra of the x and y components of the electric field in the source plane $z=0$ are equal to each other. When this is so, A_i may be written as

$$A_i \equiv I_i s^{(0)}(\omega), \quad (i = x, y), \tag{22a}$$

where the parameters I_i, $(i=x,y)$ are constants and $s^{(0)}(\omega)$ denotes the normalized spectral density in the plane $z=0$; i.e.,

$$s^{(0)}(\omega) = \frac{S^{(0)}(\boldsymbol{\rho}, \omega)}{\int S^{(0)}(\boldsymbol{\rho}, \omega)\mathrm{d}\omega}. \tag{22b}$$

We will assume that $s^{(0)}(\omega)$ is a Lorentzian line centered at the wavelength $\lambda_0 = 564$ nm ($\nu_0 = \omega_0/2\pi \cong 532$ Thz) with a FWHM of 53.2 Thz; i.e.,

$$s^{(0)}(\omega) = \frac{1}{1 + [4(\omega - \omega_0)^2/(\delta\omega)^2]}, \tag{22c}$$

where $\delta\omega$ is the FWHM. We also assume that the parameters σ_i and δ_{ij} in Eq. (5) $(i=x,y; j=x,y)$ are independent of frequency.

At a fixed distance within the fiber, the spectral shift $\Delta\omega$ is the difference between the peak frequency of the modified spectrum of the field at that distance and the peak frequency ω_0 of the source spectrum. A positive value of $\Delta\omega$ represents a blue shift, while a negative value represents a red shift. We will study the variation of the normalized spectral shifts, i.e., $\Delta\omega/\omega_0$, as a function of the distance z of propagation at a fixed radial distance from the fiber axis for different sets of source parameters, and we will illustrate the preceding analysis by examples.

As in Ref. 11, all the numerical calculations pertain to a specific fiber whose core is made of doped silica (7.9% GeO_2 at the core center) and a cladding made of pure silica SiO_2. The frequency dependence of the refractive index is taken to be given by the Sellmeier formula,

$$n^2(\omega) = 1 + \sum_{j=1}^{3} \frac{B_j\omega_j^2}{\omega_j^2 - \omega^2}. \tag{22d}$$

For pure silica, the parameters are $B_1 = 0.6961663$, $B_2 = 0.4079426$, $B_3 = 0.8974794$, $\lambda_1 = 0.0617167\ \mu$m, $\lambda_2 = 0.1162414\ \mu$m, and $\lambda_3 = 9.896161\ \mu$m, where $\lambda_j = 2\pi c/\omega_j$. To include the effect of dopant on the refractive index, the Sellmeier formula may still be used, but with somewhat different values of the parameters. For silica glass doped with 7.9% GeO_2, $B_1 = 0.7136824$, $B_2 = 0.4254807$, $B_3 = 0.8964226$, $\lambda_1 = 0.0617167\ \mu$m, $\lambda_2 = 0.1270814\ \mu$m, and $\lambda_3 = 9.896161\ \mu$m. For the fiber considered in the present analysis, we can calculate the frequency dependence of the refractive index at the core and at the fiber boundary and hence, using Eq. (6b), calculate the radial gradient $\alpha(\omega)$ of the index parameter. The core radius is taken to be 25 μm.

Figure 2 shows the variation of the spectral shifts with increasing distance of propagation for a *scalar* Gaussian Schell-model beam that propagates within the fiber at a fixed radial distance from the fiber axis ($\rho = 10/k_0$). The dashed curve shows the spectral shifts when the frequency dependence of α is ignored by setting $\alpha(\omega_0) = (4.8)10^{-4}\ k_0$. The solid curve shows the spectral shifts for a dispersive GRIN. It can be seen that the spectral shifts for a dispersion-free fiber shows only blue shifts and exhibits periodicity, while a dispersive fiber shows red shifts as well and no periodicity. Similar results have been reported in Ref. 11.

