Impact of Pump-Phase Modulation on Dual-Pump Fiber-Optic Parametric Amplifiers and Wavelength Converters

F. Yaman, Qiang Lin, *Student Member, IEEE*, Stojan Radic, *Member, IEEE*, and Govind P. Agrawal, *Fellow, IEEE*

***Abstract*—Modulation of pump phases required for suppressing stimulated-Brillouin scattering is shown to cause large power changes in both the amplified and wavelength-converted signals when dual-pump fiber-optic parametric amplifiers are used. The physical origin of power changes is related to pump-amplitude modulations caused by fiber dispersion. Analytical as well as numerical calculations show that the signal-to-noise ratio of the signal or idler may reduce to below 23 dB when pump phases change rapidly with a sharp rise time.**

***Index Terms*—Optical fiber amplifiers, optical phase conjugation, phase modulation, wavelength conversion.**

FIBER-OPTIC parametric amplifiers (FOPAs) are well known for their uniform gain over a wide bandwidth and being useful for wavelength conversion [1]–[6]. Although ultrafast response of FOPAs lends them to all-optical signal processing applications [1], it also makes them susceptible to temporal perturbations associated with the pumps. One such perturbation stems from the technique commonly used for suppressing stimulated Brillouin scattering (SBS); it consists of modulating pump phases at a relatively high rate (>1 GHz). Pump-phase modulation (PM) has been shown to cause broadening of the idler spectrum [6] and temporal variations in the signal gain [7], [8] when a single-pump FOPA is used. Such degradations can be mitigated if dual-pump FOPAs are used [3]. However, we show in this letter that a different mechanism then causes fluctuations in the amplified signal and idler powers and it can lower their signal-to-noise ratio (SNR) to below 20 dB. The group-velocity difference among the signal, idler, and the pumps, can reduce the extent of SNR degradation somewhat, depending on the fiber parameters.

Most dual-pump FOPAs use two continuous-wave pumps whose wavelengths are set 40–50 nm apart but are located almost symmetrically around the zero-dispersion wavelength of the fiber [3]–[6]. A nearly uniform gain is produced in this case in the spectral region between the pumps. Pump powers required by such FOPAs are high (>100 mW). However, the amount of pump power that can be transmitted through a fiber is limited by the SBS threshold [9]. Because SBS has a narrow gain spectrum (bandwidth <100 MHz), it is possible to increase the SBS threshold beyond the required level of pump power by broadening the spectrum of the pumps to beyond 1 GHz. In practice, pump spectra are broadened by modulating pump phases either sinusoidally at several fixed frequencies or randomly using a pseudorandom bit pattern at bit rates of 3–10 Gb/s. In this letter, we focus on the latter modulation technique and assume that pump phases ϕ_1 and ϕ_2 are modulated synchronously but in opposite directions to keep $\phi_1 + \phi_2$ constant. It is well known that spectral broadening of the idler, used for wavelength conversion, can be prevented under such conditions [3], [6].

The parametric gain in an FOPA depends on pump powers but not on pump phases. However, pumps experience a relatively large dispersion inside the fiber if they are located far from the zero-dispersion wavelength of the fiber. As a result, during propagation inside the FOPA, PM is converted into amplitude modulation (AM) through the group-velocity dispersion. Because the FOPA gain depends exponentially on the pump powers, even small changes in pump powers can cause large variations in the signal and idler powers. Such distortions of the signal and idler appear as noise when pseudorandom bit patterns are used. We use the SNR at the output end to quantify the FOPA performance.

Equations governing the evolution of the pumps, signal, and idler fields can be obtained from the scalar nonlinear Schrodinger equation, if we assume that all fields maintain the same state of polarization throughout the fiber. Assuming also that the pumps remain undepleted, we obtain the following set of equations [9]:

$$\frac{\partial A_k}{\partial z} = i\beta_{0k}A_k - \beta_{1k}\frac{\partial A_k}{\partial t} - \frac{i}{2}\beta_{2k}\frac{\partial^2 A_k}{\partial t^2} + i\gamma(P_k + 2P_{3-k})A_k, \tag{1}$$

$$\frac{\partial A_j}{\partial z} = i\beta_{0j}A_j - \beta_{1j}\frac{\partial A_j}{\partial t} - \frac{i}{2}\beta_{2j}\frac{\partial^2 A_j}{\partial t^2} + 2i\gamma(P_1 + P_2)A_j + 2i\gamma(P_1P_2)^{1/2}A^*_{7-j} \tag{2}$$

where A_k represents the pump field ($k = 1, 2$) with $A_k = \sqrt{P_k}\exp(i\phi_k)$, A_3 and A_4 are the signal and idler fields ($j = 3, 4$), respectively, γ is the nonlinear parameter, $\beta_{nj} \equiv \partial^n\beta(\omega)/\partial\omega^n|_{\omega=\omega_j}$, and β is the propagation constant. In writing (2), we have set $\phi_1 + \phi_2 = 0$. It is easy to see that pump-PM has no effect on FOPA performance in the absence of dispersive effects as all time-dependent terms disappear.

