# Ultrabroadband parametric generation and wavelength conversion in silicon waveguides

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**Abstract:** We show that ultrabroadband parametric generation and wavelength conversion can be realized in silicon waveguides in the wavelength region near 1550 nm by tailoring their zero-dispersion wavelength and launching pump wave close to this wavelength. We quantify the impact of two-photon absorption, free-carrier generation, and linear losses on the process of parametric generation and show that it is difficult to realize a net signal gain and transparent wavelength conversion with a continuous-wave pump. By investigating the transient dynamics of the four-wave mixing process initiated with a pulsed pump, we show that the instantaneous nature of electronic response enables highly efficient parametric amplification and wavelength conversion for pump pulses as wide as 1 ns. We also discuss the dual-pump configuration and show that its use permits multiband operation with uniform efficiency over a broad spectral region extending over 300 nm.

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**OCIS codes:** (130.4310) Integrated optics, nonlinear; (160.4330) Nonlinear optical materials; (190.4970) Parametric oscillators and amplifiers; (190.2620) Frequency conversion.

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### 1. Introduction

Silicon photonics has attracted much attention recently because of its potential for providing a monolithically integrated optoelectronic platform. In spite of silicon's indirect bandgap structure, optical nonlinearities in silicon-on-insulator (SOI) waveguides have been used to demonstrate functionalities such as optical modulation [1, 2], Raman amplification and lasing [3]–[8], and wavelength conversion [9]–[13]. Four-wave mixing (FWM) is promising for all-optical signal processing. In a few experiments, wavelength conversion was realized using coherent anti-Stokes Raman scattering (CARS) [9, 10]. Although the narrowband Raman spectrum of silicon enhances FWM significantly in this case, it also limits the bandwidth of wavelength conversion, and a large Raman frequency shift makes it difficult to realize phase matching [10, 14]. FWM in SOI waveguides can also occur through a nonresonant electronic response. Although the magnitude of this nonlinearity is smaller than the Raman-assisted resonant one, its response is nearly instantaneous, a feature required for parametric gain to exist over a broad bandwidth.

Although efforts have been made to take advantage of this electronic nonlinearity [11]–[13], current experiments show wavelength conversion only over a bandwidth of about 40 nm with a relatively low FWM efficiency.

In this paper we show that ultrabroadband parametric generation and wavelength conversion can be realized in an SOI waveguide in the wavelength region near 1.55  $\mu$ m by appropriately tailoring its zero-dispersion wavelength (ZDWL) and launching the pump beam close to the ZDWL. In the case of a continuous-wave (CW) pump, free-carrier absorption (FCA) limits the FWM efficiency and makes it difficult to obtain net signal gain and transparent wavelength conversion, unless the carrier lifetime can be reduced to below 100 ps. By investigating the transient dynamics of FWM with a pulsed pump, we show that the instantaneous nature of the electronic response enables highly efficient parametric amplification and wavelength conversion for pump pulses as wide as 1 ns. Moreover, multiband operation with a high and uniform efficiency can be realized through a dual-pump configuration.

## 2. Dispersion tailoring in SOI waveguides

Bulk silicon exhibits significant normal dispersion over its transparent spectral region beyond 1100 nm [15]. As a result, large-size waveguides (width and thickness  $\geq 1 \,\mu$ m) exhibit normal dispersion in the telecommunication band located near 1550 nm because of the dominance of the material dispersion. For example, numerical simulations performed using the software package BeamPROP<sup>TM</sup> (by RSoft) show that the SOI waveguide used in Ref. [10] exhibits group-velocity dispersion (GVD) of about 0.4 and 0.7 ps<sup>2</sup>/m at 1550 nm for the TM and TE modes, respectively. The corresponding values for the waveguide in Ref. [13] are about 0.7 and 0.8 ps<sup>2</sup>/m, respectively. These positive values indicate that such waveguides are not suitable for FWM in the telecommunication band because anomalous GVD is often required for efficient FWM [16]. However, this problem can be solved if the GVD resulting from a strong optical mode confinement is used to compensate for the material GVD. This approach provides an opportunity to engineer the total dispersion by changing the waveguide width and thickness or by modifying the device structure. Indeed, we recently showed using the effective-index



Fig. 1. (a) Structure of the SOI waveguide with a 50% etching depth; two insets show spatial profiles for the TE and TM modes. (b) Second- (blue) and third-order (red) dispersion parameters for the TE and TM modes.

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Fig. 2. (a) Structure of the SOI waveguide with 100% etching depth; two insets show spatial profiles for the TE and TM modes. (b) Second- (blue) and third-order (red) dispersion parameters for the TE and TM modes.

method that anomalous dispersion can be realized in the 1550-nm regime, and such waveguides can support optical solitons [17].