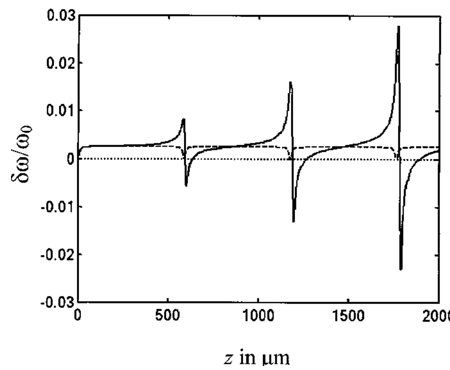

Fig. 2. Normalized spectral shift $\Delta\omega/\omega_0$ of the field generated by a *scalar* Gaussian Schell-model source, as a function of the propagation distance z, within a GRIN with a parabolic refractive-index profile. The frequency shifts are calculated for points at a fixed off-axis distance $10/k_0$, for a source with $\sigma = 20/k_0$, $\delta = 4/k_0$. The dashed curve shows the frequency shifts when the frequency dependence of α is ignored, taking $\alpha(\omega_0) = (4.8)10^{-4}\ k_0$.

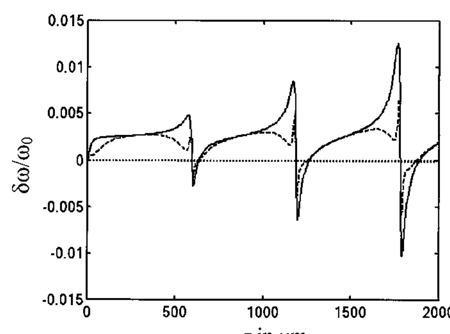

Fig. 3. Normalized spectral shift $\Delta\omega/\omega_0$ for the field generated by an *electromagnetic* Gaussian Schell-model source as function of the propagation distance z within a dispersive graded-index fiber with a parabolic refractive-index profile. The frequency shifts are calculated for points at a fixed off-axis distance $10/k_0$ from the fiber axis for a source with $\sigma_x = \sigma_y = 50/k_0$, $I_x = I_y = 0.5$. The correlation parameters for the source for the dashed curve are $\delta_{xx} = 2/k_0$, $\delta_{yy} = 4/k_0$, and those for the solid curve are taken to be $\delta_{xx} = 2/k_0$, $\delta_{yy} = 18/k_0$.

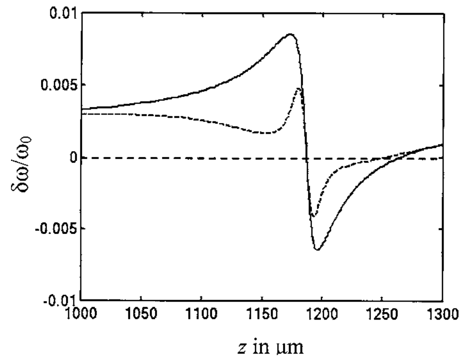

Fig. 4. The region around the second zero crossing of the normalized spectral shift of Fig. 2 shown in greater detail.

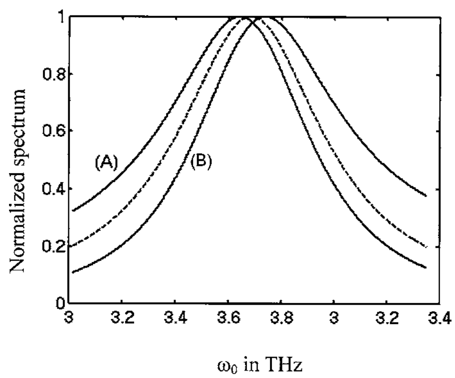

Fig. 5. Normalized spectrum of the field generated by an *electromagnetic* Gaussian Schell-model source for different values of the propagation distance within the fiber. The spectral density is calculated for points at a fixed off-axis distance $10/k_0$ from the fiber axis with a source for which $\sigma_x=\sigma_y=50/k_0$, $I_x=I_y=0.5$. The correlation parameters for the source are taken to be $\delta_{xx}=2/k_0$, $\delta_{yy}=4/k_0$. The dashed curve shows the source spectrum ($z=0$). Spectral density of the field at a propagation distance $z=1200$ μm within the fiber shows a red shift (A), and spectral density of the field at a propagation distance $z=1165$ μm within the fiber shows a blue shift (B).