Fiber dispersion distorts the pump field through PM-to-AM conversion. In general, (1) and (2) cannot be solved analytically

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F. Yaman, Q. Lin, and G. P. Agrawal are with the Institute of Optics, University of Rochester, Rochester, NY 14627 USA (e-mail: yaman@optics.rochester.edu).

S. Radic is with the Department of Electrical and Computer Engineering, University of California, San Diego, La Jolla, CA 92093 USA.

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when dispersive effects are included. However, it is possible to obtain an approximate solution with some simplifications justified for most practical dual-pump FOPA configurations. We first introduce new variables B_k and B_j with the transformation

$$A_k = B_k \exp(i\beta_{0k} z + i\gamma\psi_k)$$
$$A_j = B_j \exp[i(\beta_{0j} - \Delta\beta/2)z + 3i\gamma\psi_0/2] \tag{3}$$

where $\psi_k = \int_0^z [P_k(z') + 2P_{3-k}(z')]\, dz', \psi_0 = \int_0^z [P_1(z') + P_2(z')]\, dz'$ and $\Delta\beta = \beta_{03} + \beta_{04} - \beta_{01} - \beta_{02}$. If we also work in a reference frame moving with the pump group velocity and introduce $\tau = t - \beta_{11} z$ as a new time variable, (1) and (2) can be approximately written as

$$\frac{\partial B_k}{\partial z} = -\frac{i}{2}\beta_{2k}\frac{\partial^2 B_k}{\partial \tau^2} \tag{4}$$
$$\frac{\partial B_j}{\partial z} = \delta_{13}\frac{\partial B_k}{\partial \tau} + \frac{i}{2}\kappa(z)B_j + 2i\gamma(P_1P_2)^{1/2}B^*_{7-j} \tag{5}$$

where $\kappa(P_1, P_2) = \Delta\beta + \gamma(P_1 + P_2)$. As an approximation, we neglected the higher order effects assuming that the AM amplitude is small. Only a single group-velocity mismatch parameter $\delta_{13} = \beta_{11} - \beta_{13}$ appears in these equations because the two pumps as well as the signal and idler pair are located almost symmetrically around the zero-dispersion wavelength ($\beta_{11} \approx \beta_{12}, \beta_{13} \approx \beta_{14}$). Physically, ψ_k and ψ_0 represent the nonlinear phase shifts imposed on the four fields through self- and cross-PMs [9]. The parameter κ governs the total phase mismatch. Its linear part $\Delta\beta$ represents the contribution of fiber dispersion. Its nonlinear part $\gamma(P_1 + P_2)$ plays an important role because it makes κ to vary along the FOPA length if pump powers become z-dependent.

Equation (4) is a linear equation and can be solved in the Fourier domain. Assuming $|\beta_{2k}|\Delta\omega_k\, z \ll 1$, where $\Delta\omega_k$ is the pump bandwidth induced by PM modulation, we obtain an approximate expression for the pump powers [10]

$$P_k(z, \tau) = P_k(0)[1 + \beta_{2k} z(\partial^2\phi_k(\tau)/\partial\tau^2)]. \tag{6}$$

Equation (5) can be solved approximately even when P_1 and P_2 are z-dependent. Noting that $\kappa(P_1, P_2) \approx \gamma(\delta P_1 + \delta P_2)$, where $\delta P_k(z) = P_k(z) - \langle P_k\rangle$ and angle brackets denote time averaging, and assuming $\delta P_k \ll P_k$, the κ term in (5) can be neglected compared with the nonlinear terms. In physical terms, small variations in pump powers are transferred to signal and idler through changes in the four-wave mixing (FWM) strength rather than through phase mismatch. With this simplification, (5) can be integrated to obtain the following approximate solutions for the signal and idler fields:

$$B_3(L) = B_3(0)\cosh(\bar{g}\, L)$$
$$B_4(L) = iB_3^*(0)\sinh(\bar{g}\, L) \tag{7}$$

where L is the FOPA length. The average gain $\bar{g}$ is defined as

$$\bar{g} = \frac{2\gamma}{L}\int_0^L [P_1(z, \tau + \delta_{13}z)P_2(z, \tau + \delta_{13}z)]^{1/2}\, dz \tag{8}$$

where the pump power $P_k = |B_k|^2$ for $k = 1, 2$ is obtained from (8) for a given pump PM scheme.