In this section we show two examples of tailored dispersion for SOI waveguides. We use BeamPROP software to go beyond the effective-index method that is likely to fail for waveguides with large etching depths. We first calculate the propagation constant  $\beta(\omega)$  as a function of frequency  $\omega$  for the fundamental TE and TM modes. The resulting curve is differentiated numerically using a spline-fit procedure to calculate the *n*th-order dispersion parameters using the definition  $\beta_n(\omega) = d^n \beta / d\omega^n$ . Figure 1 shows the second- and third-order dispersion parameters for a commonly used rib-waveguide design with a 50% etching depth. The size of the waveguide is reduced to below 1  $\mu$ m to shift the ZDWL near 1.5  $\mu$ m [17]. Such a waveguide has its ZDWL located at 1546 nm for the TM mode. However, the TE mode exhibits normal dispersion over the whole telecommunication band. Extensive numerical simulations show that, if the etching depth remains small, this mode always falls in the normal-dispersion regime, regardless of the waveguide size.

It is possible to design an SOI waveguide such that the ZDWLs of both the TE and TM modes occur near 1550 nm. As the contribution of waveguide dispersion to the TE-mode dispersion depends considerably on the etching depth, anomalous dispersion can be realized for this mode by increasing this depth. Figure 2 shows such a waveguide design for which an etching depth of 100% results in a nearly square-shaped structure, with air acting as the cladding on three sides. The GVD curves for the fundamental TE and TM modes nearly coincide, indicating that the second-order dispersive properties are almost identical for them with such a design. More specifically, the TM mode has its ZDWL at 1551.3 nm, with  $\beta_3 = 3.90 \times 10^{-3} \text{ ps}^3/\text{m}$  and  $\beta_4 = -1.87 \times 10^{-5} \text{ ps}^4/\text{m}$  at this wavelength. The TE mode has its ZDWL at 1551.1 nm, with  $\beta_3 = 3.95 \times 10^{-3} \text{ ps}^3/\text{m}$  and  $\beta_4 = -4.84 \times 10^{-5} \text{ ps}^4/\text{m}$  at this wavelength. Both modes have an effective mode area of about 0.38  $\mu$ m<sup>2</sup>. Further tailoring of dispersion can be easily obtained by reducing the size of the waveguide. Because of cleaving convenience [9], SOI waveguides are usually fabricated along the [110] direction on the (001) surface, as shown in Fig. 2(a). We focus on this geometry in the following discussion and use the example shown in Fig. 2 to discuss the FWM process.

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In practice, state-of-the-art microlithography technique can provide a fabrication accuracy of < 20 nm, which enables the control of ZDWL to within 10 nm for the waveguide shown in Fig. 2(a). Although increasing the etching depth makes the waveguide to become multimoded, higher-order modes have quite different mode profiles and dispersion properties compared with the fundamental TM and TE modes. Because of such differences, higher-order modes are not likely to participate in the FWM process if the pump and signal propagate predominantly in the fundamental modes.

#### 3. FWM theory for silicon waveguides

In this section we focus on the degenerate FWM configuration in which a single pump beam is employed. FWM occurs through  $2\omega_p = \omega_s + \omega_i$ , where  $\omega_j$  is the frequency and j = p, s, i for the pump, signal, and idler waves, assumed to be identically polarized in either the fundamental TM or TE mode. Although the TE and TM modes have a polarization component along the z direction, it contains only a small fraction of incident power, and the nonlinear effects are dominated by the transverse component. For this reason, we simplify the problem considerably by adopting a scalar approach and writing the total electric field in the form

$$E(z,t) = \operatorname{Re}[A_p(z,t)e^{-i\omega_p t} + A_s(z,t)e^{-i\omega_s t} + A_i(z,t)e^{-i\omega_i t}],$$
(1)

where  $A_j(z,t)$  (j = p, s, i) is the slowly varying amplitude normalized such that  $I_j = |A_j|^2$  represents the optical intensity.

We obtain the third-order nonlinear polarization for the geometry of Fig. 2(a) through a standard procedure [18]–[20]. We use this polarization in Maxwell's wave equation, decompose this equation into its three frequency components [16, 21], and find that the pump and signal waves satisfy the following coupled equations:

$$\frac{\partial A_p}{\partial z} + \beta_{1p} \frac{\partial A_p}{\partial t} + \frac{i\beta_{2p}}{2} \frac{\partial^2 A_p}{\partial t^2} = -\frac{1}{2} [\alpha_p + \alpha_{fp}(z,t)] A_p + i\beta_{0p} A_p + i(\gamma_e + \gamma_R) I_p A_p, \tag{2}$$

$$\frac{\partial A_s}{\partial z} + \beta_{1s} \frac{\partial A_s}{\partial t} + \frac{ip_{2s}}{2} \frac{\partial A_s}{\partial t^2} = -\frac{1}{2} [\alpha_s + \alpha_{fs}(z,t)] A_s + i\beta_{0s} A_s + i(2\gamma_e + \gamma_R) I_p A_s + i\gamma_e A_p^2 A_i^* + i\gamma_R A_p \int_{-\infty}^t h_R(t-\tau) e^{i\Omega_{sp}(t-\tau)} \left[ A_p^*(z,\tau) A_s(z,\tau) + A_p(z,\tau) A_i^*(z,\tau) \right] d\tau, \quad (3)$$

where  $\beta_{0j}$  is the propagation constant at the frequency  $\omega_j$  (j = p, s, i),  $\beta_{1j}$  is inverse of the group velocity,  $\beta_{2j}$  is the GVD parameter, and  $\Omega_{sp} = \omega_s - \omega_p$  is the signal-pump frequency detuning. The values of  $\beta_{2p}$  and  $\beta_{2s}$  can be inferred from the GVD curves in Fig. 2(b), or they can be obtained approximately using the values of  $\beta_3$  and  $\beta_4$  at the ZDWL provided in Section 2. The idler equation can be obtained by exchanging the subscript *s* and *i* in Eq. (3). In Eqs. (2) and (3), we have assumed that the intensities of both the signal and idler are much smaller than that of the pump so that the pump is not depleted and higher-order (cascaded) nonlinear effects are negligible.

