## Dispersion tailoring and soliton propagation in silicon waveguides

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The dispersive properties of silicon-on-insulator (SOI) waveguides are studied by using the effective-index method. Extensive calculations indicate that an SOI waveguide can be designed to have its zero-dispersion wavelength near 1.5  $\mu$ m with reasonable device dimensions. Numerical simulations show that soliton-like pulse propagation is achievable in such a waveguide in the spectral region at approximately 1.55  $\mu$ m. The concept of path-averaged solitons is used to minimize the impact of linear loss and two-photon absorption. © 2006 Optical Society of America

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Silicon-on-insulator (SOI) waveguides have attracted considerable interest recently, as they can be used for making inexpensive, monolithically integrated, optical devices. In particular, stimulated Raman scattering (SRS) has been used to realize optical gain in SOI waveguides. This Raman gain has been used for fabricating active optical devices, such as optical modulators and silicon Raman lasers. If ultrashort pulses are used with an SOI device, one can make use of the intensity dependence of the refractive index, provided that the dispersive effects are properly accounted for. However, the dispersive properties of SOI waveguides have not been extensively studied so far, although some initial work has been done. The strategy of the strate

In this Letter we consider dispersion in SOI waveguides and show that their zero-dispersion wavelength (ZDW)  $\lambda_0$  typically exceeds 2  $\mu$ m. We also show that  $\lambda_0$  can be shifted to below 1.5  $\mu$ m with reasonable device parameters. Under such conditions, an ultrashort pulse at 1.55  $\mu$ m should form a soliton as it propagates in the waveguide. This possibility may lead to new applications of SOI waveguides related to optical interconnects and high-speed optical switching. We use a modified nonlinear Schrödinger equation to study soliton evolution inside SOI waveguides in the presence of linear loss and two-photon absorption (TPA).

Our approach makes use of the effective-index method<sup>9</sup> to obtain the dispersion relation  $\beta(\omega)$  numerically for the TM<sub>0</sub> and TE<sub>0</sub> waveguide modes, where  $\beta$  is the modal propagation constant at the frequency  $\omega$ . For our study, the three important parameters are the width W, the height H, and the etch thickness h for the waveguide geometry shown as the inset in Fig. 1; the dispersive properties should vary considerably with these parameters. We first set W =1.5  $\mu$ m, H=1.55  $\mu$ m, and h=0.7  $\mu$ m, the values used in recent experiments. The material dispersion of Si and SiO<sub>2</sub> is included using the Sellmeier relations. The modal refractive indices are determined from  $\bar{n}(\omega) = \beta(\omega)$   $c/\omega$  and are plotted in Fig. 1 as a function of wavelength. The difference between the two modal indices is related to the waveguide-induced birefringence.

Dispersion to the nth order can be calculated from  $\beta(\omega)$  using the relation  $\beta_n(\omega) = d^n \beta / d \omega^n$ . The wavelength dependence of the second- and third-order dispersion parameters is shown in Fig. 2. The ZDW of the TM<sub>0</sub> mode occurs near 2.1  $\mu$ m, and that of the TE<sub>0</sub> mode near 2.3  $\mu$ m. In the wavelength region near 1.55  $\mu$ m,  $\beta_2 \approx 0.7$  ps²/m is positive (normal dispersion) for both modes. The third-order dispersion is relatively small with a value of  $\beta_3 \approx 0.002$  ps³/m. We stress that our results are approximate because of the use of the effective-index approximation.

The important question from a practical standpoint is whether SOI waveguides can be designed to exhibit anomalous dispersion ( $\beta_2 < 0$ ) near 1.55  $\mu$ m. This is possible if  $\lambda_0$  is reduced to below 1.55  $\mu$ m by choosing the appropriate device parameters. We have performed extensive numerical calculations to study how the ZDW depends on W, H, and h and how it can be controlled by designing the SOI waveguide suitably. The results are shown in Fig. 3. Figures 3(a) and 3(b) indicate that  $\lambda_0$  decreases as W and H are reduced. Figure 3(c) shows that there is an optimum value of h for the TM<sub>0</sub> mode for minimizing  $\lambda_0$ . The contours of this optimum value of h are shown in Fig. 3(d) in the W-H plane. Note that  $\lambda_0$  is almost always

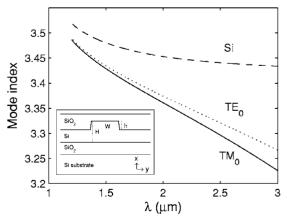


Fig. 1. Modal refractive indices of the  $\text{TE}_0$  (dotted curve) and  $\text{TM}_0$  (solid curve) modes for  $W=1.5~\mu\text{m}$ ,  $H=1.55~\mu\text{m}$ , and  $h=0.7~\mu\text{m}$ . The material dispersion of silicon is shown by a dashed curve. The inset shows the waveguide geometry.

