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Linear optical bullets

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Abstract

We describe a class of spatio-temporal optical pulses that spread neither spatially nor temporarily in linear dispersive media over long propagation distances. These new spatio-temporal pulses, referred to as linear optical bullets, can have any spatio-temporal profile and temporal width, and they carry a finite amount of energy. We also discuss in detail a technique for the experimental realization of linear optical bullets.

Spatial and spatio-temporal wave packets that can resist spreading due to diffraction and dispersion play a special role in optics because of their fundamental particle-like behavior with a variety of potential applications. It is often believed that the formation of such wave packets requires a nonlinear medium. Indeed, the entire field of optical solitons is based on the balance of dispersion or diffraction with the nonlinear response of an optical medium [1]. If the nonlinearity of such a medium is capable of balancing both dispersion and diffraction, the resulting spatio-temporal soliton is referred to as an optical bullet [2].

Perhaps somewhat surprisingly, localized wave packets can also form in a linear medium. Many such solutions of the paraxial and non-paraxial wave equations have been found in the context of both optics [3–12] and ultrasonics [13–15]. Localized spatio-temporal solutions propagating either in free space or in linear dispersive media are known as X waves [12,15] because of the particular shape of their spatio-temporal profiles. However, most of the localized waves discussed in the literature thus far are solutions of the wave equations with an infinite amount of energy. Similarly, the so-called non-diffracting beams, first studied in Refs. [5] and existing even in free space, have field envelopes containing an infinite power [16]. Recently found spatio-temporal nonspreading X waves [11,12] propagate in linear dispersive media, but they carry an infinite amount of energy as well. Thus, none of the nonspreading waves studied to date can play a role similar to that of optical bullets [2], which represent pulses with a finite energy and owe their very existence to the *anomalous dispersion* of the medium. A natural question thus arises: Can one construct a linear analog of a finite energy optical bullet that will not spread, at least to good accuracy, on propagation in a linear dispersive medium?

In this communication, we answer this question in the affirmative. More specifically, we show that dispersion and diffraction can act in concert to produce spatio-temporal wave packets with a *finite energy*, in the *normal dispersion* regime of a linear medium. We call such waves, generated by a source of finite spatial size, *linear optical bullets* (LOB) because they carry a finite energy and are not affected by either diffraction or dispersion over long distances. We show that the characteristic distance over which the proposed LOBs become affected by diffraction and dispersion can be controlled by adjusting the size of the source aperture. We also emphasize that unlike the conventional optical bullets [2], whose spatio-temporal shape is determined by the trade-off between spreading due to diffraction and dispersion and self-focusing produced by Kerr nonlinearity, the LOBs proposed in this paper can have any

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spatio-temporal shape. In this work, we propose a specific scheme for the experimental realization of LOBs by analyzing in detail the spectrum of the source that produces them. An important advantage of our scheme is the capability of generating LOBs of a desirable temporal width within the limits of applicability of the slowly varying envelope approximation.

We begin by considering an optical pulse propagating in the normal dispersion regime in a linear optical medium characterized by a frequency-dependent refractive index $n(\omega)$. On expanding the propagation constant $\beta(\omega) = n(\omega)\omega/c$ in a Taylor series around the carrier frequency ω_0 of the pulse and truncating the series after the quadratic term, we arrive at the following paraxial wave equation for the slowly varying field envelope $U(x, y, z, t)$ [1]:

$$\mathrm{i}\frac{\partial U}{\partial z} + \mathrm{i}\beta_1\frac{\partial U}{\partial t} + \frac{1}{2\beta_0}\left(\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2}\right) - \frac{\beta_2}{2}\frac{\partial^2 U}{\partial t^2} = 0. \tag{1}$$

Here $\beta_m = (\mathrm{d}^m\beta/\mathrm{d}\omega^m)_{\omega=\omega_0}$, β_1 is the inverse of the group velocity, and β_2 governs the dispersive properties of the linear medium ($\beta_2 > 0$ for normal dispersion). This equation can be written in terms of the dimensionless variables as

$$\mathrm{i}\frac{\partial U}{\partial Z} + \frac{1}{2}\left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2}\right) - \frac{\partial^2 U}{\partial T^2} = 0, \tag{2}$$

where $X = x/\sigma_{\mathrm{I}}$, $Y = y/\sigma_{\mathrm{I}}$, $Z = z/\beta_0\sigma_{\mathrm{I}}^2$, and $T = (t - \beta_1 z)/T_{\mathrm{p}}$. Physically, T is the retarded time normalized to the pulse width T_{p}. The transverse spatial scale σ_{I} is related to the temporal pulse width T_{p} as $\sigma_{\mathrm{I}}^2 = 2T_{\mathrm{p}}^2/(\beta_0\beta_2)$.