In the preceding equations, the nonlinear effects are included through the parameters  $\gamma_e$  and  $\gamma_R$  representing the contributions of bound electrons and optical phonons (Raman process), respectively. The Raman response  $h_R(t)$  is introduced through its spectrum  $\widetilde{H}_R(\Omega)$  defined as  $h_R(t) = \int_{-\infty}^{+\infty} \widetilde{H}_R(\Omega) e^{-i\Omega t} d\Omega/2\pi$ . These nonlinear parameters are given by

$$\gamma_e = \xi_e(\gamma_0 + i\beta_T/2), \qquad \gamma_R = \frac{\xi_R g_R \Gamma_R}{\Omega_R}, \qquad \widetilde{H}_R(\Omega) = \frac{\Omega_R^2}{\Omega_R^2 - \Omega^2 - 2i\Gamma_R \Omega}, \tag{4}$$

where  $\gamma_0 = n_2 \omega_p / c$  with  $n_2 = 6 \times 10^{-5} \text{ cm}^2/\text{GW}$  [22] and  $\beta_T = 0.45 \text{ cm/GW}$  is the coefficient of two-photon absorption (TPA) [3, 22]. Lacking experimental data, we assume both  $n_2$  and  $\beta_T$ 

are constant over the telecommunication band. Although  $\beta_T$  may change with wavelength, in practice, TPA induced by the signal and idler is negligible compared with that induced by the pump (near 1.55  $\mu$ m). For this reason, the wavelength dependence of  $\beta_T$  does not affect our results much.  $g_R = 20$  cm/GW is the Raman gain coefficient [3] in the 1550-nm regime and the Raman gain spectrum is assumed to have a Lorentzian shape with a peak frequency shift of  $\Omega_R/2\pi = 15.6$  THz and a half-width of  $\Gamma_R/2\pi = 52.5$  GHz [14]. The constants  $\xi_e$  and  $\xi_R$  are polarization factors related to the intrinsic symmetry of silicon. For the waveguide configuration shown in Fig. 2,  $\xi_e = 1$  and  $\xi_R = 0$  for the TM mode, but  $\xi_e = 5/4$  and  $\xi_R = 1$  for the TE mode. In the latter case, the factor of 5/4 results from the nonlinear anisotropy of silicon leading to  $\chi_{1212}/\chi_{1111} \approx 0.5$  in the 1550-nm regime [23, 24]. Note that stimulated Raman scattering (SRS) does not occur for the TM mode [3], and FWM is due solely to the electronic response in this case. In contrast, FWM is assisted by SRS in the case of the TE mode, if pump-signal frequency separation is close to  $\Omega_R$  [9, 10].

In Eqs. (2) and (3),  $\alpha_j$  accounts for the linear (scattering) loss and  $\alpha_{fj}(z,t) = 1.45 \times 10^{-17} (\lambda_j / \lambda_{ref})^2 N_{eh}(z,t)$  represents FCA [25], where  $\lambda_{ref} = 1550$  nm and  $N_{eh}(z,t)$  is the density of electron-hole pairs created through pump-induced TPA.  $N_{eh}$  is obtained by solving

$$\frac{\partial N_{eh}}{\partial t} = \frac{\xi_e \beta_T |A_p(z,t)|^4}{2\hbar\omega_p} - \frac{N_{eh}}{\tau_0},\tag{5}$$

where  $\tau_0$  is the carrier lifetime. It depends on the waveguide design and can be reduced to < 1 ns by changing the waveguide geometry to enhance the carrier recombination [1, 26] or by applying an external field to remove carriers [2, 8]. The polarization factor  $\xi_e$  in Eq. (5) results from the polarization-dependent TPA for the pump, as shown in Eqs. (2) and (4).

Equations (2) and (3) are quite general as they take into account all the effects of self-phase modulation (SPM), cross-phase modulation (XPM), SRS, FWM, TPA, FCA as well as GVD and linear scattering losses. The integration in Eq. (3) accounts for the Raman amplification [21, 27] as well as XPM and FWM effects resulting from the slow Raman response. Since the Raman response time is  $1/\Gamma_R \approx 3$  ps, for pulses much longer than 3 ps, the Raman response can be treated as being instantaneous and Eq. (3) reduces to

$$\frac{\partial A_s}{\partial z} + \beta_{1s} \frac{\partial A_s}{\partial t} + \frac{i\beta_{2s}}{2} \frac{\partial^2 A_s}{\partial t^2} = -\frac{1}{2} [\alpha_s + \alpha_{fs}(z,t)] A_s + i\beta_{0s} A_s + i [2\gamma_e + \gamma_R + \gamma_R \widetilde{H}_R(\Omega_{sp})] |A_p|^2 A_s + i [\gamma_e + \gamma_R \widetilde{H}_R(\Omega_{sp})] A_p^2 A_i^*.$$
(6)

In particular, this equation is simplified further if FWM induced by the electronic response dominates, since all terms including  $\gamma_R$  would disappear.