Equation (8) has a simple physical interpretation. The PM-to-AM conversion process makes pump powers to vary with time in a pseudorandom fashion. These time-dependent fluctuations in pump powers get transferred to the signal and idler through the FWM process, and the net parametric gain is determined by the length-averaged quantity $\bar{g}$ in (8). Note that the signal and idler interact with different temporal regions of the pumps at different locations of the fiber because of the walkoff effects induced by group-velocity mismatch. As a result, (8) can also be seen as a temporal averaging that reduces the extent of signal and idler fluctuations if the walkoff between the signal and the pumps is large enough.

To make further progress, a specific shape for PM profile has to be assumed. If the nonreturn-to-zero (NRZ) format is employed, the phase remains constant except near leading and trailing edges. We assume the leading edges to follow a raised-cosine shape, i.e., $\phi(\tau) = \pi[1 - \cos(\pi\tau/T_r)]/2$, where $0 < \tau < T_r$. Using (6), (8), and $T_w \equiv \delta_{13}L$, we obtain the following expression for $\bar{g}(\tau)$ near each leading edge:

$$\bar{g} = 2\gamma\sqrt{P_1(0)P_2(0)}\left[L + \frac{\pi\beta_{21}L^2}{T_w^2} \times \left(\cos\left[\frac{\pi}{T_r}(\tau - T_w)\right] - \cos\left[\frac{\pi}{T_r}\tau\right] - \frac{\pi T_w}{T_r}\sin\left[\frac{\pi}{T_r}(\tau - T_w)\right]\right)\right]. \tag{9}$$

A similar approach is used to find $\bar{g}$ near the trailing edges. In the absence of walkoff effects ($T_w \Rightarrow 0$), changes in $\bar{g}$ scale with fiber length as L^2 and with rise time as T_r^{-2}

To test validity of the approximate solution in (7), we performed numerical simulations for an FOPA using realistic parameters and compared them with the analytical results. To make the PM profile realistic, we used a filtered "rect" function to simulate the NRZ bit stream. We consider a 1-km-long fiber with its zero-dispersion wavelength at 1556 nm and $\gamma = 10$ W^{-1}/km. Other fiber parameters, taken from [11], are $\beta_3 = 0.049$ ps^3/km (dispersion slope $= 0.03$ ps/nm^2/km), and $\beta_4 = -5.810^{-5}$ ps^4/km. Two pumps are located at 1531 and 1581 nm and are launched with 150 mW of power at the input end. The signal is launched with 0.1 μW of power at 1557 nm. With these parameters, the FOPA provides a gain of 20 dB in the spectral region located between the two pump wavelengths.

In numerical simulations, phases were modulated using a pseudorandom bit stream of NRZ pulses at a bit rate of 10 Gb/s. Fig. 1(a) shows the modulated phase profile of one of the pumps for a duration of 6 bits. Fig. 1(b) shows how $P_1/\langle P_1\rangle$ (or $P_2/\langle P_2\rangle$) varies as a function of time after propagation through the fiber. Solid lines represent the analytical predictions and circles show the numerical results. The effect of different rise times is shown by using $T_r = 25$ and 40 ps. The rise time T_r is defined as the time during which pump phases change from 10% to 90% of their peak value. It is clear from Fig. 1 that pump powers are distorted at the locations where pump phases change rapidly. A shorter rise time ($T_r = 25$ ps) increases the level of distortion significantly.

Fig. 2 shows temporal variations in the normalized signal power $P_3/\langle P_3\rangle$ after amplification in the same time window used for Fig. 1. Similar variations occur at the idler wavelength.