It is easy to show that any wave of the from $F(X + Y - T)$, with $F(x)$ being an arbitrary function of x, is a non-spreading solution of Eq. (2). However, no such solution can be an optical bullet because the energy, $E = \int \mathrm{d}T \int \mathrm{d}X \int \mathrm{d}Y \mid F(X + Y - T)|^2$, associated with such a wave is infinite. Motivated by this observation, we propose as a candidate for the LOB the spatio-temporal envelope whose profile U_0 in the source plane is given by

$$U_0(X, Y, T) = \theta(L_X - |X|)\theta(L_Y - |Y|)F(X + Y - T), \tag{3}$$

where $\theta(x)$ is a unit step-function, and $2L_X$ and $2L_Y$ are the dimensions of the source aperture in units of σ_{I}. The solution to Eq. (2) with the initial condition (3) is readily obtained by first introducing a Fourier transform $\widetilde{U}(X, Y, Z, \Omega)$ of U and noting that each frequency component of the pulse diffracts according to the Fresnel diffraction theory. By taking the inverse Fourier transform, we obtain for the LOB field in any transverse plane $Z > 0$ the expression

$$U(X, Y, Z, T) = \int_{-\infty}^{\infty} \frac{\mathrm{d}\Omega}{2\pi}\widetilde{F}(\Omega)\exp[\mathrm{i}\Omega(X + Y - T)] \times \Psi(X, \Omega, Z)\Psi(Y, \Omega, Z). \tag{4}$$

Here the Fourier transform in the source plane is defined as $\widetilde{F}(\Omega) \equiv \int_{-\infty}^{\infty} \mathrm{d}T F(T)\exp(\mathrm{i}\Omega T)$ and the auxiliary function $\Psi(S, \Omega, Z)$ is defined as

$$\Psi(S, \Omega, Z) \equiv \frac{1}{2}\left\{\mathrm{erf}\left[\frac{(1-\mathrm{i})}{2\sqrt{Z}}(L_S - S + \Omega Z)\right] + \mathrm{erf}\left[\frac{(1-\mathrm{i})}{2\sqrt{Z}}(L_S + S - \Omega Z)\right]\right\}, \tag{5}$$

where $\mathrm{erf}(z) = (2/\sqrt{\pi})\int_0^z \mathrm{e}^{-x^2}\mathrm{d}x$ is the error function.

Physically, Eq. (5) implies that each monochromatic component of the pulse undergoes Fresnel diffraction at the source aperture with the Fresnel number depending on the corresponding frequency. However, if the fields of such monochromatic components are combined with the appropriate spectral weights and phase chirps according to Eq. (4), the resulting spatio-temporal pulse is a non-spreading optical bullet. Further analysis of Eqs. (4) and (5) leads to the following conclusions. First, as $L_S \to \infty$, $(S = X, Y)$, Fresnel diffraction reproduces the profile of the aperture, i. e., the step-functions, and the field amplitude approaches that of an ideal non-spreading wave of the form $F(X + Y - T)$. Second, in the case of a finite-aperture source, a characteristic distance over which the LOB becomes affected by diffraction increases with L_S, and hence, it can be controlled by adjusting the size L_S of the source aperture. These semi-qualitative considerations are confirmed by the behavior of the exact solution (4). The intensity distributions of the bullet at distances $Z = 0$ and $Z = 5$ from the source plane are displayed in Fig. 1, assuming a square aperture of the size $L = 30$. The normalized spectral distribution of the bullet field in the source plane is taken to be a Gaussian, $\widetilde{F}(\Omega) = \exp(-\Omega^2/2)$. It can be seen from Fig. 1 that the spatio-temporal profile of the LOB remains virtually immune to spreading over distances that exceed the diffraction length $z_0 = \beta_0\sigma_{\mathrm{I}}^2$ of a beam of width σ_{I} by at least a factor of 5.