FWM is a coherent process and requires phase matching among the interacting waves. Equations (2) and (6) show that the phase mismatch is given by

$$\kappa = \Delta \beta_0 + 2|A_p|^2 \operatorname{Re}[\gamma_e + \gamma_R H_R(\Omega_{sp})], \tag{7}$$

where the first term represents the linear part,  $\Delta\beta_0 = \beta_{0s} + \beta_{0i} - 2\beta_{0p}$ , and the second term is the nonlinear part resulting from the SPM and XPM introduced by the pump wave [16]. As the signal and idler are located symmetrically around the pump frequency, the linear phase mismatch depends only on even-order dispersion parameters as [28]

$$\Delta\beta_0 = \beta_{2p}\Omega_{sp}^2 + \frac{\beta_{4p}}{12}\Omega_{sp}^4 + \cdots, \qquad (8)$$

where  $\beta_{4p}$  is the fourth-order dispersion at the pump frequency. As the nonlinear part of the phase mismatch is positive, phase matching can be realized by locating the pump in the



Fig. 3. Signal gain (a) and conversion efficiency (b) as a function of signal wavelength at three pump wavelengths in the vicinity of the ZDWL of the TM mode. Input pump intensity is 0.2 GW/cm<sup>2</sup> in all cases. The dashed vertical line shows the location of ZDWL.

anomalous-dispersion regime ( $\beta_{2p} < 0$ ) such that the linear phase mismatch compensates the nonlinear one. The quasi-phase-matched bandwidth is inversely proportional to  $|\beta_{2p}|^{1/2}$  [28]. Thus, if the pump is located close to the ZDWL such that  $|\beta_{2p}|$  is small, FWM will occur over a broad bandwidth set by the  $\beta_{4p}$  term in Eq. (8). Such quasi-phase matching can be realized by tailoring the waveguide ZDWL, as we proposed in the last section, and it provides broadband and efficient parametric generation, as we show in the following sections.

In the case of a CARS configuration [10], linear phase mismatch dominates over the nonlinear one because of a large pump-signal detuning. As a rough estimate, quasi-phase matching is satisfied when  $\Delta\beta_0 \approx 0$ , or when  $\beta_{2p} \approx -\beta_{4p}\Omega_{sp}^2/12$ . Since  $\beta_{4p} \sim 10^{-5}$  ps<sup>4</sup>/m for SOI waveguides (see the preceding section), efficient CARS requires pump GVD to be around  $10^{-2}$  ps<sup>2</sup>/m for a pump-signal detuning of 15.6 THz. For a third-order dispersion of  $\sim 10^{-3}$  ps<sup>3</sup>/m at the ZDWL, this value limits the pump wavelength to be within 20 nm from the ZDWL. It is clear that dispersion tailoring is also required for wavelength conversion based on CARS.

## 4. Parametric generation with a CW pump

We first consider parametric generation with a CW pump, as it is the most practical situation for wavelength conversion. In this case, the steady-state solution of Eq. (5) provides  $N_{eh} = \tau_0 \xi_e \beta_T |A_p|^4 / (2\hbar\omega_p)$ . If we assume the signal to be in the form of a CW beam, the terms with time derivatives in Eqs. (2) and (6) disappear. The resulting equations can be easily solved to provide the signal gain  $G_s$  and the wavelength-conversion efficiency  $G_i$ , defined as

$$G_j = 10\log_{10}[|A_j(L)|^2 / |A_s(0)|^2] \quad (j = s, i),$$
(9)

for a waveguide of length *L*. Figure 3 shows  $G_s$  and  $G_i$  spectra in the TM mode for a 3-cm-long waveguide with a free-carrier lifetime of 1 ns and a linear loss of 0.2 dB/cm (assumed to be same for all three waves). The waveguide is pumped with a CW beam of intensity 0.2 GW/cm<sup>2</sup> (or a pump power of 0.76 W for the waveguide shown in Fig. 2).

When the pump is launched in the normal-dispersion regime (green curves in Fig. 3), phase matching limits parametric generation to a relatively small bandwidth. This case corresponds to a recent experimental configuration [13] in which pump wavelength fell in a regime with significant normal dispersion. In contrast, if pump wavelength coincides with the ZDWL (blue curves), parametric generation occurs over a broad bandwidth extending from 1450 to 1670 nm



Fig. 4. (a)  $G_s$  (dashed curves) and  $G_i$  (solid curves) as a function of input pump intensity for several values of carrier lifetime. (b) Upper limit of carrier lifetime  $\tau_0$  and linear propagation loss  $\alpha_s$  for efficient FWM. In the TE case, pump-signal detuning is set to be 15.54 THz, where the contribution of Raman nonlinearity is maximum.

with nearly uniform efficiency. This 220-nm bandwidth covers the entire S, C, L, U telecommunication bands. If the pump wavelength moves far into the anomalous-GVD regime (red curves), the bandwidth decreases considerably. However, the balance between linear and nonlinear phase mismatch still enables quasi-phase matching over a 90-nm-wide spectral region (1530 to 1620 nm), covering the C and L telecommunication bands.