Figure 3 shows the variation of the spectral shifts with increasing distance as an *electromagnetic* Gaussian Schell-model beam propagates through a dispersive GRIN fiber. The figure illustrates the influence of the parameters δ_{xx} and δ_{yy} that characterize the correlation properties of the two orthogonal Cartesian components of the electric field in the source plane. A comparison with Fig. 1 shows that the electromagnetic beam exhibits more complicated spectral shifts than does its scalar counterpart. Figure 4 shows the variation of spectral shifts in greater detail for propagation distances at which the blue shifts switches over to red shifts. Figure 5 shows the behavior of the spectral density of the field generated by an EGSM beam within the fiber for two different values of the distance of propagation.

6. CHANGES IN THE SPECTRAL DEGREE OF POLARIZATION OF AN ELECTROMAGNETIC GAUSSIAN SCHELL-MODEL BEAM ON PROPAGATION THROUGH A GRIN FIBER

Next we will consider the changes in the spectral degree of polarization of an EGSM beam on propagation through a GRIN fiber. The degree of polarization at a point $\mathbf{r}$ is given by Eq. (12) and obviously requires knowledge of all the elements of the cross-spectral density matrix for $\mathbf{r}_1=\mathbf{r}_2=\mathbf{r}$. The diagonal elements of the cross-spectral density matrix at the point $\mathbf{r}$ are given by Eqs. (20) and (21). The off-diagonal element at a point $\mathbf{r}\equiv(\boldsymbol{\rho},z)$ may be expressed in the form

$$W_{xy}(\boldsymbol{\rho},\boldsymbol{\rho},z;\omega)=A_iA_jB_{xy}\frac{1}{\Delta_{xy}}\exp\left[-\frac{4\alpha_{xy}\rho^2}{\Delta_{xy}}\right], \tag{23}$$

where

$$\Delta_{xy}=\left(\frac{\mathcal{B}}{k}\right)^2\left[16\alpha_{xy}\gamma_{xy}-\left(\beta_{xy}+\frac{ik\mathcal{A}}{\mathcal{B}}\right)^2\right] \tag{24a}$$

and

$$\alpha_{xy}=\frac{1}{16}\left(\frac{1}{\sigma_x^2}+\frac{1}{\sigma_y^2}\right), \quad \beta_{xy}=\frac{1}{4}\left(\frac{1}{\sigma_x^2}-\frac{1}{\sigma_y^2}\right), \quad \gamma_{xy}=\alpha_{xy}+\frac{1}{2\delta_{xy}^2}. \tag{24b}$$

In Eq. (23), $\boldsymbol{\rho}$ represents, of course, the radial position vector of an off-axis point in any cross-section of the fiber.

For simplicity, we will consider a source that has the same spectral widths for the x and y components of the electric field in the source plane (i.e., when $\sigma_x=\sigma_y\equiv\sigma$). Then,

$$\beta_{ii}=0, \quad \alpha_{xy}=\frac{1}{8\sigma^2}, \quad \gamma_{xy}=\frac{1}{8\sigma^2}+\frac{1}{2\delta_{xy}^2}, \tag{25a}$$

and

$$\Delta_{xy}=\left[\cos^2(\alpha z)+\frac{\sin^2(\alpha z)}{(2nk\alpha\sigma^2)^2}\left(1+\frac{4\sigma^2}{\delta_{xy}^2}\right)\right]. \tag{25b}$$

Substituting from Eqs. (25) into Eq. (23) and using Eqs. (20) and (21) in formula (12), we can determine the spectral degree of polarization of the field at a point within the fiber. Figures 5 and 6 show the changes in the spectral de-