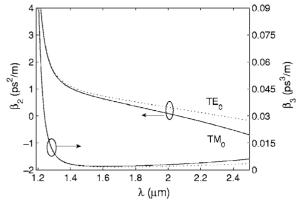


Fig. 2. Wavelength dependence of  $\beta_2$  and  $\beta_3$  for the  $TE_0$  (dotted curves) and  $TM_0$  (solid curves) modes.

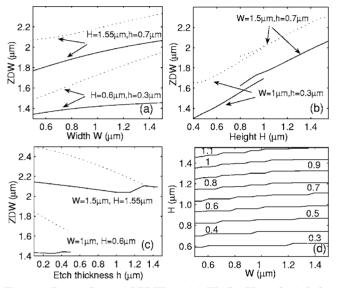


Fig. 3. Dependence of ZDW on (a) W, (b) H, and (c) h for the  $TE_0$  (dotted curves) and  $TM_0$  (solid curves) modes; (d) contours of optimum h in the range  $0.3-1.1~\mu m$  in W-H plane for the  $TM_0$  mode.

lower for the  $TM_0$  mode compared with the  $TE_0$  mode. In fact,  $\lambda_0$  cannot be reduced to below 1.5  $\mu m$  for the  $TE_0$  mode because the required width of  $W<0.3~\mu m$  becomes impractical. In the case of the  $TM_0$  mode,  $\lambda_0$  can be reduced to below 1.55  $\mu m$  for W in the range of 0.5–1.5  $\mu m$  provided that H and h are chosen properly.

To find the range of W and H for realizing a specific value of  $\lambda_0$ , we depict in Fig. 4 the contours of constant  $\lambda_0$  in the W-H plane with h optimized in each case. It follows that the dispersion can be made anomalous at 1.55  $\mu$ m for a wide range of device parameters. As an example,  $\beta_2 < 0$  at 1.55  $\mu$ m when  $W = 1 \ \mu$ m,  $H = 0.6 \ \mu$ m, and  $h \approx 0.3 \ \mu$ m. These device parameters, although on the low side, are realistic for SOI waveguides.

One should expect an optical soliton to form inside an SOI waveguide if  $\beta_2$ <0. A rough estimate of the pulse parameters can be obtained by using the standard soliton theory. According to this theory, a fundamental soliton can be excited if  $\gamma P_0 L_D = 1$ , where  $\gamma = 2\pi n_2/(\lambda A_{\rm eff})$  is the nonlinear parameter,  $P_0$  is the

peak power, and  $L_D=T_0^2/|\beta_2|$  is the dispersion length for a pulse of width  $T_0$ . The nonlinear refractive index of silicon is  $n_2\!\approx\!4.4\!\times\!10^{-18}\,\mathrm{m}^2/\mathrm{W}.^{11}$  Using  $W\!=\!1~\mu\mathrm{m}, H\!=\!0.6~\mu\mathrm{m}$ , and  $h\!=\!0.3~\mu\mathrm{m}$ , the parameters of the  $TM_0$  mode at  $1.55~\mu\mathrm{m}$  are found to be  $\beta_2\!=\!-0.56~\mathrm{ps}^2/\mathrm{m}$ ,  $\beta_3\!=\!5.2\!\times\!10^{-3}~\mathrm{ps}^3/\mathrm{m}$ ,  $\lambda_0\!=\!1.42~\mu\mathrm{m}$ ,  $A_{\mathrm{eff}}\!=\!0.38~\mu\mathrm{m}^2$ , and  $\gamma\!=\!47~\mathrm{W}^{-1}/\mathrm{m}$ . If we assume  $L_D$  =1 cm, then  $T_0\!=\!75~\mathrm{fs}$ , corresponding to a full width at half maximum of 130 fs for the pulse shape governed by  $P(t)\!=\!P_0\,\mathrm{sech}^2(t/T_0)$ . The required peak power for  $\gamma\!P_0L_D\!=\!1$  is approximately 2.1 W, a relatively low value for 130 fs pulses.