Figure 3 shows that TPA affects the FWM process by generating free carriers. Unless these carriers are removed by some techniques, such as by applying an external electric field [2, 8] or by introducing non-radiative centers, FCA produced by them becomes a major limiting factor for CW pumping. Because of FCA, the signal gain is negative on a dB scale, indicating a net signal loss. A slight tilt in Fig. 3(a) also stems from FCA, which scales with signal wavelength as  $\lambda_s^2$ . The situation is worse for wavelength conversion efficiency, which has a maximum value of about -8 dB at a pump level of 0.2 GW/cm<sup>2</sup> when the carrier lifetime is 1 ns.

One may ask whether it is possible to realize net gain by increasing pump intensity. The answer for a CW pump turns out to be negative because of the detrimental effects of FCA. Figure 4(a) shows  $G_s$  and  $G_i$  as a function of pump intensity for several values of the carrier lifetime  $\tau_0$ . The pump is launched at the ZDWL of 1551.3 nm, and the signal is located at 1601.3 nm. Both  $G_s$  and  $G_i$  decrease with increasing  $\tau_0$ . This is so because the magnitude of FCA scales linearly with the carrier lifetime. The rate of TPA scales quadratically with pump intensity and makes the FCA problem worse at higher intensity levels. For  $\tau_0 = 10$  ns, the peak conversion efficiency is only -17.5 dB, but it increases to -8 dB if  $\tau_0$  is reduced to 1 ns. For a CW pump, positive signal gain and transparent wavelength conversion are possible only if the carrier lifetime can be reduced to below 100 ps.

Linear propagation losses also affect the FWM process. If the use of CW pumps is required, one needs to understand the ultimate limit imposed by FCA and linear losses. We deduce from Eqs. (2) and (6) that, when the phase-matching condition is satisfied, FWM efficiency in a relatively small section of waveguide scales with pump intensity  $I_p$  as

$$\eta_f \propto 2I_p \operatorname{Re}[\gamma_e + \gamma_R H_R(\Omega_{sp})] - 2\xi_e \beta_T I_p - \sigma_s I_p^2 - \alpha_s, \tag{10}$$

where the four terms represent FWM, TPA, FCA, and linear loss for the signal, respectively, and  $\sigma_s$  is the FCA coefficient defined as  $\sigma_s = 1.45 \times 10^{-17} (\lambda_s / \lambda_{ref})^2 \xi_e \beta_T \tau_0 / (2\hbar\omega_p)$ . Efficient



Fig. 5. (a) Signal gain (a) and conversion efficiency (b) for the TE mode under the same conditions as in Fig. 3.

parametric generation requires  $\eta_f$  to be positive. This is possible only when  $\alpha_s \sigma_s < \{\text{Re}[\gamma_e + \gamma_R \tilde{H}_R(\Omega_{sp})] - \xi_e \beta_T\}^2$ . This criterion imposes an upper limit on the magnitude of linear loss and free-carrier lifetime for the waveguide. Figure 4(b) shows this upper limit for the TM case as well as the TE case with CARS. Efficient FWM is impossible in the region above the curves. For example, the case of  $\tau_0 = 10$  ns and  $\alpha_s = 0.2$  dB/cm is located above the blue curve, resulting in very low efficiency of wavelength conversion [green curves in Fig. 4 (a)]. Similarly, it is not possible to realize efficient wavelength conversion in commonly used waveguides for which  $\alpha_s \sim 1$  dB/cm and  $\tau_0 \sim 10$  ns typically.

In the case of CARS, the situation improves because of the contribution of the Raman nonlinearity. This makes it possible to realize wavelength conversion with net gain by using a CW pump, although over a relatively narrow bandwidth. Figure 5 shows the  $G_s$  and  $G_i$  spectra in the case of TE mode, under the same pumping conditions used for Fig. 3. The situation is similar to the TM-mode case over a broad spectral region in which electronic response dominates. However, as SRS can occur in this case, it assists FWM by enhancing  $G_s$  and  $G_i$  when the pump-signal detuning is close to the Raman shift. When the pump wavelength is far into the normal (green curves) or anomalous (red curves) dispersion regime, phase matching cannot occur at the Raman frequency shift. As a result, the signal experiences large Raman gain or loss, depending on whether it is on the Stokes or anti-Stokes side of the pump.