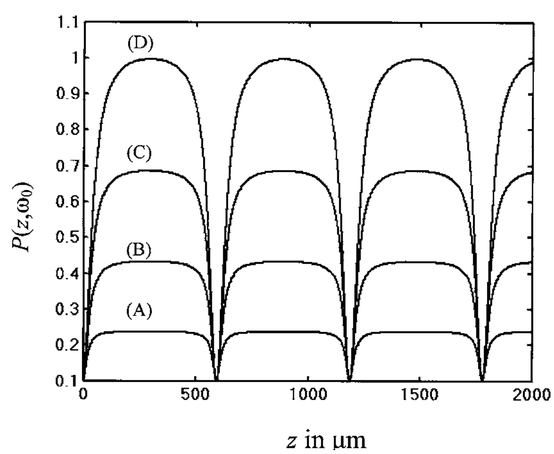

Fig. 6. Spectral degree of polarization of the field on-axis as a function of propagation distance z within a dispersive GRIN fiber with a parabolic refractive-index profile. The spectral and correlation parameters of the EGSM source are given by $\sigma_x=\sigma_y=50/k_0$, $I_x=I_y=0.5$, $\delta_{xx}=2/k_0$, $\delta_{yy}=2/k_0$, $I_{xy}=0.1$. (A) $\delta_{xy}=3.0811/k_0$, (B) $\delta_{xy}=4.1623/k_0$, (C) $\delta_{xy}=5.2434/k_0$, (D) $\delta_{xy}=6.3246/k_0$.

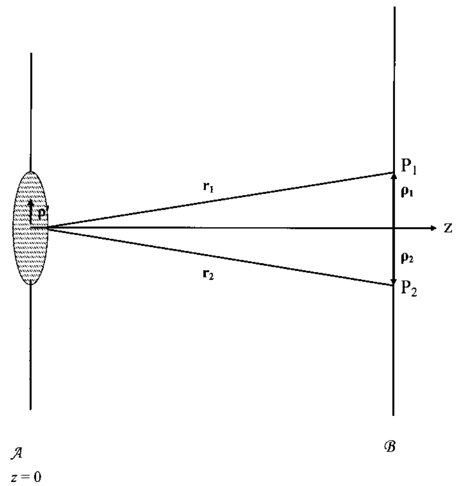

Fig. 7. Illustration of the notation.

gree of polarization on-axis as the EGSM propagates within the GRIN fiber. The periodicity of the spectral degree of polarization is due to the imaging property of a medium with a quadratic variation of the refractive index.

As already mentioned in Section 2, the parameters characterizing the spectral and the correlation properties of the source have to be chosen with care to ensure that the source is physically realizable. The sufficiency condition, chiefly due to the nonnegative definiteness of the cross-spectral density matrix, may be expressed as inequality (Ref. 19):

$$\max\{\delta_{xx},\delta_{yy}\} \leqslant \delta_{xy} \leqslant \min\left\{\frac{\delta_{xx}}{\sqrt{|Q_{xy}|}},\frac{\delta_{yy}}{\sqrt{|Q_{xy}|}}\right\}. \tag{26}$$

As a consequence of this constraint, the spectral degree of polarization at a point in the fiber may assume values only within a well-defined range, as can be seen from Fig. 6. The lower and upper bounds of the degree of polarization of the electric field at any point are determined by the spectral properties and by the correlation properties of the source. Additional constraints on the parameters imposed by the requirement that the source generate a beam were derived in Ref. 22.

7. CHANGES IN THE SPECTRAL DEGREE OF COHERENCE OF AN ELECTROMAGNETIC GAUSSIAN SCHELL-MODEL BEAM ON PROPAGATION THROUGH A GRIN FIBER

The spectral degree of coherence of the electric field for a pair of points in the cross section of the fiber is given by formula (11). It is evident that it depends only on the diagonal elements of the cross-spectral density matrix. We will choose the two points to be located radially symmetrically with respect to the fiber axis (i.e., $\boldsymbol{\rho}_2=-\boldsymbol{\rho}_1$; see Fig. 7). With this choice we have

$$\boldsymbol{\rho}=\frac{\boldsymbol{\rho}_1+\boldsymbol{\rho}_2}{2}=0 \quad \text{and} \quad \boldsymbol{\rho}'=\boldsymbol{\rho}_2-\boldsymbol{\rho}_1\equiv 2\boldsymbol{\rho}_2. \tag{27}$$

On substituting from Eq. (27) into Eq. (16), one finds that

$$W_{ii}(\boldsymbol{\rho},\boldsymbol{\rho}',z;\omega)=A_i\frac{1}{\Delta_{ii}}\exp\left(-\frac{4\gamma_{ii}\rho_2^2}{\Delta_{ii}}\right), \tag{28a}$$

where

$$\Delta_{ii}=\left[\cos^2(\alpha z)+\frac{\sin^2(\alpha z)}{(2nk\alpha\sigma_i^2)^2}\left(1+\frac{4\sigma_i^2}{\delta_{ii}^2}\right)\right] \tag{28b}$$