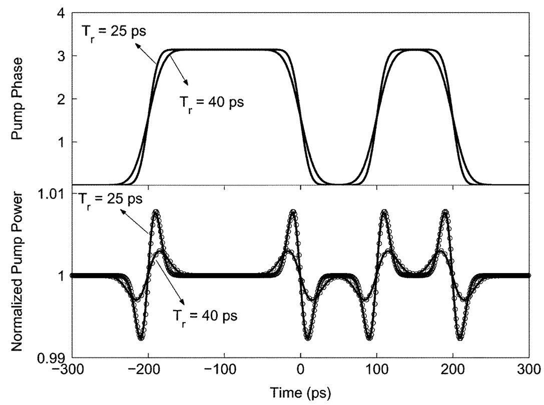

Fig. 1. (a) Modulated pump phase over a duration of 6 bits at 10 Gb/s. (b) Normalized pump power as a function of time. Numerical results (circles) are also shown for comparison.

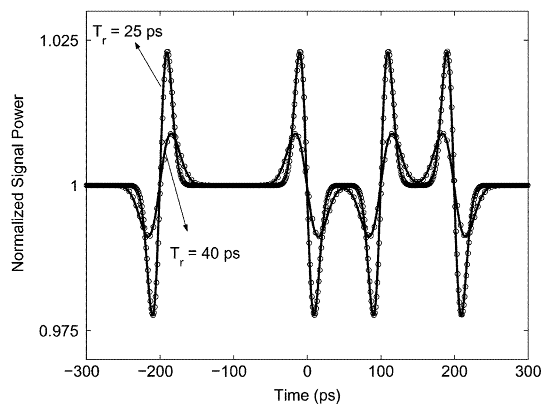

Fig. 2. Normalized signal power as a function of time.

A comparison of Figs. 1(b) and 2 shows that variations in the signal power follow pump variations and are enhanced considerably by the FWM process. The agreement between analytical theory and numerical simulations is excellent. In most cases, the curves are indistinguishable, justifying the approximations made in deriving (8).

To quantify the extent of degradation induced by PM-to-AM conversion, we define the SNR of signal as $\mathrm{SNR} = \langle P_3 \rangle / \sigma_3$, where $\sigma_3^2 = \langle (P_3(t) - \langle P_3 \rangle)^2 \rangle$ is the variance of signal fluctuations. Fig. 3 shows the signal SNR as a function of bit rate for different rise times. The SNR values obtained in the absence of walkoff effects ($\delta_{13} = 0$) are shown by dashed lines for comparison. FOPA parameters are the same as those used for Fig. 1. Clearly, signal SNR decreases rapidly as the bit rate of PM is increased. The SNR also depends on rise time and it becomes $<$23 dB even for a relatively small gain of 20 dB, if NRZ pulses with a short rise time ($T_r \leq 25$ ps) are employed. The situation is worse for FOPAs with higher gains; a 40-dB gain will reduce the SNR below 20 dB.

In conclusion, PM-to-AM conversion of pumps lowers the SNR of both the amplified and wavelength-converted signals when dual-pump FOPAs are employed. The problem can be solved to some extent by optimizing the bit rate and the rise time of the bit stream used for pump PM. One should choose PM parameters such that the desired level of SBS suppression is achieved without inducing large variations in the signal and idler powers. The length of the FOPA is also an important design parameter since it determines both the extent of PM-to-AM conversion and the walkoff effects. Although we have focused in this letter on the nondegenerate FWM process, several other nonlinear effects produce additional idlers [4] whose growth is also affected by the PM-to-AM conversion of the pumps. Our treatment can easily be extended to study these effects. Dispersion fluctuations, although not included in this letter, would not affect our conclusion as long as the contribution of these fluctuations to the phase mismatch κ remains negligible.

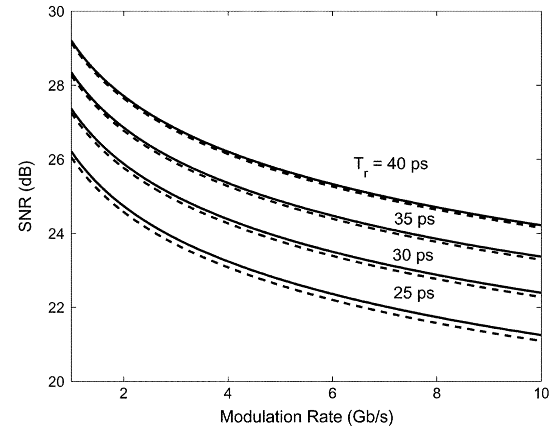

Fig. 3. Signal SNR as a function of PM bit rate for $T_r = 25$ to 40 ps. Dashed curves show the SNR without the walkoff effects.

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