When the pump is located at the ZDWL (blue curves), broadband quasi-phase matching can partially cover the Raman regime, leading to enhanced CARS (see the sharp blue peak at 1435.6 nm). The phase-matching condition can be further optimized by launching the pump with slightly normal dispersion (1541.3 nm, black curves) such that second- and fourth-order dispersions compensate each other near the Raman frequency shift. As seen in Fig. 5, this configuration results in a net gain of 5 dB on the anti-Stokes side and a wavelength conversion efficiency of about 9 dB. It follows that tailoring of the waveguide ZDWL can help in realizing phase matching for efficient CARS. Note that an enhanced peak always occurs for the idler on the anti-Stokes side, regardless of the phase-matching condition. Such a peak cannot be used as an indication of quasi-phase matching. Figure 5 shows that phase matching occurs only when an enhanced parametric generation is realized for the signal located on the anti-Stokes side of the pump.



Fig. 6. Signal gain (a) and conversion efficiency (b) for the TM mode at three pump wavelengths for pump pulses with a peak intensity of  $0.6 \text{ GW/cm}^2$  when FCA is neglected.

#### 5. Parametric generation with a pulsed pump

FCA can be reduced significantly by using pump pulses that are shorter than the carrier lifetime  $\tau_0$ , as shown in the experiments on Raman amplification [6, 7]. Because of the instantaneous nature of the third-order electronic response, FWM efficiency remains nearly intact in such a transient regime. Thus, the use of a pulsed pump provides a practical solution to the FCA limitation encountered in the preceding section. Note that even though FCA is negligible, TPA still occurs and adds to the linear losses. However, the FWM efficiency scales with pump intensity as  $\gamma_0 |A_p|^2$ , while TPA scales as  $\beta_T |A_p|^2$  for the signal and idler [see Eq. (4) and (10)]. As  $\gamma_0 / \beta_T \approx 5.4$  in the 1550-nm regime, FWM dominates, and gain increases with increasing pump intensity. Figure 6 shows the  $G_s$  and  $G_i$  spectra for the TM mode when pump pulses are much shorter than  $\tau_0$  and are launched with a peak intensity of 0.6 GW/cm<sup>2</sup>.

In the absence of FCA, the pump intensity remains at a relatively high level along the waveguide, and the nonlinear phase mismatch becomes significant. Phase matching occurs when nonlinear part of phase mismatch compensates its linear part. As is well known [16], gain spectra in this case exhibit two peaks near which both  $G_s$  and  $G_i$  grow exponentially with pump intensity and waveguide length [28]. In contrast, they only increase quadratically with pump intensity when the signal wavelength is located close to the pump wavelength. When the SOI waveguide is pumped in the normal-dispersion regime (green curves), parametric amplification in Fig. 6 occurs over a bandwidth of 60 nm with a peak value of 4.8 dB. However, if we launch the pump at the ZDWL (blue curves), amplification occurs over an extremely broad band ranging from 1402 to 1737 nm, with a peak value of about 15 dB. The same value of peak gain can be obtained by pumping in the anomalous-GVD regime, but the bandwidth is reduced if the pump is detuned too far from the ZDWL. For example, parametric amplification occurs over 1475–1681 nm when pumping at 1571.3 nm.

Experimentally, such highly efficient amplification and wavelength conversion can be realized even for a CW signal by launching a train of pump pulses. Of course, FWM will amplify the signal and idler only in time slots in which a pump pulse is present, thus converting a weak CW signal into a pulse train. Figure 7 shows the temporal profiles of the pump, signal, and idler for the TM mode, obtained numerically by solving Eqs. (2), (3), and (5) with the split-step Fourier method [16]. The pump is launched into the anomalous-GVD regime at a wavelength of 1571.3 nm in the form of 16.7-ps (FWHM) Gaussian pulses, and the 1645-nm CW signal is located at the parametric gain peak. Because of TPA, pump intensity decreases by about 50%



Fig. 7. Input and output temporal profiles of (a) pump and (b) signal for a carrier lifetime of  $\tau_0 = 1$  ns; red curve shows the idler pulse. The 16.7-ps pump pulses at 1571.3 nm have a peak intensity of 0.6 GW/cm<sup>2</sup>. Both signal and idler profiles are normalized by the input signal intensity.

at the waveguide output. However, it still amplifies the signal by a factor of 30 and generates an idler pulse. The 8.6-ps width of signal and idler pulses is shorter than the pump width because of the exponential dependence of gain on the pump intensity. Unlike Raman amplification, which is very sensitive to pump spectral width, the 1.33-nm width of the pump spectrum does not affect the FWM efficiency because of the broadband nature of parametric amplification.

Further numerical simulations show that FCA is negligible for pump pulses as wide as 1 ns. Figure 8 shows the temporal profiles of the pump, signal, and idler pulses for  $\tau_0 = 1$ , 10, and 50 ns under the conditions of Fig. 7 except for a Gaussian pump pulse of 16.7 ns width. When  $\tau_0 = 1$  ns, the pump shape is still dominated by TPA, but FCA reduces the signal gain to 20. If  $\tau_0$  increases to 10 ns, FCA starts to affect the pump itself and introduces temporal asymmetry. The amplification factor decreases to 7, and a long tail is formed in the signal pulse [see Fig. 8(d)]. The dip at the leading edge of the signal pulse is produced by TPA. It is interesting to note that the idler pulse shows neither this TPA dip nor the long FCA tail.