and

$$\gamma_{ii}=\frac{1}{8\sigma_i^2}+\frac{1}{2\delta_{ii}^2}. \tag{28c}$$

Substituting from Eqs. (28), (20) and (21) into Eq. (11), we obtain for the spectral degree of coherence at the chosen pair of points, the expression

$$\eta(\boldsymbol{\rho},-\boldsymbol{\rho},z;\omega)=\frac{A_x\frac{1}{\Delta_{xx}}\exp\left(-\frac{4\gamma_{xx}\rho^2}{\Delta_{xx}}\right)+A_y\frac{1}{\Delta_{yy}}\exp\left(-\frac{4\gamma_{yy}\rho^2}{\Delta_{yy}}\right),}{A_x\frac{1}{\Delta_{xx}}\exp\left(-\frac{\alpha_{xx}\rho^2}{\Delta_{xx}}\right)+A_y\frac{1}{\Delta_{yy}}\exp\left(-\frac{\alpha_{yy}\rho^2}{\Delta_{yy}}\right)}. \tag{29}$$

Figure 8 shows the variation of the spectral degree of coherence with increasing distance of propagation within the fiber when the ratio of the parameters A_x/A_y is varied while the correlation parameters δ_{xx} and δ_{yy} are kept constant. Figure 9 shows the changes in the spectral degree of coherence as a function of the distance of propagation when the correlation width of the y component of the electric field is varied while all the other source parameters are kept constant. The periodic variation of the degree of coherence is again a consequence of the imaging property of the gradient fiber.

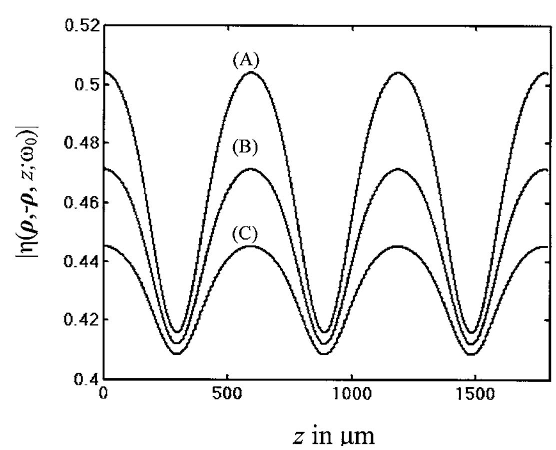

Fig. 8. Spectral degree of coherence as a function of propagation distance z for a pair of points located radially symmetrically at a distance $1/k_0$ from the fiber axis. The different curves show the variation as the spectral density of the x component of the field is changed while that of the y component is kept fixed. The parameters of the source are taken to be $\sigma_x=\sigma_y=50/k_0$, $I_y=1$, $\delta_{xx}=40/k_0$, $\delta_{yy}=30/k_0$, $\omega=\omega_0$. (A) $I_x=1$, (B) $I_x=0.5$, (C) $I_x=0.25$.

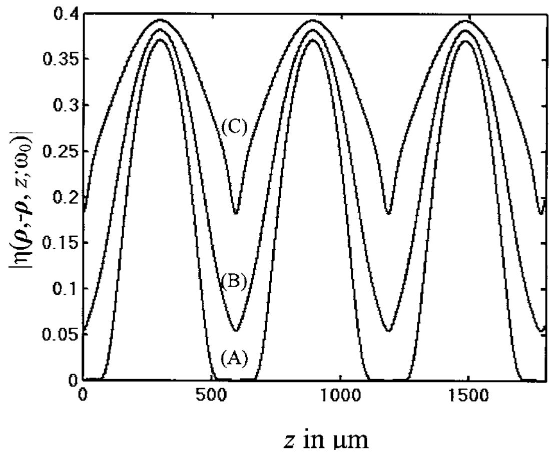

Fig. 9. Spectral degree of coherence as a function of the propagation distance z for a pair of radially symmetric points at a distance $1/k_0$ from the fiber axis. The different curves show the variation of the spectral degree of coherence as the parameter that characterizes the correlations of the x component of the field is changed, keeping that of the y component fixed. The source is specified by the following parameters: $\sigma_x=\sigma_y=50/k_0, I_x=0.5, I_y=1, \delta_{yy}=4/k_0, \omega=\omega_0$. (A) $\delta_{xx}=3/k_0$, (B) $\delta_{xx}=18/k_0$, (C) $\delta_{xx}=25/k_0$.