Before concluding that a soliton would form when such pulses are launched into the waveguide, we should consider the impact of linear loss, TPA, and free-carrier absorption (FCA). SRS can be ignored for 130 fs pulses because their bandwidth ( $\approx$ 2.4 THz) is much less than the Raman shift of 15.6 THz for Si. We modify the standard nonlinear Schrödinger equation to include TPA and FCA, and obtain

$$\frac{\partial A}{\partial z} + \frac{\alpha}{2}A + \frac{i\beta_2}{2}\frac{\partial^2 A}{\partial t^2} - \frac{\beta_3}{6}\frac{\partial^3 A}{\partial t^3} = i\kappa |A|^2 A - \frac{\alpha_f}{2}A, \quad (1)$$

where  $\alpha$ =0.22 dB/cm is the linear loss, and  $\kappa$ = $\gamma$ + $i\Gamma/2$ . The imaginary part of  $\kappa$  is related to the TPA coefficient,  $\beta_{\text{TPA}}$ =5 $\times$ 10<sup>-12</sup> m/W, as  $\Gamma$ = $\beta_{\text{TPA}}/A_{\text{eff}}$ =13 W<sup>-1</sup>/m. FCA is included by  $\alpha_f$ = $\sigma N$ , where  $\sigma$ =1.45 $\times$ 10<sup>-21</sup> m<sup>2</sup> for silicon, and N is the density of carriers produced by TPA. It is obtained by solving

$$\frac{\partial N}{\partial t} = \frac{\beta_{\text{TPA}}}{2h \nu_0} \frac{P^2(z)}{A_{\text{eff}}^2} - \frac{N}{\tau},\tag{2}$$

where  $\tau \approx 25$  ns is the effective carrier lifetime. For  $T_0 \ll \tau$ , and at relatively low repetition rates, N can be approximated as  $N \approx 2\beta_{\text{TPA}} P_0^2 T_0/(3h \nu_0 A_{\text{eff}}^2)$ . For the device parameters used,  $N \approx 6.1 \times 10^{19} \text{ m}^{-3}$ . Since  $\alpha_f \approx 8.8 \times 10^{-4} \text{ cm}^{-1} \ll \alpha$  for this value of N, we can ignore the FCA in Eq. (1).

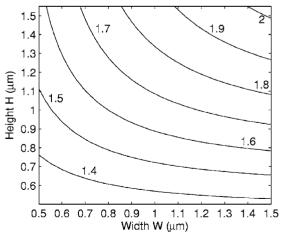


Fig. 4. Contours of constant ZDW for the TM<sub>0</sub> mode as a function of W and H in the range of  $\lambda_0 = 1.4 - 2 \mu m$ ; etch thickness h is optimized for each set of W and H and is in the range h/H = 0.4 - 0.8.

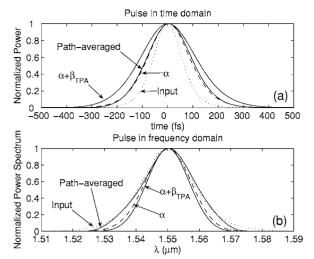


Fig. 5. Output (a) pulse shape and (b) spectrum with (solid curves) and without (dashed curves), TPA effects with third-order dispersion and linear loss included in both cases; dotted curves show input profiles. The curve marked path-averaged shows that loss-induced pulse broadening can be reduced by increasing the input peak power suitably.

We solve Eq. (1) with the split-step Fourier method  $^{10}$  and study soliton propagation inside a 5 cm long SOI waveguide. The input and output pulse shapes and the corresponding spectra are plotted in Fig. 5 under several different conditions. The pulse does not maintain its width because of  $\beta_3$ ,  $\alpha$ , and  $\beta_{\text{TPA}}$ . The impact of  $\beta_3$  is found to be relatively minor. Even without TPA, linear loss leads to pulse broadening, and TPA enhances this broadening further. However, we should stress that the pulse would broaden by a factor of 4 in the absence of nonlinear effects. Clearly, soliton effects help because the pulse broadens by a factor of less than 2.

We can reduce pulse broadening even further by using the concept of a path-averaged soliton. <sup>10</sup> In this approach, the input peak power is increased by averaging the pulse peak power over the waveguide length,  $\bar{P}_0 = (1/L) \int_0^L P_0(z) dz$ , and requiring  $\gamma \bar{P}_0 L_D = 1$ . This amounts to enhancing the input peak power by a factor of  $F_e = \alpha_t L/[1 - \exp(-\alpha_t L)]$ , where  $\alpha_t = \alpha + \Gamma \bar{P}_0$  is the total effective loss. For the parameters used,  $F_e$  equals 1.58. As shown in Fig. 5, the broadening in-

duced by linear loss and TPA can be reduced considerably when the input power is increased by this factor. Moreover, the pulse spectrum becomes almost identical to that of the input pulse.

In summary, we studied the dispersive properties of SOI waveguides and found that the ZDW of the  $TM_0$  mode is approximately 2.1  $\mu m$  for the device used in Ref. 4. We used numerical calculations to reveal the dependence of the ZDW on the three design parameters of the device. The results show that  $\lambda_0$  can be reduced to below 1.5  $\mu m$  with reasonable waveguide dimensions. Propagation of a 130 fs pulse in the spectral region near 1.55  $\mu m$  reveals that such a pulse can nearly maintain its shape and spectrum over a 5 cm long waveguide because of the solitonlike effects in the anomalous-dispersion regime.

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