#### 6. Parametric generation with a dual-pump configuration

Although efficient parametric amplification and wavelength conversion can be realized with the use of a single pulsed pump, the gain spectrum is not uniform and exhibits a dip because of the dispersive nature of phase matching. For the example of Fig. 6, signal gain changes from 15 dB near the gain peak to only 5 dB when the signal wavelength is close to the pump wavelength. As is known in the context of fiber-optic parametric amplifiers [29]–[31], this problem can be solved simply by employing a two-pump configuration in which parametric gain is produced through non-degenerate FWM such that  $\omega_l + \omega_h = \omega_s + \omega_i$ , where  $\omega_l$  and  $\omega_h$  are the frequencies of the low- and high-frequency pumps. In this case, Eqs. (2) and (3) change as follows:

$$\begin{aligned} \frac{\partial A_l}{\partial z} &+ \beta_{1l} \frac{\partial A_l}{\partial t} + \frac{i\beta_{2l}}{2} \frac{\partial^2 A_l}{\partial t^2} = -\frac{1}{2} [\alpha_l + \alpha_{fl}(z,t)] A_l + i\beta_{0l} A_l + i(\gamma_e + \gamma_R) |A_l|^2 A_l \\ &+ i(2\gamma_e + \gamma_R) |A_h|^2 A_l + i\gamma_R A_h \int_{-\infty}^t h_R(t-\tau) e^{i\Omega_{lh}(t-\tau)} A_h^*(z,\tau) A_l(z,\tau) d\tau, \end{aligned}$$
(11)  
$$\begin{aligned} \frac{\partial A_s}{\partial z} &+ \beta_{1s} \frac{\partial A_s}{\partial t} + \frac{i\beta_{2s}}{2} \frac{\partial^2 A_s}{\partial t^2} = -\frac{1}{2} [\alpha_s + \alpha_{fs}(z,t)] A_s + i\beta_{0s} A_s + i(2\gamma_e + \gamma_R) I_p A_s + 2i\gamma_e A_l A_h A_i^* \end{aligned}$$

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Fig. 8. Temporal profiles of (a) pump, (b) idler, and (c) signal pulses for three values of carrier lifetime. All Other parameters are the same as in Fig. 7. (d) Signal temporal profiles plotted on a log scale.

$$+ i\gamma_R \sum_{j,k=l,h} A_j \int_{-\infty}^t h_R(t-\tau) e^{i\Omega_{sj}(t-\tau)} \left[ A_j^*(z,\tau) A_s(z,\tau) + A_k(z,\tau) A_i^*(z,\tau) \right] d\tau,$$
(12)

where  $j \neq k$ ,  $A_l$  is the field amplitude for low-frequency pump at  $\omega_l$ , and  $I_p = |A_l|^2 + |A_h|^2$ is the total pump intensity. The equation for  $A_h(z,t)$ , governing the high-frequency pump at  $\omega_h$ , is obtained by exchanging the subscript *l* and *h* in Eq. (11). Similarly, the idler equation is obtained by exchanging the subscript *s* and *i* in Eq. (12). Also,  $|A_p|^4$  in the TPA term of Eq. (5) should be replaced by  $I_p^2$ .

For pulses much longer than the 3-ps Raman response time, SRS can be treated as being instantaneous and Eqs. (11) and (12) reduces to

$$\frac{\partial A_{l}}{\partial z} + \beta_{1l} \frac{\partial A_{l}}{\partial t} + \frac{i\beta_{2l}}{2} \frac{\partial^{2}A_{l}}{\partial t^{2}} = -\frac{1}{2} [\alpha_{l} + \alpha_{fl}(z,t)]A_{l} + i\beta_{0l}A_{l} + i(\gamma_{e} + \gamma_{R})|A_{l}|^{2}A_{l} + i[2\gamma_{e} + \gamma_{R} + \gamma_{R}\widetilde{H}_{R}(\Omega_{lh})]|A_{h}|^{2}A_{l}, \qquad (13)$$
$$\frac{\partial A_{s}}{\partial z} + \beta_{1s} \frac{\partial A_{s}}{\partial t} + \frac{i\beta_{2s}}{2} \frac{\partial^{2}A_{s}}{\partial t^{2}} = -\frac{1}{2} [\alpha_{s} + \alpha_{fs}(z,t)]A_{s} + i\beta_{0s}A_{s} + i[2\gamma_{e} + \gamma_{R} + \gamma_{R}\widetilde{H}_{R}(\Omega_{sl})]|A_{l}|^{2}A_{s} + i[2\gamma_{e} + \gamma_{R} + \gamma_{R}\widetilde{H}_{R}(\Omega_{sh})]|A_{h}|^{2}A_{s} + i[2\gamma_{e} + \gamma_{R}\widetilde{H}_{R}(\Omega_{sl}) + \gamma_{R}\widetilde{H}_{R}(\Omega_{sh})]A_{l}A_{h}A_{i}^{*}. \qquad (14)$$

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Fig. 9. Spectra of parametric gain (blue curve) and conversion efficiency (red curve) for the TM mode pumped with two waves of a same input intensity of  $0.3 \text{ GW/cm}^2$ . The gain exceeds 10 dB over a 350-nm bandwsidth when two pump wavelengths are 255.3 nm apart.