We should stress that our general exact solution (4) predicts that a wide variety of LOBs can exist, distinguished from each other on the basis of their spectra. As an interesting example, we display in Fig. 2 the intensity distribution at the distance $Z = 5$ for an LOB with a finite-bandwidth spectrum. More specifically, we take $\widetilde{F}(\Omega) = \theta(B - \mid\Omega\mid)$ and choose $B = 1/2$. Similar to the Gaussian case, the intensity distribution after 5 diffraction lengths almost coincides with that in the source plane ($Z = 0$), even though this LOB contains a number of satellite peaks in addition to the dominant central peak. The oscillatory structure seen in Fig. 2 is expected for a finite-bandwidth beam and follows a pattern of the form $\sin^2(X - T)/(X - T)^2$. Clearly, LOBs that we have found can have a multipeak structure in space and time that is governed solely by their source spectrum.

It remains to consider the issue of the LOB generation in a laboratory setting. To address this issue, we calculate the spectral amplitude of U_0. Assuming, for simplicity, a Gaussian spectral profile of the LOB in the source plane, taking the Fourier transform of Eq. (3) with respect to T, and converting the result to the dimensional units, we obtain

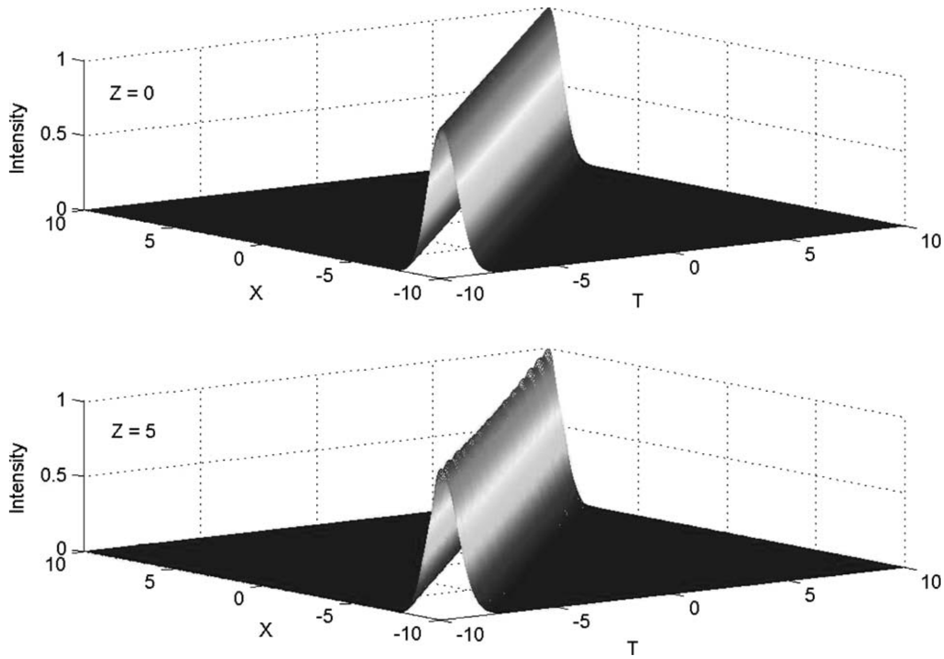

Fig. 1. Intensity profile of the LOB, evaluated at $Y = 0$ and $Z = 0$, within a square aperture of the size $L = 30$ (top) and at the distance $Z = 5$ from the aperture (bottom). The spectral distribution of the LOB field in the source plane is taken to be a Gaussian, centered at the frequency ω_0.