8. CONCLUSIONS

The analysis presented in this paper shows that the spectral density, the spectral degree of polarization, and the spectral degree of coherence of an electromagnetic Gaussian Schell-model (EGSM) beam all change on propagation through a gradient-index fiber that supports a large number of low-order modes. The transverse degree of coherence at two points within the fiber and the degree of polarization at a point in the fiber exhibit periodicity, caused by the focusing property of square-law media. The spectral, polarization, and coherence properties of the EGSM beam in any cross section at a certain propagation distance within the gradient-index fiber depend on the correlation properties and on the spectral properties of the source generating the beam. It is to be noted that coherence and polarization properties of the electric field change as any one of the parameters characterizing the spectral and the correlation properties of the source is varied while all the other parameters are kept fixed. Since the nonnegative definiteness of the cross-spectral density matrix of the electric field in the source plane imposes restriction on the physically allowable range of values of the source parameter, the degree of polarization and the degree of coherence of the electric field at a fixed propagation distance are also restricted within a certain range.

APPENDIX A: DERIVATION OF EQ. (16) FOR THE ELEMENTS OF THE CROSS-SPECTRAL DENSITY MATRIX ACROSS THE BEAM

According to formula (1), the elements of the cross-spectral density matrix in any plane $z>0$ are given by the formula

$$W_{ij}(\boldsymbol{\rho},\boldsymbol{\rho}',z;\omega) = \left(\frac{k}{2\pi\mathcal{B}}\right)^2 A_i A_j B_{ij} \exp\left(-\frac{ik}{\mathcal{B}}(\mathcal{D}\boldsymbol{\rho}\cdot\boldsymbol{\rho}')\right)\int\int \mathrm{d}^2\xi \times\exp\left[-\xi^2\left(\frac{1}{4\sigma_i^2}+\frac{1}{4\sigma_j^2}\right)\right]\exp\left(-\frac{ik}{\mathcal{B}}\boldsymbol{\xi}\cdot\boldsymbol{\rho}'\right) \times\int\int \mathrm{d}^2\xi' \exp\left[-\xi'^2\left(\frac{1}{16\sigma_i^2}+\frac{1}{16\sigma_j^2}+\frac{1}{2\delta_{ij}^2}\right)\right] \times\exp\left[\boldsymbol{\xi}'\cdot\left(\frac{\boldsymbol{\xi}}{4\sigma_i^2}-\frac{\boldsymbol{\xi}}{4\sigma_j^2}+\frac{ik\mathcal{A}}{\mathcal{B}}\boldsymbol{\xi}-\frac{ik}{\mathcal{B}}\boldsymbol{\rho}\right)\right]. \tag{A1}$$

Introducing the parameters

$$\alpha_{ij}=\frac{1}{16}\left(\frac{1}{\sigma_i^2}+\frac{1}{\sigma_j^2}\right),\quad \beta_{ij}=\frac{1}{4}\left(\frac{1}{\sigma_i^2}-\frac{1}{\sigma_j^2}\right),\quad \gamma_{ij}=\alpha_{ij}+\frac{1}{2\delta_{ij}^2}, \tag{A2}$$

the integral over the parameter $\boldsymbol{\xi}'$ takes the form

$$I(\boldsymbol{\xi},\boldsymbol{\rho}) = \int\int \mathrm{d}^2\xi' \exp(-\gamma_{ij}\xi'^2) \times\exp\left\{\boldsymbol{\xi}'\cdot\left[\left(\beta_{ij}+\frac{ik\mathcal{A}}{\mathcal{B}}\right)\boldsymbol{\xi}-\frac{ik}{\mathcal{B}}\boldsymbol{\rho}\right]\right\}. \tag{A3}$$