As two pumps participate in the FWM process, the phase-matching condition in Eq. (7) also changes to become

$$\kappa = \Delta \beta_0 + I_p \operatorname{Re}[\gamma_e + \gamma_R \dot{H}_R(\Omega_{sl}) + \gamma_R \dot{H}_R(\Omega_{sh}) - \gamma_R \dot{H}_R(\Omega_{hl})], \qquad (15)$$

where the linear part of the phase mismatch is now given by  $\Delta\beta_0 = \beta_{0s} + \beta_{0l} - \beta_{0l} - \beta_{0h}$  and is related to fiber dispersion as [29]

$$\Delta\beta_0 = \left[\beta_{2c}\Omega_{sc}^2 + \frac{\beta_{4c}}{12}\Omega_{sc}^4\right] - \left[\beta_{2c}\Omega_d^2 + \frac{\beta_{4c}}{12}\Omega_d^4\right] + \cdots.$$
(16)

Here  $\beta_{2c}$  and  $\beta_{4c}$  are the second- and fourth-order dispersion at the center frequency  $\omega_c = (\omega_l + \omega_h)/2$ ,  $\Omega_{sc} = \omega_s - \omega_c$ , and  $\Omega_d = (\omega_h - \omega_l)/2$ . In the case of dual pumping, additional terms appear in the linear phase mismatch that are only determined by the pump frequency separation and is independent of the signal frequency. We can use it to compensate for the frequency-independent nonlinear phase mismatch by adjusting the pump spacing, while maintaining the total phase mismatch at a low level by locating the pump center close to the ZDWL.

Figure 9 shows the  $G_s$  and  $G_i$  spectra for the TM mode when FCA is negligible and two pumps with each intensity of 0.3 GW/cm<sup>2</sup> are located at wavelengths of 1431 and 1686.3 nm. The new feature is that the 12.6-dB gain remains constant over a 110-nm wide region extending from 1490 to 1600 nm. Moreover, the total spectra cover an extremely broad region extending from 1390 nm to 1730 nm, with less than 2-dB ripple. Clearly, the gain spectrum is much more uniform compared with the single-pump case discussed earlier. We stress that such a broad and uniform gain spectrum can be realized with multiple combinations of the two pump wavelengths, as long as these wavelengths are chosen appropriately to ensure quasi-phase matching. For example, similar spectra are obtained when the SOI waveguide is pumped at 1466.3 and 1631.3 nm, or at 1486.3 and 1611.3 nm. Thus, the two-pump configuration allows considerable flexibility in realizing broadband wavelength conversion with uniform efficiency. Furthermore, although the non-degenerate FWM  $\omega_l + \omega_h = \omega_s + \omega_i$  dominates in a dual-pump configuration, several other FWM processes also participate in the parametric process [30] and lead to the generation of multiple idlers. This feature provides an opportunity for multiband operation and fast optical switching, as recently demonstrated in the context of highly nonlinear fibers [32].

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## 7. Conclusions

In this paper we have developed a comprehensive treatment of FWM in SOI waveguides. We first show that the ZDWL in such waveguides can be shifted toward the 1550 nm region with a suitable waveguide design. Moreover, by using an almost square-shaped cross section with a 100% etching depth, it is possible to match the ZDWL for the TE and TM modes. We derive a set of coupled nonlinear equations that describe FWM in such waveguides under quite general conditions. These equations include the contributions of both the electronic and Raman nonlinearities, the SPM, XPM, and TPA effects introduced by the pump waves, and FCA occurring inside the SOI waveguide.

We then show that ultrabroadband parametric generation and wavelength conversion can be realized by launching a pump beam close to the ZDWL of the SOI waveguide. In the case of a CW pump, FCA is found to limit the FWM efficiency, and it is difficult to realize a net signal gain and transparent wavelength conversion, unless the carrier lifetime can be reduced to below 100 ps. We quantify the upper limit of FCA and linear losses required for parametric generation. By investigating the transient dynamics of FWM with a pulsed pump, we show that the instantaneous nature of electronic response enables highly efficient parametric amplification and wavelength conversion for pump pulses as wide as 1 ns. The single-pump configuration suffers from a central dip in the gain spectrum occurring close to the pump wavelength. This dip can be eliminated by adopting a dual-pump configuration. With a suitable choice of pump wavelengths, multiband operation can be realized in the dual-pump configuration over a broad spectral region extending from 1390 nm to 1730 nm, while maintaining less than 2-dB ripple in the gain spectrum. The >300 nm bandwidth covers the entire spectrum that is likely to be used for future ultradense multichannel telecommunication systems. This feature points to the potential application of SOI waveguides for lightwave systems.

# 8. Acknowledgements

We thank F. Yaman and L. Yin for fruitful discussions. This work was supported in part by National Science Foundation under grant ECS-0320816 and ECS-0334982 and a grant from Intel Corporation.