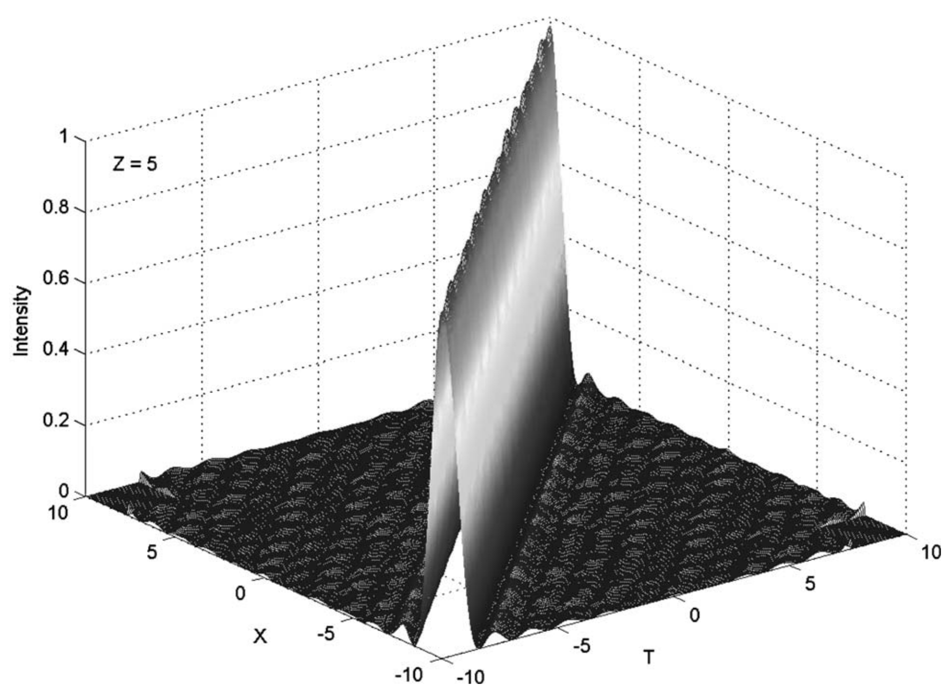

Fig. 2. Intensity profile, evaluated at $Y = 0$, at the distance $Z = 5$ for the LOB generated by a source with a finite-bandwidth spectrum of the form $\widetilde{F}(\Omega) = \theta(B - |\Omega|)$ with $B = 1/2$ and $L = 30$.

$$\widetilde{U}_0(x, y, \omega) \propto \exp(-\omega^2 T_{\mathrm{p}}^2/2) \exp[\mathrm{i}\omega T_{\mathrm{p}}(x + y)/\sigma_{\mathrm{I}}]. \qquad (6)$$

It can be inferred from Eq. (6) that to form an LOB at the source plane, the spectral amplitude of the pulse must have a linear chirp whose magnitude depends on the spatial coordinates through the variable $x + y$.

Fig. 3 shows schematically an experimental setup that can be used to generate the LOB with such a spectral amplitude. The experimental procedure is as follows. An optical beam with a Gaussian spatio-temporal intensity profile is placed at the focus of a thin lens so that the output field is collimated to a desired spatial size in both transverse dimensions. The Gaussian spectrum of the pulse is then separated spatially into individual frequency components (say, along the x-axis) using a bulk grating. The resulting uniform pulse is then transmitted through a

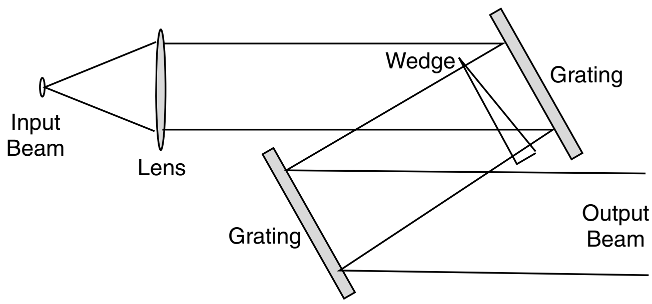

Fig. 3. Schematics of the setup for experimental realization of linear optical bullets.

planar wedge whose thickness varies along x in a linear fashion. Different monochromatic components of the pulse are assumed to propagate close to the z-axis such that the maximum angle the direction of propagation of a given component makes with the z-axis is much smaller than the inclination angle of the wedge. The wedge imparts an x-dependent phase shift (a linear chirp) onto the spectral amplitude of the pulse. A second identical grating is then used to reconstruct the optical beam. This procedure is repeated to impose a y-dependent linear chirp on the pulse by using another wedge and a grating pair oriented orthogonal to the first one. We assume that dispersion of the grating pairs and the wedge material is either negligible or is compensated using a suitable dispersion-compensation scheme.