Using the formula

$$\int_{-\infty}^{\infty}\exp(-p^2x^2 \pm qx)dx = \frac{\sqrt{\pi}}{p}\exp\left(\frac{q^2}{4p^2}\right), \tag{A4}$$

the integral in Eq. (A3) may be evaluated and gives the following expression for $I(\boldsymbol{\xi},\boldsymbol{\rho})$:

$$I(\boldsymbol{\xi},\boldsymbol{\rho}) = \frac{\pi}{\gamma_{ij}}\exp\left\{\frac{\left[\left(\beta_{ij}+\frac{ik\mathcal{A}}{\mathcal{B}}\right)\boldsymbol{\xi}-\frac{ik}{\mathcal{B}}\boldsymbol{\rho}\right]^2}{4\gamma_{ij}}\right\}. \tag{A5}$$

After expanding the exponent, we obtain the formula

$$I(\boldsymbol{\xi},\boldsymbol{\rho}) = \frac{\pi}{\gamma_{ij}}\exp\left(-\frac{k^2}{4\gamma_{ij}\mathcal{B}^2}\rho^2\right) \times\exp\left[-\frac{ik}{2\gamma_{ij}\mathcal{B}}\left(\beta_{ij}+\frac{ik\mathcal{A}}{\mathcal{B}}\right)\boldsymbol{\xi}\cdot\boldsymbol{\rho}\right] \times\exp\left[\frac{1}{4\gamma_{ij}}\left(\beta_{ij}+\frac{ik\mathcal{A}}{\mathcal{B}}\right)^2\xi^2\right]. \tag{A6}$$

On substituting from Eq. (A6) into (A1), the integral I' taken over the variable $\boldsymbol{\xi}$ takes the form

$$I'(\boldsymbol{\rho},\boldsymbol{\rho}',z;\omega) = \int\int \mathrm{d}^2\xi \times\exp\left\{-\left[4\alpha_{ij}-\frac{1}{4\gamma_{ij}}\left(\beta_{ij}+\frac{ik\mathcal{A}}{\mathcal{B}}\right)^2\right]\xi^2\right\} \times\exp\left\{-\left[\frac{ik}{\mathcal{B}}\boldsymbol{\rho}'+\frac{ik}{2\gamma_{ij}\mathcal{B}}\left(\beta_{ij}+\frac{ik\mathcal{A}}{\mathcal{B}}\right)\boldsymbol{\rho}\right]\cdot\boldsymbol{\xi}\right\}, \tag{A7}$$

which again can be evaluated by using formula (A4). Performing the integration yields the expression

$$I'(\boldsymbol{\rho},\boldsymbol{\rho}',z;\omega)=\frac{\pi}{\Delta_{ij}}\left(\frac{4\gamma_{ij}\mathcal{B}^2}{k^2}\right)\exp\left[-\frac{1}{4\Delta_{ij}\gamma_{ij}}\left(\beta_{ij}+\frac{ik\mathcal{A}}{\mathcal{B}}\right)^2\rho^2\right]\times\exp\left(-\frac{\gamma_{ij}}{\Delta_{ij}}\rho'^2\right)\exp\left[-\frac{1}{\Delta_{ij}}\left(\beta_{ij}+\frac{ik\mathcal{A}}{\mathcal{B}}\right)\boldsymbol{\rho}\cdot\boldsymbol{\rho}'\right], \tag{A8}$$

where

$$\Delta_{ij}=\left(\frac{\mathcal{B}}{k}\right)^2\left[16\alpha_{ij}\gamma_{ij}-\left(\beta_{ij}+\frac{ik\mathcal{A}}{\mathcal{B}}\right)^2\right]. \tag{A9}$$

On substituting from Eqs. (A6)–(A9) into Eq. (A1), we finally obtain, after some algebraic manipulations, the following expression for the elements of the cross-spectral density matrix in any transverse plane $z>0$:

$$W_{ij}(\boldsymbol{\rho},\boldsymbol{\rho}',z;\omega)=A_iA_jB_{ij}\frac{1}{\Delta_{ij}}\exp\left(-\frac{4\alpha_{ij}\rho^2}{\Delta_{ij}}\right)\times\exp\left\{\left[\frac{ik\mathcal{D}}{\mathcal{B}}-\left(\beta_{ij}+\frac{ik\mathcal{A}}{\mathcal{B}}\right)\frac{1}{\Delta_{ij}}\right]\boldsymbol{\rho}\cdot\boldsymbol{\rho}'\right\}\times\exp\left(-\frac{\gamma_{ij}\rho'^2}{\Delta_{ij}}\right). \tag{A10}$$

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