It follows from the geometry of Fig. 3 that the spectral amplitude of the modified beam can be written as

$$\widetilde{U}_s(x,y,\omega) \propto \exp(-\omega^2 T_p^2/2)\exp[i(\omega/c)n_w(x+y)\tan\alpha], \quad (7)$$

where n_w is a linear refractive index of the wedge at the carrier frequency ω_0 and α is the angle of inclination of the plane of the wedge. On comparing Eqs. (6) and (7), we conclude that in order for the proposed setup to realize an LOB, one has to adjust the angle α such that

$$\tan\alpha = \frac{c}{n_w}\frac{T_p}{\sigma_I} = \frac{c}{n_w}\sqrt{\frac{\beta_0\beta_2}{2}}. \quad (8)$$

Note that the inclination angle of each wedge is independent of the parameters of the pulse, and hence the same experimental setup can generate LOBs of any temporal width. It is also worth pointing out that LOBs of different spatio-temporal profiles can be generated starting from a pulse of a given temporal profile $F(T)$ and following the procedure just described.

In summary, we have presented a novel class of spatio-temporal pulses, which carry a finite amount of energy and do not spread either spatially or temporarily on propagation in linear, normally dispersive media over long distances. Consequently, these pulses qualify as linear optical bullets. Such bullets can have any spatio-temporal profile and any temporal width as long as the slowly varying envelop approximation remains valid. We have also proposed a concrete way for realizing LOBs experimentally. The proposed LOBs are expected to have many applications ranging from high definition metrology to telecommunications.

References

[1] Yu.S. Kivshar, G.P. Agrawal, Optical Solitons: From Fibers to Photonic Crystals, Academic Press, Boston, 2003.
[2] Y. Silberberg, Opt. Lett. 15 (1990) 1282, For an up-to-date review on the subject, see Chap. 7 of Ref. [1].
[3] J.M. Brittingham, J. Appl. Phys. 54 (1983) 1179.
[4] R.W. Ziolkowski, J. Math. Phys. 26 (1985) 861.
[5] J. Durnin, J. Opt. Soc. Am. A 4 (1987) 651.
[6] R.W. Ziolkowski, I.M. Besieris, A.M. Shaarawi, J. Opt. Soc. Am. A 10 (1993) 75.
[7] J. Fagerholm, A.T. Friberg, J. Huttunen, D.P. Morgan, M.M. Salomaa, Phys. Rev. E 54 (1996) 4347.
[8] J. Salo, J. Fagerholm, A.T. Friberg, M.M. Salomaa, Phys. Rev. E 62 (2000) 4261.
[9] P. Saari, K. Reivelt, Phys. Rev. Lett. 79 (1997) 4135.
[10] H. Sonajalg, M. Ratsep, P. Saari, Opt. Lett. 22 (1997) 310.
[11] M.A. Porras, Opt. Lett. 26 (2001) 1364.
[12] D.N. Christodoulides, N.K. Efremidis, P. Di Tripani, B.A. Malomed, Opt. Lett. 29 (2004) 1446.
[13] J. Lu, J.F. Greenleaf, IEEE Trans. Ultrasonics Ferroelec. Freq. Contr. 39 (1992) 19;
J. Lu, J.F. Greenleaf, IEEE Trans. Ultrasonics Ferroelec. Freq. Contr. 39 (1992) 441.
[14] J. Lu, J.F. Greenleaf, IEEE Trans. Ultrasonics Ferroelec. Freq. Contr. 44 (1997) 181.
[15] N.V. Sushilov, J. Tavakkoli, R.S.C. Cobbold, IEEE Trans. Ultrasonics Feroelec. Freq. Control 48 (2000) 274.
[16] Finite-energy realizations of approximately nondiffracting X-wave fields have also been realized in several experiments [6,9,10].